Collaboration between Mathematicians and Math Educators at Sonoma State University

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California State University System
23 campuses

- Humboldt
- Chico
- Sacramento
- Sonoma
- Maritime
- East Bay
- Stanislaus
- Fresno
- Bakersfield
- San Francisco
- San José
- Monterey Bay
- Northridge
- Pomona
- San Bernardino
- San Luis Obispo
- Channel Islands
- Los Angeles
- Fullerton
- San Marcos
- San Diego
- Dominguez Hills
- CSU Chancellor's Office
- Long Beach

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Collaboration between Mathematicians and Math Educators at SSU

Ben Ford - mathematician with math ed interest

Edie Mendez - math educator in math department (just retired – hopefully replaced soon)

Brigitte Lahme - mathematician with math ed interest

Kathy Morris - elementary math educator

Megan Taylor – secondary math educator

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We are not unique

- At teaching institutions it is common to have math educators in the math department and mathematicians teach courses for future teachers.

- Across the CSU, mathematicians work closely with math educators in professional development.

- Mathematics education counts as scholarship in many math departments.

How do we collaborate?

- Teacher Professional Development
- Curriculum Development
- Program Development
- Joint Presentations at Conferences
- Co-teach classes:
    - Edie and Brigitte (Math 300)
    - Edie and Kathy (Math 300A)
    - Ben and Rick (several courses)
    - Brigitte and Elaine (Math 300B)
    - Brigitte and Megan (Math 490)

Example 1: Teacher Professional Development 1999 – 2014

- California Professional Development Institute 2000 - 2003

- Three multi-year California Math and Science Partnership grants 2005 – 2013

- Project LEAD (2010 – 2013), 120 teachers, PD focus on math content, leadership, lesson study, assessment, technology, equity

- CPEC grant (2011 – 2014): Regional CCSS leadership

Example 2: Curriculum Development - Math for Elementary Teachers at SSU

Math Department is taking content courses seriously.

Math 300a – Elementary Number Systems

- Developed together in the 90s
- Close coordination of content and teaching (regular meetings, common assignments, regular revision)

Math 300b – Data, Chance, and Algebra

- Developed together in the early 2000s
- Taught by other math faculty (especially stats)

Common Core State Standards – Revision 2013

Traditional strengths of Math 300a

- experiences creating viable arguments and critiquing the reasoning of others (active classroom, group work, students explain thinking)

- using tools strategically (diagrams, manipulatives etc.)

- perseverance (alphabitia, take-home exams, "chunky problems")

- making sense (looking at multiple representations etc.)

Changes due to CCSS

- We more explicitly encourage meta-level thinking (community agreements/norms for discussion; final reflection on SMPs)

- Approach to fractions

- Modeling (first attempts, need more in the future)

- SMPs: Moving from capacity to inclination

CCSS Fractions snapshot

- Unit fractions as "basic building block"

- Fraction as a *number* on number line

- "Putting together" meaning for addition, consistent with whole number

- Whole number * fraction: repeated addition

- Multiplication via unit fractions: 4/3 x 5 means "4 of 1/3 of 5"

Our undergraduates and fractions

- "Just invert and multiply"
- Lots of pizzas and cookies
- Lots of common denominators
- Every fraction must be simplified or it's "wrong"
- See a fraction as two numbers with a bar between
- Little capacity for reasoning about fractions
- All true for "good at fractions" as well as others

Fractions warmup

- Small groups, 2 teams
- Generate two random integers 1-20
- One team uses integers as numerator and denominator, place on 0-3 number line
- Justify placement
- Repeat, generating new strategies for comparing when placing

Adapted by Kathy Morris from *MathLand* grade 5 and *Making Meaning for Operations*

