

Mathematicians Engaged in Mathematics Education

James Epperson (University of Texas at Arlington)

Yvonne Lai (University of Nebraska-Lincoln)

Dev Sinha (University of Oregon)

Sunita Vatuk (City College of NY)

MSRI Critical Issues in Mathematics Education • 28 March 2014

Combining mathematics and mathematics education

- *Percolation on the randomized Sierpinski Carpet* (Epperson, 1996)
- *On the structure of equivariant bordism rings for cyclic groups of prime order* (Sinha*, 1999)
*current MSRI resident
- *An effective compactness theorem for Coxeter groups* (Lai, 2008)
- *An exploration of hyperbolic exterior differential systems and their integrability by the method of Darboux* (Vatuk, 2009)
- Texas Mathematics Standards and Assessments, Teacher professional development
- Consultant for local school districts and Smarter Balanced Assessment Consortium
- Reshaping program for preparing secondary mathematics teachers at University of Nebraska-Lincoln
- Professional development of math teachers in NYC schools

How does engagement in mathematics teacher education already resonate with the typical training and inclinations of professional mathematicians?

On the other hand ...

What new perspectives, skills, and attitudes do we need to develop in order to be productive in this work?

Panel presentation plan

- We will each reflect upon formative experiences in mathematics education with an eye on these two questions.
- We then summarize what may resonate and where there may be learning curves.
- Finally, we offer some conclusions and proposals for moving the community forward.

How does it resonate?

Finding structure, exploring connections

Sunita

Early experiences: the naïve mathematician?

What I got right:

the activities,
the mathematical connections,
the children

What I missed:

connections to the school
Connections to the teachers

What do we need to learn?

Communication, communication, communication...

Sunita

Later insights:

We are from at least

3 Cultures.

What each of us sees clearly may be invisible to the others.

Filling in some missing links

Sunita

Being explicit about my choices

Meta-level assignments in math classes.

- having students explore representations

- having students do task analysis

- have students find the connections

Lesson study with in-service teachers.

How does it resonate?

Conveying and clarifying nature of mathematics

Dev

Personal background in this area – having assumptions challenged

- Experience teaching at the PROMYS program, early in my career (and the program's existence.)
- Awareness of cognitive dimensions to reasoning (Wason Selection Task).

How does it resonate?

Conveying and clarifying nature of mathematics

Dev

Broader setting for work in this area

- K-12 mathematics education is our “front porch”, but has often suppressed mathematical reasoning rather than promoting it.
- The CCSSM supports and emphasizes mathematical reasoning.

How does it resonate?

Conveying and clarifying nature of mathematics

Dev

Questions for our communities:

- What constitutes mathematical reasoning?
- How can that change across grades/
experience?

Example: properties of multiplication.

Writing number sentences based on rectangular arrays → writing an argument.

How does it resonate?

Conveying and clarifying nature of mathematics

Dev

Deeper example:

Illustrative Mathematics work with Diane Briars
(President NCSM, NCTM).

Exponents lesson – why are negative exponents
defined as they are?

My contribution: the phrase “If the law of
exponents is to continue to hold...”

How does it resonate?

Conveying and clarifying nature of mathematics

Huge Moment!

Dev

In middle grades in the Common Core, reasoning turns from being based on models, to being based on properties (which were previously established with models).

This is a great moment to convey aspects of the nature of mathematics to for pre- and in-service teachers and by extension with wider public.

How does it resonate?

James

- Writing and reviewing curricular and assessment materials and standards

How can one invite and address understanding of a mathematical topic appropriate to time, level, etc.?

How does it resonate?

Writing and reviewing curricular and assessment materials

James

The place of the expertise of a mathematician in education is only as good as his or her ability to communicate with a broad audience.

How does it resonate?

