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Adventures of Mathematicians in School Education

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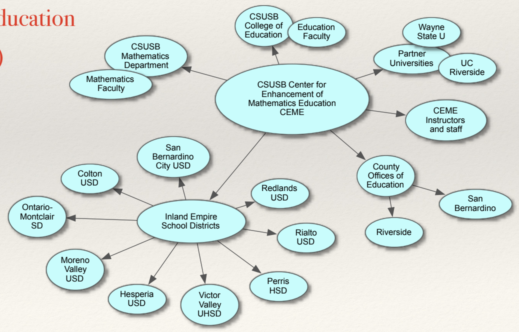
Agenda

- Who are we?
- In-service projects
- Pre-service programs and initiatives
- Research
- Questions?

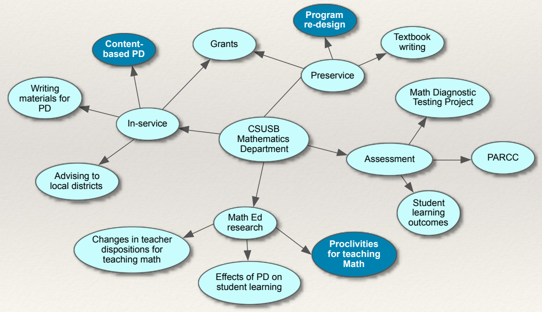
Who are we?

CSU San Bernardino
Center for Enhancement of
Mathematics Education
(CEME)



CSUSB Mathematics Department

- 26 research mathematicians, 1 mathematics educator
- Some of our education work:



Inservice Projects:
ACES
Algebraic Concepts for Elementary Students

- 5-years, teachers of grades 4-8
- Funding:
  - National Science Foundation
  - CSUSB colleges, department
  - Ontario-Montclair School District

ACES: Algebraic Concepts for Elementary Students

- Who plans <u>and</u> implements?
  - 3 mathematicians
  - 3 math/science educators
  - 2 K-12 experts
  - 1 graduate student
- Monthly meetings to review and plan

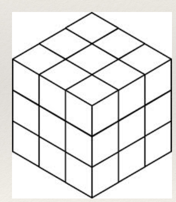
Teacher activities
- Summer Learning Institute
- Monthly seminars
- Monthly lesson study
- Monthly un-facilitated collaboration time

A day in the Summer Learning Institute

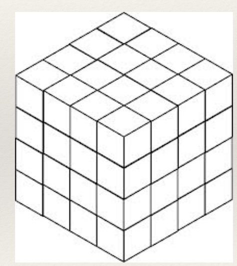
- Three 2-hour segments:
  - Plenary - focus on math concepts
  - Grade-span - focus on grade-span implementation
  - Breakout topic sessions - e.g. formative assessment, problem solving, mathematical modeling, classroom discussion
- Overall focus: understanding arithmetic in a way that leads to algebra naturally; transition to algebra

Sample Mathematician-Educator Collaboration

- M: What is the same? What is different?



A



B

Mental math of a different sort...

- For the following problem, put your pencils/pens down. Respond
  - without writing
  - without talking, and
  - without counting one by one
- You have a 4x4x4 cube made up of unit cubes, and you dunk it in a pail of paint. When you take it out, how many unit cubes have paint on them?
- Individually: Write down your answer and your reasoning.
- In groups: compare and contrast solutions
- Generalize to an *n x n x n* cube

Scripting the session

S-(See *Figure 1*, below.) I had a hard time visualizing this, so I drew a diagram.
T- A net. That's great.
S-So the red part is cubes that are painted on only one side. There's 4 on each face so that was 4x6=24 cubes. Then I found the corners. I figured that the corners have three faces in common, so that's 8 more cubes. See the boxes that have 1 in them? That's all cube number 1. I kept lining them up, and that was hard to see. I think I counted them all correctly.
T-That was amazing! You said you didn't know what to do, so you were very methodical!

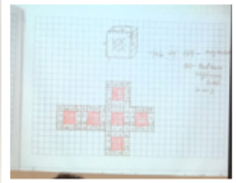


Figure 1 – Net of Cube

Figure 2 – 4x4 cube

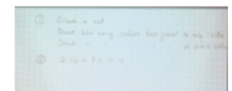


Fig. 3 – Strategies Chart

T- (Writes strategy—Create a net. Count how many cubes have paint on only 1 side. Count how many cubes have paint on 2 or 3 sides.)

Scripting the session (cont'd)

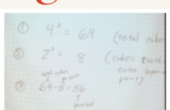


Figure 5

S- (See *Figure 5*, above.
4^3= 64 (total cubes)
2^3= 8 (cubes within outer layer—no paint)
64 (total)-8 (no paint)=56 (painted)

T-(Writes the student's strategy. 4^3- 2^3.)