You roll 6 and 19
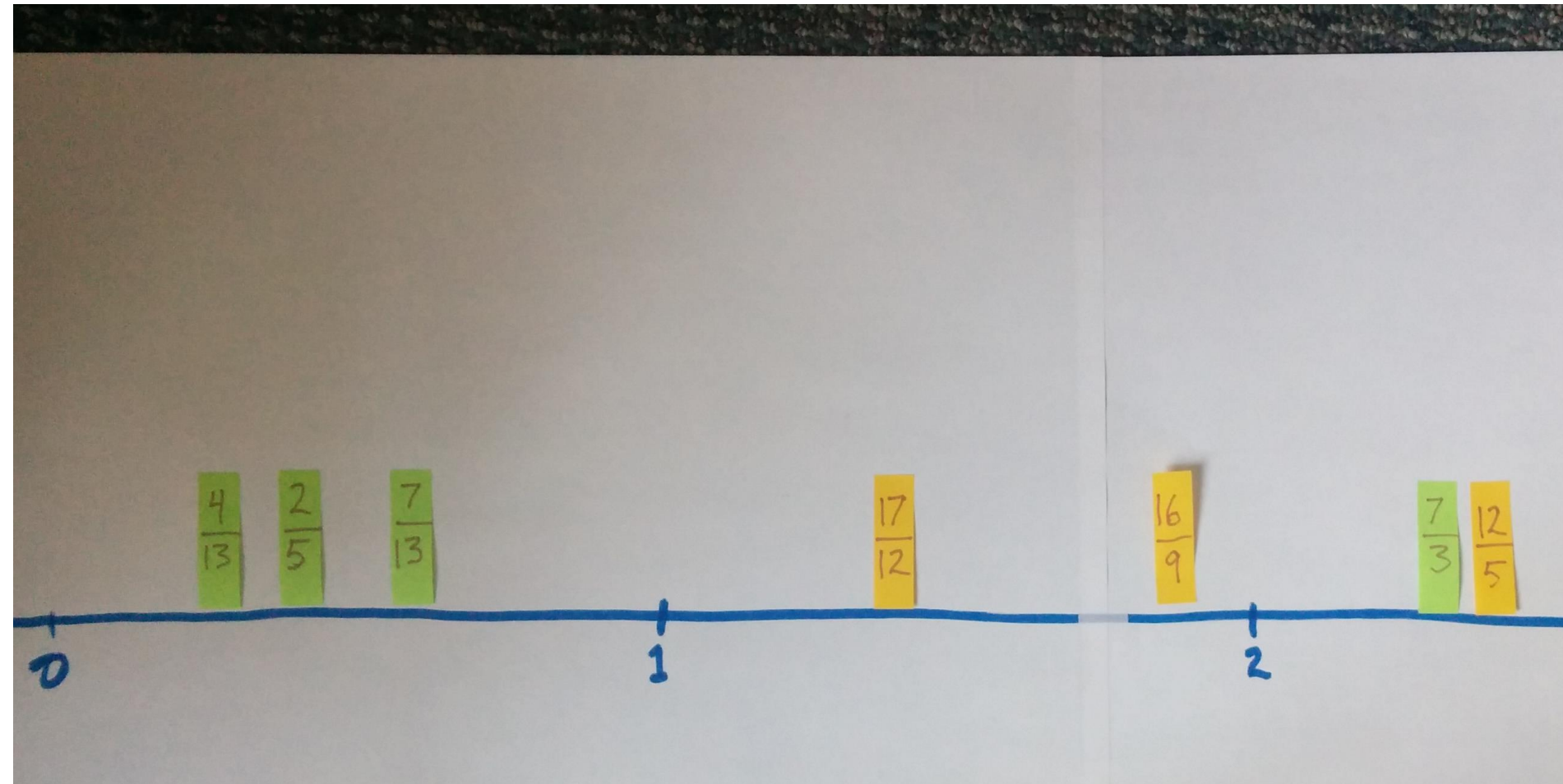
Fraction multiplication

- First: Want to build on number line work
- Lends itself to area models on a Cartesian plane

- Represent each product, indicating where each factor and the product are represented:

