Developing Teachers as Professionals: Park City Mathematics Institute Summer School Teachers Program

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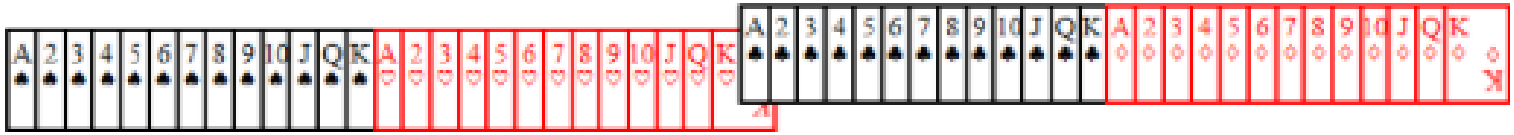
Moving Things Around: Card Shuffles, Repeating Decimals and Geometric Transformations

Math Content Class - Bowen Kerins and Darryl Yong

click on each animated graphic to view larger version

The Developing Mathematics class is being led by Bowen Kerins and Darryl Yong. The focus of the class this year is "Moving Things Around: Card Shuffles, Repeating Decimals and Geometric Transformations."



Professional development often

- Has as a goal to “change” teacher practice/beliefs/…
- Is developed around content that teachers teach
- Is dependent on “professional” professional development leaders
- Is localized – school, region, or state based
- Does not involve mathematicians

What is known about effective mathematical preparation of teachers, including curriculum, instructional approaches, and assessments?

- Little evidence that programs "work" (Gerstein et al, 2013)
- Infrequent professional development focusing on content (METII)

MET II

- Recommendation 4. All courses and professional development experiences for mathematics teachers should develop the habits of mind of a mathematical thinker and problem-solver, such as reasoning and explaining, modeling, seeing structure, and generalizing. Courses should also use the flexible, interactive styles of teaching that will enable teachers to develop these habits of mind in their students. (CBMS, 2013)

IAS/Park City Math Institute

- Three-week residential Summer Session comprised of Core Program and Cross-Program activities
- Publication series

Mathematics
Research
Graduate
Undergraduate
Undergraduate Faculty

Mathematics Education
Teachers
International Seminar

Past Research Themes

2013: Geometric Analysis
2012: Geometric Group Theory
2011: Moduli Spaces of Riemann Surfaces
2010: Image Processing
2009: Arithmetic of L-functions
2008: Analytic and Algebraic Geometry
2007: Statistical Mechanics
2006: Low Dimensional Topology
2005: Mathematical Biology
2004: Geometric Combinatorics
2003: Harmonic Analysis and Partial Differential Equations
2002: Automorphic Forms and Applications
2001: Quantum Field Theory, Supersymmetry, and Enumerative Geometry
2000: Computational Complexity Theory
1999: Arithmetic Algebraic Geometry
1998: Representation Theory of Lie Groups

Summer School Teachers' Program* (SSTP)

- 55-60 grade 2-12 teachers from across the US
  - Math for America
  - At large
  - Teachers as Professionals
- Diverse backgrounds with respect to mathematical content knowledge, experience, schools, cultures
- E-tables link in participants from remote sites

*Currently Funded by Math for America, Past funding from NSF MSP, TCP, Noyce Programs

SSTP Goals

- Deepen content knowledge
- Reflect on practice:
- Become a resource to colleagues

Math Course: Design Process

- Tied to PCMI mathematics research theme
- Original framing done by Bowen Kerins, Al Cuoco from EDC and Glenn Stevens (BU)
- Field tested in Boston area with teachers
- Refined **at** PCMI by instructors (Bowen Kerins and Darryl Yong)