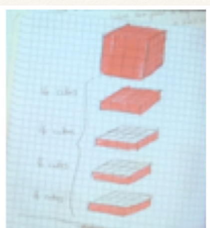


Figure 6

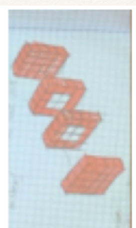


Fig. 7

T-Another strategy?
S-(See *Figure 6.*)Red represents the paint. We can think of the 4 stacks of cubes.
T-Tell me what you're counting.
S-There's a total of 64. Looking at the sides. This particular cube is painted. (Colors in all visible parts of cube.) This cube is painted...Here are the cubes that are not painted. (See *Figure 7.)*
T- Look at that. They both got to the same point...The last student peeled the layers off like an onion. This one took cross sections...

Painted cubes - Educator debrief

"We purposely did not provide you with cubes. Keep in mind that this is on purpose." How did this approach help or hinder your learning?

Why do you think M made this instructional decision?

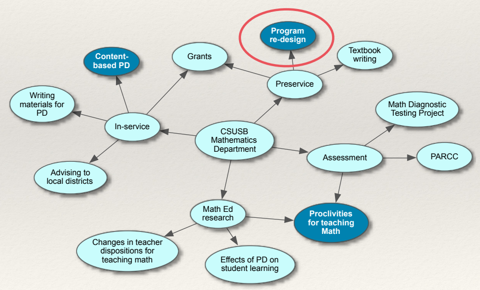
"We were way off. We misread the question. We saw it as how many full cubes rather than faces…"

What happened here? How did M turn it into a learning opportunity for the small group? For the entire group?

"We don't have enough time to close it today, but we're going to have one representation shared today...We'll come back and finish it tomorrow, because just like today, there will be multiple ways to represent this problem."

What was the value of this approach to running short of time?

Pre-service
Programs and Initiatives



Writing course materials

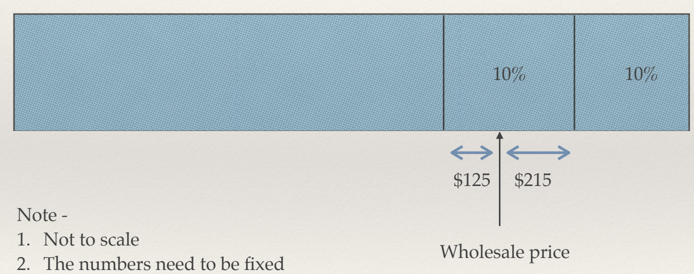
- Seeking problems that will
  - be thought-provoking
  - highlight concepts and lead to generalization
  - reveal mathematical structure
  - bring to light misconceptions

Sample problem

If a washing machine is sold at 10% discount, a store makes $215 profit; but if it is sold at 20% discount, the store loses $125. What is the original price of this washing machine?

- Solve the problem in two different ways, at least one of them visual.
- What concepts are involved in this problem, and how?
- What kinds of misconceptions might students have about the content of this question?
- If students have difficulty solving this problem, what questions might you ask to help them move forward? Make sure that your questions do not lower the cognitive complexity of the problem.

Washing machine problem

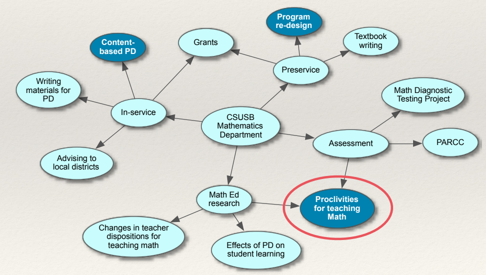


Note -
1. Not to scale
2. The numbers need to be fixed so they are more realistic.

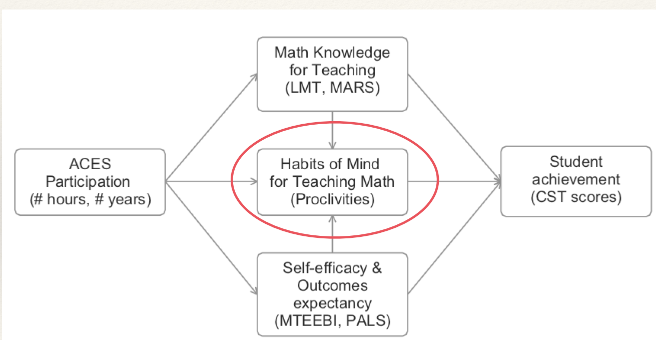
Noyce Math and Science Scholars

- Prospective math and science secondary teachers
- MENTORING by lead teachers and faculty
- $10,000/year for up to three years
- Commitment to teach in the district 2 years for every year of support
- COMMUNITY to support beginning and experienced teachers
- Funded by NSF, San Bernardino City USD, CSUSB