  2•6

  2•1½

  ⅓•5

  ⅓•½

  ⅓•⅘

- For the rest, generate a description of the process you are using so someone else could understand
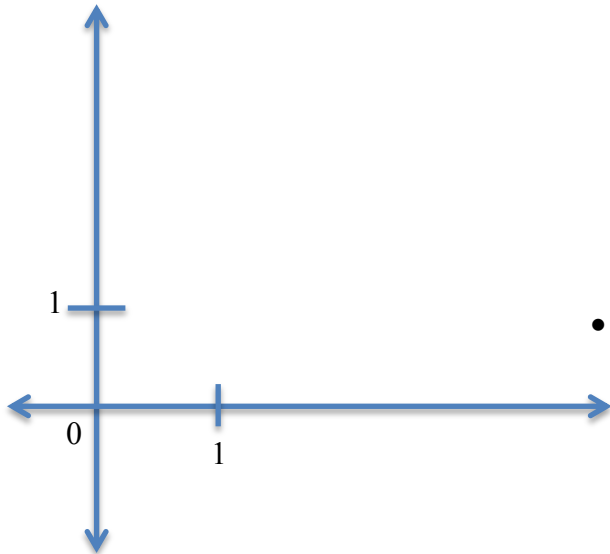
  ⅗•⅚

  ⅙•⅖

  ⅗•½

Goal: Shifting our thinking from Capacity vs. Inclination

OVERARCHING HABITS OF MIND
1. Make sense of problems and persevere in solving them
6. Attend to precision

REASONING AND EXPLAINING
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS
4. Model with mathematics
5. Use appropriate tools strategically

SEEING STRUCTURE AND GENERALIZING
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

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Goal: Shifting our thinking from Capacity vs. Inclination

- Goal Layer 1:
  - How do we build students' capacity? (necessary but insufficient)
  - Given some capacity, how do we build their to inclination?
- Goal Layer 2:
  - How do we build teachers' pedagogical capacity to develop students' capacity? To develop students' inclination?
  - Given some capacity, how do we build teachers' INCLINATION to develop students' capacity and their inclination?

Example 3: Program Development

Masters Program in Math Education

- Cohort of 19 teachers from Northern California

- 3 math courses designed/taught by math faculty

- Math ed research course designed/taught by math ed faculty plus addition ed courses

- Advisors from Math ed; committee members from math

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Math – School of Ed connection

- Accelerating Academic Achievement for English Learners (AAAEL), led by Kelly Estrada, Curriculum Studies and Secondary Education at SSU

- School of Education enlisted our help to redesign the student teaching experience and the math-specific courses in the credential program.

- Connecting fall courses (math methods and "critical pedagogies") with student teaching

Math – School of Ed connection

- Math has developed a large cadre of teacher leaders through PD work.

- Math teacher leaders videotape lessons to share model with other disciplines

- The project helps to make our students' experience here more coherent, in math and other subject areas.

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Example 4: Co-teaching courses

- Math 300A Elementary Number Systems
  - Edie and Brigitte (Mentor/Inductee)
  - Edie and Kathy (Math/Math Ed)
  - Ben and Rick (Math/Math Ed)
- Math 300B Data Chance and Algebra
  - Brigitte and Elaine (Math/Math)
- Math 490 Secondary Math Capstone Course
  - Brigitte and Megan (Math/Math Ed)

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Math Education Influence in Math & Stats Department

- Introduced pedagogy and lesson study to other mathematicians in the department.

- Revised our tenure criteria to reflect that we value the work done in teacher preparation.

- Participated in a Carnegie Foundation project on collaboration between mathematicians and math educators.

- Talk to other mathematicians about changing the experiences we give to our math students.

Lesson Study Cycle

- Professional development process to examine practice of teaching and become more effective.

- Instructors plan lesson together

- Observe the lesson, collect evidence of student thinking

- Debrief lesson

- Revise lesson

- Teach lesson again (different section or later).

Lesson Study at SSU

- 2001 - present: As part of PD with SCOE (Joan Easterday), Lesson Study conferences

- 2007: Lesson study in Math for Elementary teachers at SSU – place value (part of Carnegie grant)

- 2009: Real Analysis – first cycle (with pure mathematicians)

- 2011: Lesson study in Math for Elementary teachers at SSU – division of fractions

- 2012: Real Analysis – second cycle

- 2013: Elementary Statistics (only math & stats faculty, supported by NBMP)

More info at: tinyurl.com/ssu-lesson-study

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Lesson Study Examples

Elementary Number Systems:

(1) Introduction of base number system

(2) Fraction division

Real Analysis:

Introduction to Riemann integral

Elementary Statistics:

Understanding of p-value (un-scaffolded task)

Pedagogy Workshops

- Kick off the semester by talking about good teaching.
- All math and stats faculty, lecturer and TT faculty participate.
- Topics:
  - Dan Meyer 3-act math
  - Standards for Mathematical Practice
  - 5 Practices for Orchestrating Product Mat Discussions

