

The Mathematical Education of Teachers, II

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The double discontinuity

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Felix Klein, Elementary Mathematics from an Advanced Standpoint

Summary of the 6 recommendations

Mathematics for teachers

1. Teachers should know the mathematics they teach
2. Amount of mathematics coursework:
 - ▶ 12 units for elementary teachers
 - ▶ 24 units for middle school teachers
 - ▶ math major including 9 units high school mathematics from an advanced standpoint for high school teachers
3. Teachers should have continued opportunities to learn mathematics throughout their career
4. Courses should develop mathematical habits of mind

Roles for Mathematics in Teacher Education

5. Departments should encourage and reward faculty who work in teacher education
6. Mathematicians should be involved in a community of mathematics educators

Mathematics for teachers

- ▶ Recommendation 1. Prospective teachers need mathematics courses that develop a solid understanding of the mathematics they will teach.
- ▶ Recommendation 2. [Specific recommendations on coursework]
- ▶ Recommendation 3. Throughout their careers, teachers need opportunities for continued professional growth in their mathematical knowledge.
- ▶ Recommendation 4. All courses and professional development experiences for mathematics teachers should develop the habits of mind of a mathematical thinker and problem-solver, such as reasoning and explaining, modeling, seeing structure, and generalizing. Courses should also use the flexible, interactive styles of teaching that will enable teachers to develop these habits of mind in their students.

Recommendation 2

Coursework that allows time to engage in reasoning, explaining, and making sense of the mathematics that prospective teachers will teach is needed to produce well-started beginning teachers. Although the quality of mathematical preparation is more important than the quantity, the following recommendations are made for the amount of mathematics coursework for prospective teachers.

Recommendation 2

- i. Prospective elementary teachers should be required to complete at least 12 semester-hours on fundamental ideas of elementary mathematics, their early childhood precursors, and middle school successors.
- ii. Prospective middle grades (58) teachers of mathematics should be required to complete at least 24 semester-hours of mathematics that includes at least 15 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers.
- iii. Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics that includes three courses with a primary focus on high school mathematics from an advanced viewpoint.

Roles for Mathematicians in Teacher Education

- ▶ Recommendation 5. At institutions that prepare teachers or offer professional development, teacher education must be recognized as an important part of a mathematics department's mission and should be undertaken in collaboration with mathematics education faculty. More mathematics faculty need to become deeply involved in PreK–12 mathematics education by participating in preparation and professional development for teachers and becoming involved with local schools or districts.

Roles for Mathematicians in Teacher Education

- ▶ Recommendation 6. Mathematicians should recognize the need for improving mathematics teaching at all levels. Mathematics education, including the mathematical education of teachers, can be greatly strengthened by the growth of a mathematics education community that includes mathematicians as one of many constituencies committed to working together to improve mathematics instruction at all levels and to raise professional standards in teaching.

Essential topics and experiences for preparation

- ▶ Single and multi-variable variable calculus
- ▶ Vectors and matrices
- ▶ Analytic geometry
- ▶ Abstract linear algebra
- ▶ Statistics and probability
- ▶ Geometry and transformations
- ▶ The real number system
- ▶ The complex number system
- ▶ Experience with reasoning and proof
- ▶ Algebraic structures in high school mathematics
- ▶ Modeling
- ▶ Trigonometry
- ▶ History of mathematics
- ▶ Experience with technology
- ▶ Research experience

Desirable topics for preparation, essential in early-career education

- ▶ Further statistics
- ▶ Discrete mathematics and computer science
- ▶ Further geometry
- ▶ Further experience with algebra

Topics and experiences for mid- and late- career teachers

- ▶ The mathematics of high school
- ▶ Differential equations
- ▶ Group theory
- ▶ Number theory
- ▶ Advanced calculus
- ▶ Further history of mathematics
- ▶ Math circles
- ▶ Immersion experiences
- ▶ Lesson study

Short program

- I Lower-division courses taken by students in a variety of majors (15+ semester-hours)
 - ▶ Single and Multivariable Calculus (9+ semester-hours)
 - ▶ Linear Algebra (3 semester-hours)
 - ▶ Introduction to Statistics (3 semester-hours)
- II Courses intended for all math majors (9 semester-hours)
 - ▶ Introduction to Proofs (lower division) (3 semester-hours)
 - ▶ Abstract Algebra (approach emphasizing rings and polynomials) (3 semester-hours)
 - ▶ A third course for all mathematics majors (e.g. Differential Equations) (3 semester-hours)
- III Courses taken primarily by preservice high school mathematics teachers (9 semester-hours).

Long program

- I Lower-division courses taken by students in a variety of majors (21+ semester-hours)
 - ▶ Single and Multivariable Calculus (9+ semester-hours)
 - ▶ Linear Algebra (3 semester-hours)
 - ▶ Introduction to Computer Programming (3 semester-hours)
 - ▶ Introduction to Statistics (6 semester-hours)
- II Courses intended for all math majors (12 semester-hours)
 - ▶ Introduction to Proofs (lower division) (3 semester-hours)
 - ▶ Advanced Calculus (3 semester-hours)
 - ▶ Abstract Algebra (approach emphasizing rings and polynomials) (3 semester-hours)
 - ▶ Geometry or Mathematical Modeling (3 semester-hours)
- III Courses taken primarily by preservice high school mathematics teachers (9 semester-hours).