Writing and reviewing curricular and assessment materials

James

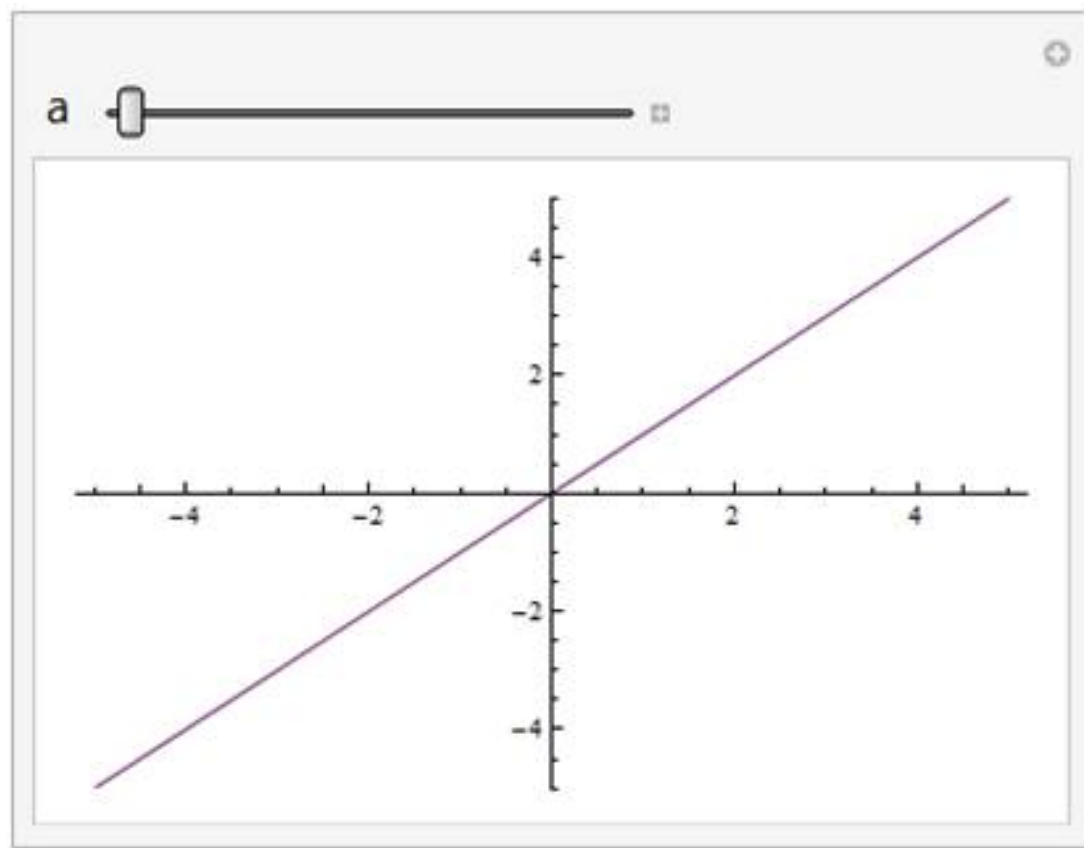


How does it resonate?

Writing and reviewing curricular and assessment materials

James

Identify the effect on the graph of replacing $f(x)$ with $a \cdot f(x)$ where $a \neq 0$.



How does it resonate?

Writing and reviewing curricular and assessment materials

James

28. Yes _____ No _____

Which equation is the parent function of a quadratic equation?

A $y = \sqrt{x}$

B $y = x$

C $y = |x|$

D $y = x^2$

Sample item with conflicting wording from textbook or internet sources—can our preservice or inservice teachers identify the inconsistencies?

This motivates the following question to my preservice and inservice students:

If a student asked you to explain the difference between a function and an equation, what explanation would you offer?

How does it resonate?

Writing and reviewing curricular and assessment materials

James

Many teachers believe that $f(x) = \frac{1}{x}$ is a “parent function” for all rational functions.

Can preservice or inservice mathematics teachers construct a convincing argument or proof to conclude it is not?

How does it resonate?

Building knowledge through collaboration

Yvonne

Questions for the community:

What is the knowledge needed to teach the knowledge needed?

Where are the sources for learning this knowledge?

“These courses are the most emotionally hard to teach because you’re teaching teachers and they are the future. I don’t know how to wonder about whether my course is even useful.”

How does it resonate?

Building knowledge through collaboration

Yvonne

Five questions we have, can, and need to become

more skilled at asking and answering through collaboration

1. What is the math that the teachers are teaching? *e.g., CCSS*
2. What are the teaching practices that teachers can use to teach this math? *Should account for interactions between students, teacher, environment, goals ... and mathematics.*
3. What is the mathematical knowledge that is needed to carry out these teaching practices well? *mathematical meaning/definition, question/solution spaces, connections*
4. What are the teaching practices that we could use to teach this math?
5. What is the knowledge that is needed to carry out these teaching practices to teach this math?

How does it resonate?

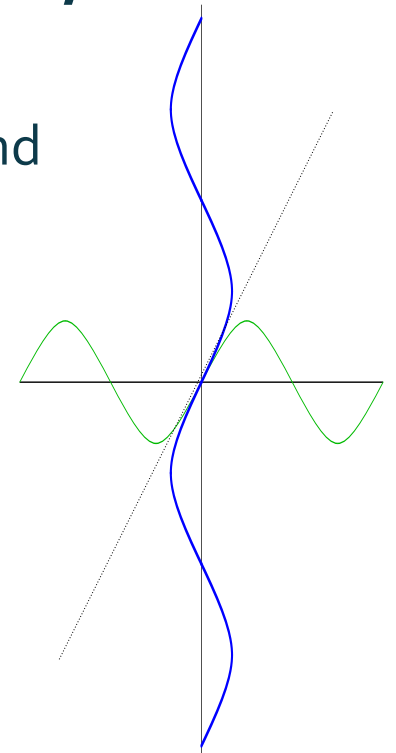
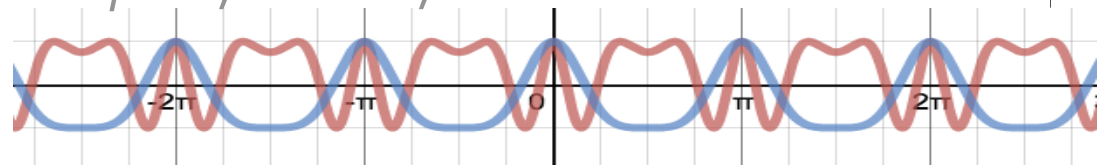
Building knowledge through collaboration