SMP 3: Construct viable arguments and critique the reasoning of others

Use assumptions, definitions, and previous results

Make a conjecture

Build a logical progression of statements to explore the conjecture

Distinguish correct logic

Explain flaws

Analyze situations by breaking them into cases

Ask clarifying questions

Recognize and use counter examples

Communicate conclusions

Justify conclusions

Respond to arguments

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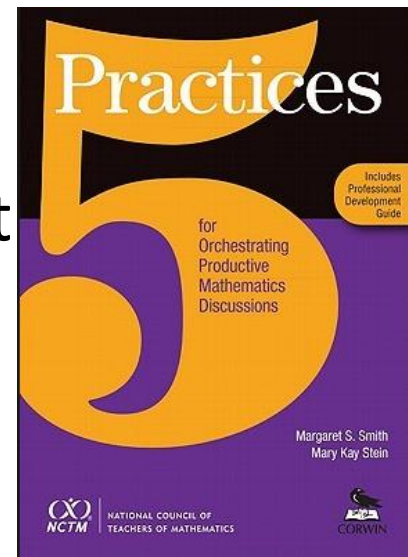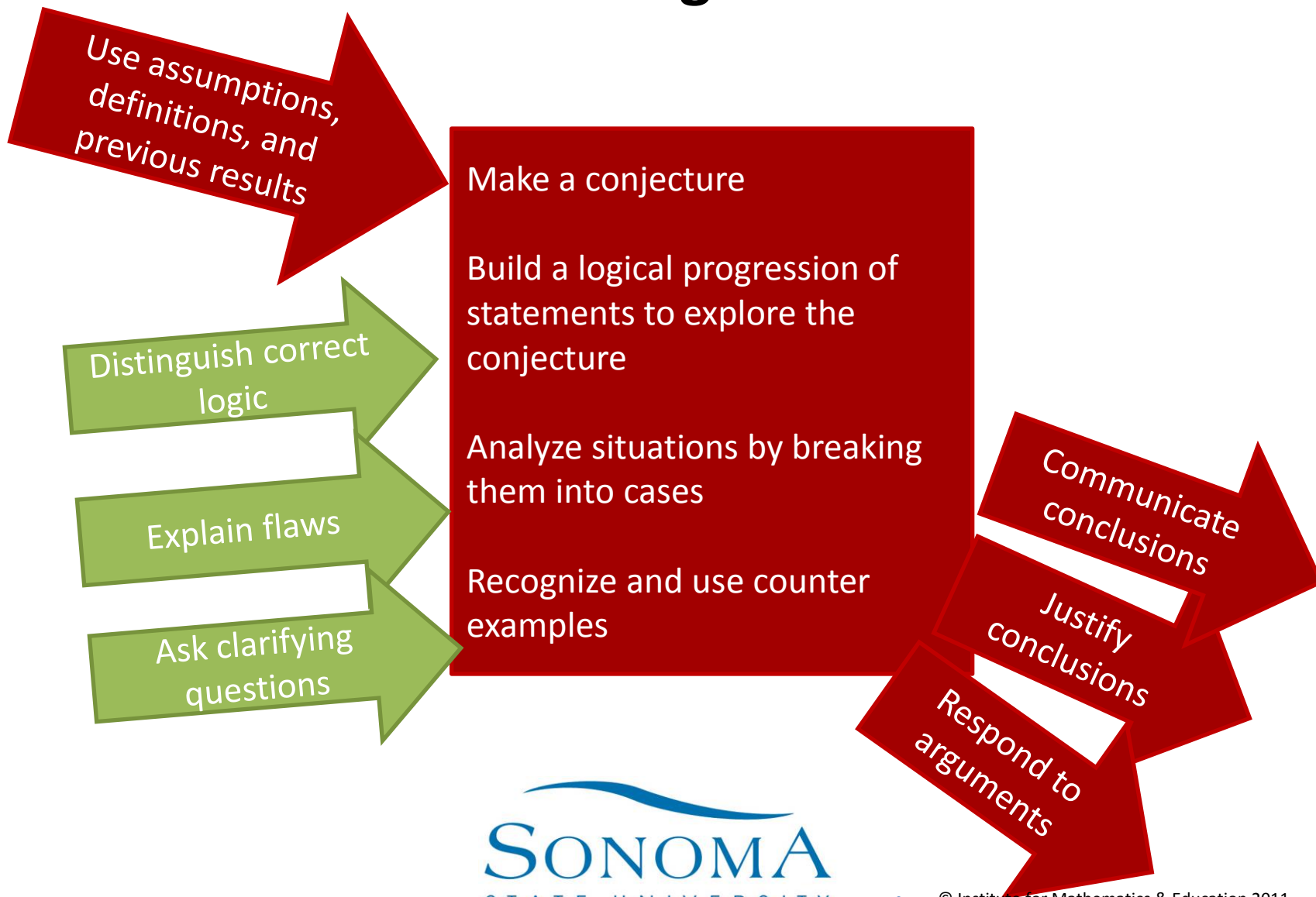
Posters on the 1st Day of Class