Mathematics Education Research



Bird's eye view of ACES logic



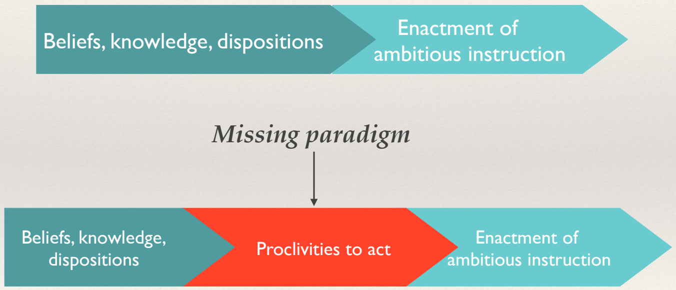
Connecting to Habits of Mind Literature

Example: <u>Questioning</u>

- Costa & Kallick: Questioning and posing problems
- Cuoco/Goldenberg/Marks: Sniffing patterns, conjecturing
- Project Zero: Skepticism and appreciation for evidence; consideration of alternatives; curiosity
- Adding it Up: Strategic competence
- Driscoll et.al: Balancing exploration and reflection
- CCSS/SMPs: Critique the reasoning of others
- Charbonneau et.al.: Critical thinking; curiosity
- Ennis: Look for alternatives; use one's critical thinking abilities

Proclivities for Teaching Mathematics

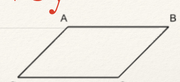
with Jennifer Lewis



Levels of Proclivities to Teach Math

Levels of Proclivities	Example: Persistence
Level 1: Teachers' proclivities to DO mathematics	Persistence in understanding a mathematical context and in solving a mathematical problem
Level 2: Teachers proclivities in thinking about student thinking in mathematics	Persistence in figuring out what a student is thinking, and in helping students to do mathematics
Level 3: Students' habits of mind that teachers wish to develop	Helping students to be persistent in understanding a math'l context and in solving a math'l problem

Sample item in PTM survey



Context: A class is learning about area of parallelograms.
On the board is a non-rectangular parallelogram ABCD.

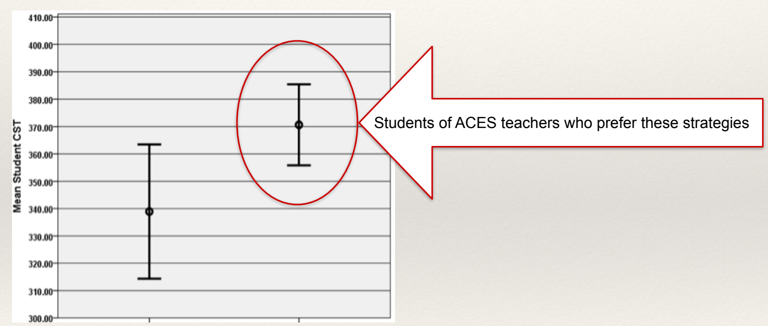
The teacher comments to the class: "Let's see if we can use our knowledge of the area of a rectangle to figure out the area of the parallelogram ABCD."

Student: "I know that the area of a parallelogram is base times height, so I will multiply AD times DC."

To what degree would you be likely to use each of the following actions in response?

- *Explain that AD is not the height of the parallelogram, and show how to solve correctly.*
- *Ask the student to define an altitude of a parallelogram, to recall prior knowledge and then apply it here.*
- *Give the student paper and scissors, and ask him to try to use them to justify his conclusion.*
- *Ask the class to show various ways in which they solved the problem, so this student, as well as others, can see multiple solution paths.*
- *Ask the student to explain his thinking, so the class can critique his reasoning.*
- *Ask the student to explain his thinking, so you can better understand what was in his mind.*

Effects of *critiquing correct/incorrect solutions, comparing and contrasting, and classroom discussion* on student achievement



What got me hooked?

- Curiosity about how people think about mathematics
- The excitement of seeing the lightbulb go on
- A feeling that we are making a difference - for example…

From an ACES teacher

I began working with the ACES grant a year later than most participants. Those that had already been in ACES would often excitedly begin conversations by reflecting on their personal math experiences. These tended to fall into two categories. It was either that math had always been difficult and unenjoyable for them or that they had received good grades in math but never really understood it. Though their pasts varied, all ended their anecdote with how enjoyable math had become. After a year with the grant, I often found myself doing the same. I didn't recognize this pattern until just recently. Last week, I was watching an interview with a physics professor from Stanford. You could see his enthusiasm as he talked about how beautiful he found a particular theory to be. I was surprised when he said that he had sometimes worried that his theoretical work might have been blinded by how beautiful he found the science to be. The beauty of mathematics (and certainly physics) eludes me, but I can now appreciate that it exists. I'm sure that I wouldn't have understood this a few years ago, and I think this may have been what many of those personal stories were about. We're beginning to discover an understanding that had been hidden, and we're enthusiastic.

ACES has affected each of us in a profound way. We're not just learning instructional techniques to better teach students; we're developing and deepening our own understandings and content knowledge. We have discovered that math is exciting. It makes sense, and it should. Discovering mathematical structure and its connections to the world has the beginnings of beauty.

Questions?

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