Yvonne

Tasks resulting from collaboration between
mathematicians, teachers, teacher educators,
and education researchers

1. Explain what the vertical line test has to do with inputs and outputs.
2. Draw the image of the reflection of the green graph over the line $y=x$. Why is the image congruent while not appearing to be congruent to the original?
3. Explore the function $f(x)=\cos(k \sin(x))$ for different constants k . Explain using the meanings of radian, sine, cosine, and period, why a period of $f(x)$ is always π .

[Variation on task by Thompson, Carlson, and Silverman (2007)]



How does it resonate?

Building knowledge through collaboration

Yvonne

The need to build knowledge individually and collectively

- So we can specify learning goals for our classes and courses.
- So we talk with each other about how and why we think that our class activities should impact the learning goals and would allow us to detect the impact.
- So we can build collective knowledge (Morris & Hiebert, 2009):
 - Shared goals across the community
 - Visible, tangible, changeable products (our tasks and outcomes)
 - Sources of innovation throughout community
 - Building shared knowledge is seen as part of essential and daily work

Summary: How does it resonate?

Close study of mathematics – search for pattern, structure, connection, representations, generalization ... in the context of K-12 teaching.

*What are mathematical sources of student errors?
What mathematical tools are available to address them?*

Conveying and clarifying the nature of mathematics

*How does the nature of mathematical argument change
as learners progress from PK-12-20?*

Writing and reviewing curricular and assessment materials

*How can one invite and address understanding of a mathematical topic
appropriate to time, level, etc.?*

Building mathematical/pedagogical knowledge through collaboration

*What is the knowledge needed to teach the knowledge needed?
Where are the sources for learning this knowledge?*

Summary: Where is there a learning curve?

- Where the mathematical issues arise in the curriculum.
- Relevance of the mathematics to their work with children.
- Outcomes based on mathematical practices.
- Teaching quality has dimensions beyond mathematical validity.
- Teaching is complex and has to do with interaction between students, instructor, environment, and the content.
- Schools are complex cultural institutions.

Conclusions and Proposals

Orientation for working in mathematics education

- Humility
- Openness to multiple perspectives
- We need to reach out to others, and not expect others to come to our door
- We cannot and should not attempt to develop knowledge in isolation
 - ... need collaborate and learn from teachers, teacher educators, policy makers, and each other.

Changing the rhetoric

Unproductive rhetoric (“us” versus “them”):

“they (we) have mathematical experience”

“they (we) have classroom experience”

No one person or profession holds all the knowledge; we hold the needed knowledge together and can only build on it together.

While being respectful of expertise, we need to cultivate curiosity and productive rhetoric:

“we must learn and teach each other about mathematical and classroom experience and knowledge”

Need for an intellectual home, and moving beyond moral imperative

We often feel that we work in idiosyncratic and isolated ways. But this doesn't have to be true.

Question for the community: how do we support and promote the intellectual community around “mathematical analysis of teaching and learning”?

Need for an intellectual home, and moving beyond moral imperative

Being mindful of the intellectual infrastructure which is already in the Mathematics Education community including the MAA, it seems there is some need to capture, vet, refine, collect and recognize some important work (e.g. by people like Wu, Howe, Patterson).

The Illustrative Mathematics Journal, anyone?

Need for an intellectual home, and moving beyond moral imperative

Any healthy community needs younger and older members, and ways for them to interact (mentoring relationships, conferences and seminar visits, reviewing of work, etc.).

Need for programs for early- and mid-career mathematicians to build community and knowledge, analogous to NeXT or STaR, but inclusive of mathematics PhD holders with teaching experience AND mathematics education faculty.

And of course there is a need for departments and their institutions to recognize the value of people so trained and set up career paths accordingly.

Thank you!

Links and references

Illustrative Mathematics:

- <http://www.illustrativemathematics.org/>

Mathematics of K-12 mathematics:

- Progressions Documents for the CCSS <http://ime.math.arizona.edu/progressions/>
- Howe, R. (2011). Three pillars of first grade mathematics. *The De Morgan Journal* 1(1), 3-17.
- Wu, H.-H. (many articles). <http://math.berkeley.edu/~wu/>

Mathematical knowledge for teaching:

- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching what makes it special?. *Journal of Teacher Education*, 59(5), 389-407.

Tasks designed to develop mathematical meaning:

- Thompson, P. W., Carlson, M. P., & Silverman, J. (2007). The design of tasks in support of teachers' development of coherent mathematical meanings. *Journal of Mathematics Teacher Education*, 10(4-6), 415-432.

Building collective knowledge:

- Morris, A. K., & Hiebert, J. (2009). Introduction: Building knowledge bases and improving systems of practice. *The Elementary School Journal*, 109(5), 429-441.