Math 316: Combinatorics and Graph Theory

Introductory Exercise. Fill in the squares below with the numbers 1 through 8 so that no two consecutive numbers appear in squares that touch one another. This means, for example, that the number 2 cannot occupy a square that touches another square containing either 1 or 3. Two squares are considered touching if they are directly above, below, left, right, or diagonally across from each other.
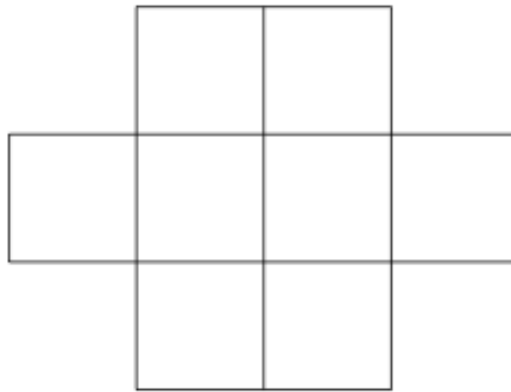
Follow-up question. Once you have found one solution, determine the maximum number of different solutions that there are, and prove that your answer is correct.

Math 316 Combinatorics

The numbers 1 and 8 must be placed in the two middle boxes. Since 1 and 8 are the only numbers with only one other adjacent number, and any number placed into either of the two middle boxes will have only one box that it is not adjacent to, it follows that only 1 and 8 can be placed in the middle.
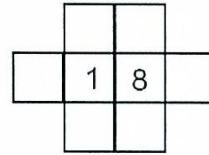

Next, we know that there is only one box that each of the two boxes in the middle is not adjacent to. In these boxes, we place the two adjacent numbers of 1 and 8 in opposite order. So if 1 and 8 are placed like so: 1 | 8, then 2 and 7 need to be placed like so: 7 | 1 | 8 | 2. If 1 and 8 are flipped vertically, then 7 and 2 are also flipped vertically.
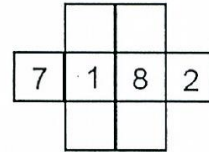
Next, we know that 3 must be placed in one of the boxes not touching 2, and 6 must be placed in one of the boxes not touching 7, yet 3 and 6 cannot be placed next to each other, since that would force 4 and 5 to be placed next to each other. This means that 3 and 6 must be placed in opposite sides (top & bottom) of their respective adjacent number. Note that this can be done in two ways as well. In the example below, 3 can be on top and 6 on bottom.
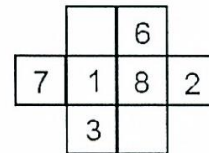
Now we are left with 4 and 5. The number 4 cannot be placed next to 3 and the number 5 cannot be placed next to 6. There is only one choice for these two numbers and it is shown below:
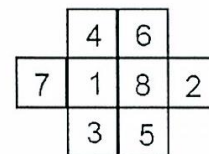
Given that 1 and 8 can be placed in one of two ways, and 3 and 6 can be placed in one of two ways, we end up with 2 x 2 = 4 different possible solutions to this problem.

Why do this activity?

Our students write proofs and have to show their work all the time, why is this activity useful?

- Sets the tone for the semester.

- Give feedback and then <u>use</u> feedback to revise.

- The goal is communicating solutions, not just answer getting.

- You can see yourself doing this at any grade level.

What makes collaboration work?

- Respectful culture
- Math Ed Committee – continuous work together
- Math ed work is shared with Math and Stats department.
- Math and Stats department views the role in math education as important.
- School of Ed values what we do, too.

Thank You

Brigitte Lahme

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Swimming Pool

Remember those lazy days of summer. You were hanging out at the pool with your kids and all of a sudden your 9-year old asked:

"I wonder if a person could drink the amount of water in a swimming pool."

Thanks to Patrick Callahan and the California Mathematics Project

The CCSS Modeling Cycle

1. identifying variables in the situation and selecting those that represent essential features,

2. formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables,

3. analyzing and performing operations on these relationships to draw conclusions,

4. interpreting the results of the mathematics in terms of the original situation,

5. validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable,

6. reporting on the conclusions and the reasoning behind them.

Which CCSS Practice Standards might a teacher work on with a problem like this to?

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Fermi Problems

An estimation problem designed to teach dimensional analysis, approximation, and the importance of clearly identifying one's assumptions. Named after physicist Enrico Fermi, such problems typically involve making justified guesses about quantities that seem impossible to compute given limited available information.
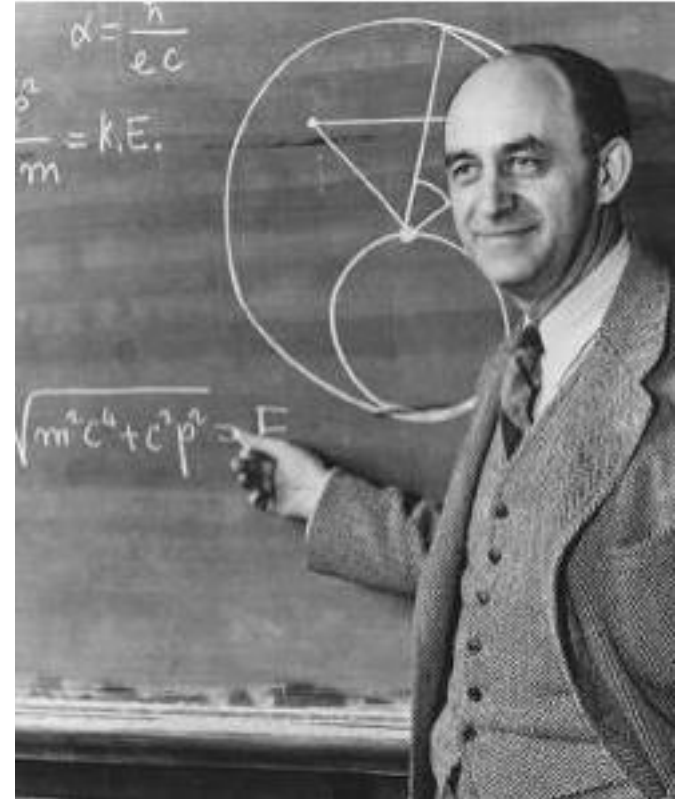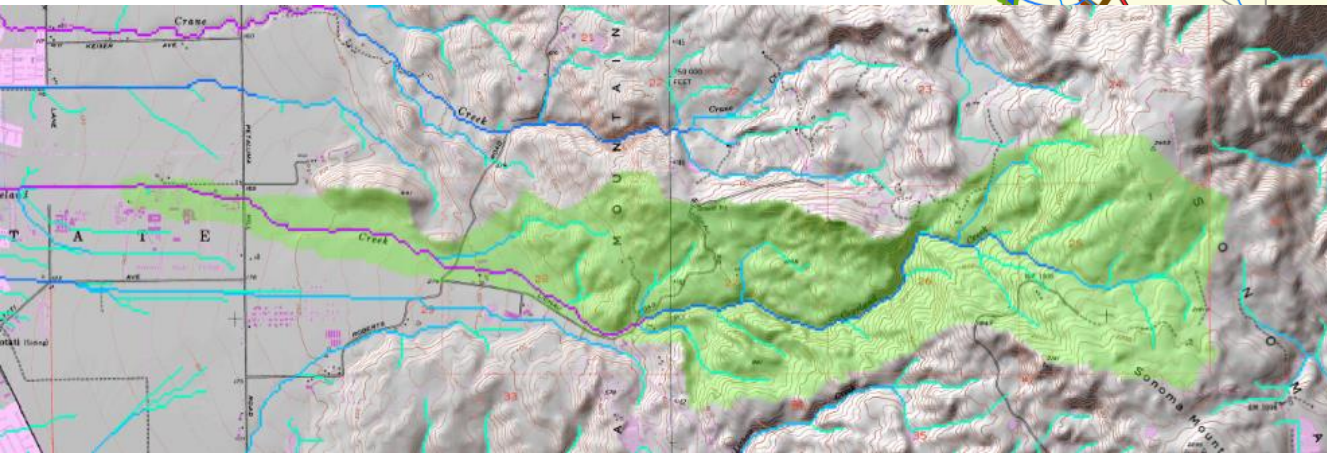

Enrico Fermi (1901-1954)
Nobel Prize Physics 1938

Science 121 Fermi Problem

A USGS Water Data Report says that in an average year about 8000 acre-feet of water flow down Copeland Creek through Rohnert Park. Does this seems reasonable?

Make a guess

What information do you need to check?

Poster – First Attempt

Inaccurate USGS Reading
- 82,000 acre ft. of water falls on the area yearly, therefore
 8,000 is too small.

SUPPORTING ARG.
- Area of Watershed: 9 km²
- 9 km² = 2,211 acres
- Avg. yearly rainfall = 30-45 in.
- Avg. of 30 and 45 = 37.5 in.
- $T_{rain} = 2,211(37.5)$
  $= 82,912$ acre/ft

ASSUMPTIONS
- Lack of tools ⟹ inaccurate watershed measurements
- Yearly rainfall of 37.5 in is just an avg.
- Area of watershed = 9 km²
- Regular rainfall that year

Item	1Pt Missing or substandard	2Pts Meets minimum standards	3Pts Meets standard expectations	4Pts High quality professional work
The poster is visually constructed to clearly showcase your work	Hard to read or follow. Graphics missing or illegible and/or largely handwritten	Text is organized but still obscure. Images misplaced. Some handwriting.	Organized text that makes sense Images help reader.	As 3 but Poster is also compelling to look at
Includes an accurate description of your problem	Can't tell what the poster is for	Topic is there but still confusing	Clear thesis statement	Compelling thesis statement
Includes an accurate description of your assumptions	Authors completely unaware of their background assumptions	Authors make unstated assumptions	Assumptions are stated	Assumptions are stated and mitigated
Includes an accurate description of your calculations, methods and conclusion	Calculations are wrong or mysterious	Calculations have small errors or seem suspect	Calculations are solid	The viewer actually wants to look at the calculations which are easy to read and follow
Illustrations and Graphs	Missing or illegible. Missing labels or scales	Some errors on illustrations or graphs	Everything is labeled correctly. Illustrations aid	As 3 but high quality illustrations and graphs

Poster – After Revision
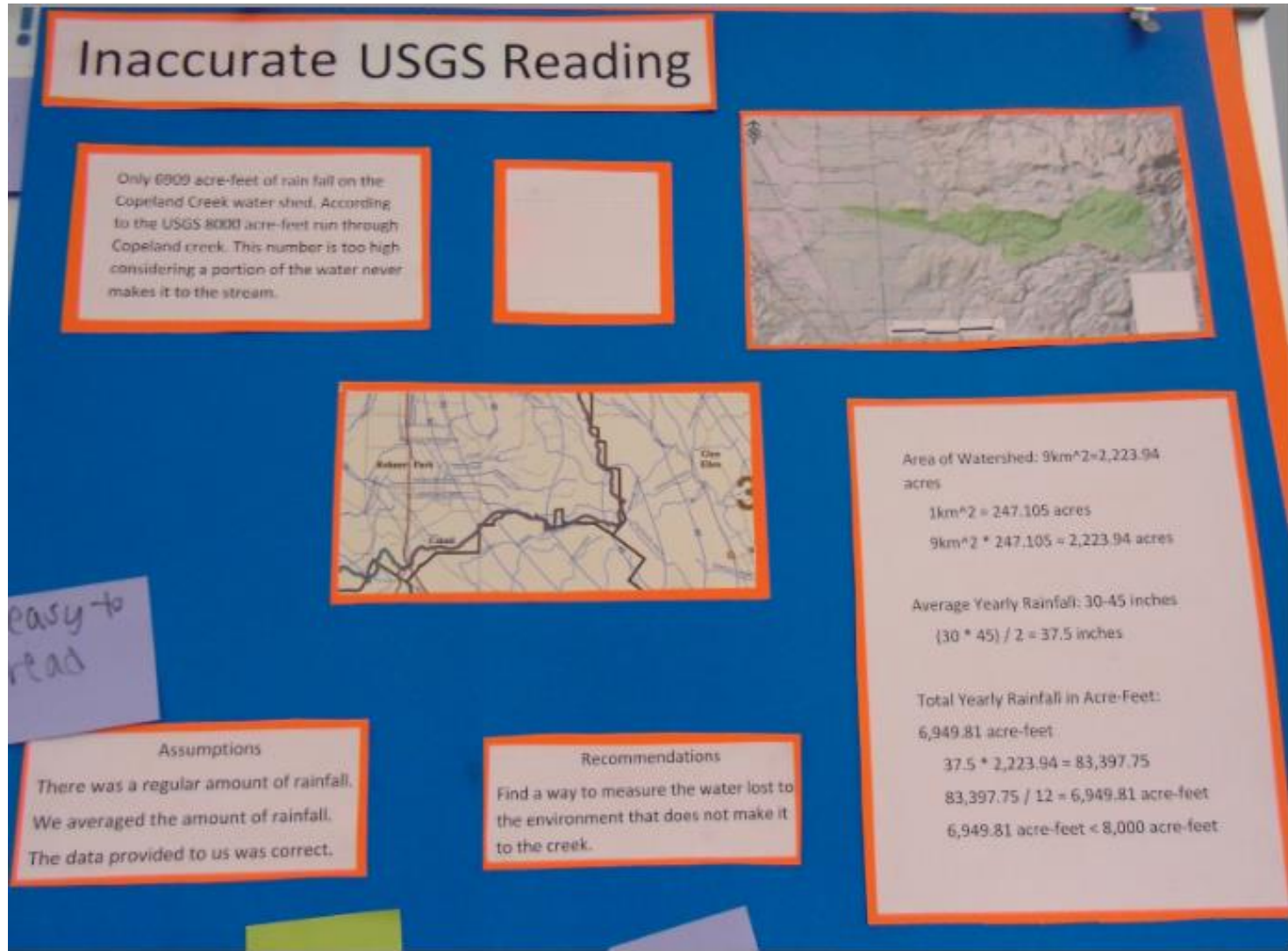
Copeland Creek: Flows More Than You Know

USGS CLAIM: In an average year, about 8000 acre-feet of water flow down Copeland Creek through Rohnert Park.

ASSUMPTIONS

1. We assume that all the rain that collects in the watershed drains into Copeland Creek
2. We assume that the average annual rainfall in this area is 36 inches.

COMPUTATIONS

STEP 1: 1 Square Kilometer = 247.5 acres

STEP 2: Find Area of Watershed

Area = 2,394.4 acres

STEP 3: Multiply Area by Average Annual Rainfall

2,394.4 acres x 3.0 ft. =

7,183.2 acre-feet

This shows the annual rainfall in the watershed area. We estimated the average annual rainfall to be 36 inches (3ft.).

The watershed is divided into different sections to find the area.
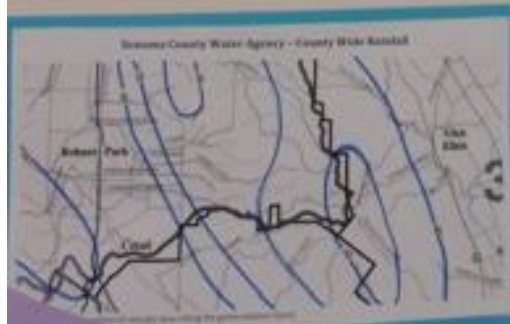
CONCLUSION

Based on our assumptions and calculation, the USGS report is reasonable. The answer we got is relatively close to 8000 acre-feet. The reason our answer is less than 8000 acre-feet is because there may be other factors besides just the rainfall in the watershed that contribute to the total annual flow of the creek.