Math Course: Design Principles

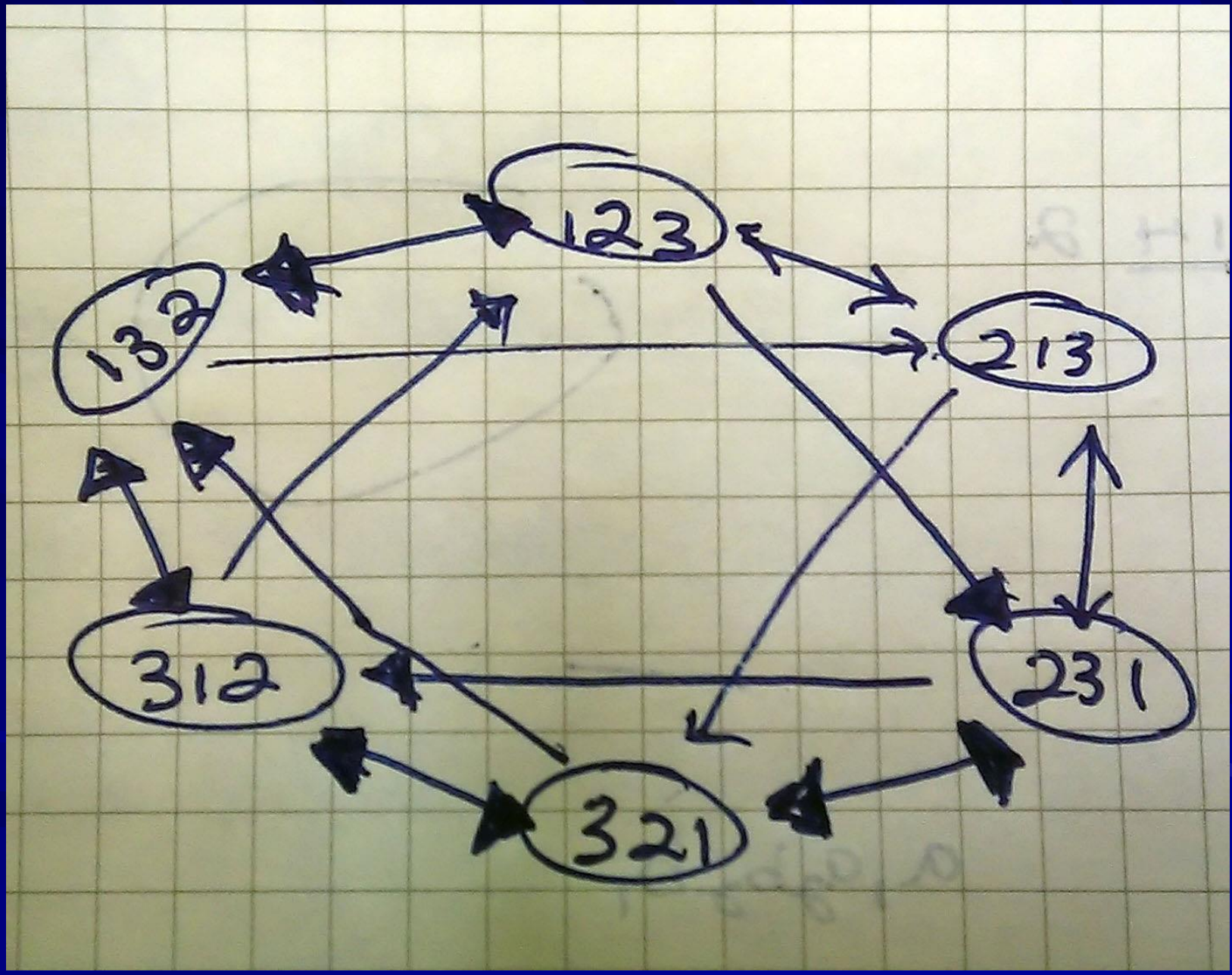
- Exposure before closure
- Multiple entry points into the mathematics
- Emphasis on connections and relationships among problems
- Problems repeated in different contexts and forms (e.g., geometry-algebra)
- Key ideas foreshadowed but not formalized until late in course
- Stuff: Important stuff, neat stuff, tough stuff
- Learning about mathematics and teaching mathematics by doing mathematics

Opener

Problem Set 2: Slhi′unfg

Opener

Can perfect shuffles restore a deck with 9 cards to its original state? If so, how many perfect shuffles does it take? If not, why not?


123
132
213
312
231
321

Day 13

pcmi-2012-problems.pdf

1. Draw the eight orientations of the square from Set 12's opener. Use arrows in one color to connect two orientations if the $(2\ 3)$ transformation takes one orientation to the other. Use arrows in a different color to connect two orientations if the $(1\ 2\ 4\ 3)$ transformation takes one orientation to another. Don't draw arrows for other transformations. Notice anything?

2. Use your two diagrams to argue that the group generated by in- and out-shuffles on 4 cards is isomorphic to D_4, the group of symmetries of the square.

Implementation

- Participants, in groups of 4 or 5, engage with a set of carefully sequenced and scaffolded tasks.
- Groups are facilitated by a “table leader”.
- Groups are reformed every three-four days to account for participant needs and ways of working.
- Time is spent primarily doing math.
- Occasional large-group discussions are facilitated by the instructors.
- Instructors observe participants doing mathematics and adjust the tasks accordingly.

Norms

- **Don’t worry about answering all the questions.** If you’re doing that, we haven’t written the problem sets correctly.
- **Don’t worry about getting to a certain problem number.** Some participants have been known to spend the entire session working on one problem (and perhaps a few of its extensions or consequences).
- **Stop and smell the roses.** Getting the correct answer to a question is not a be-all and end-all in this course. How does the question relate to others you’ve encountered? How do others think about this question?
- **Respect everyone’s views.** Remember that you have something to learn from everyone else. Remember that everyone works at a different pace.
- ….

Reflecting on Practice: Design Process

- Teacher leaders selected from past participants and serve as support staff for SSTP
- Brainstorm weekend with teacher leaders
- Meet in teams of two to develop course for the three weeks, facilitated by SSTP leadership team
- During SSTP, meet daily to debrief and revise and Sundays to assess scope for coming week

Reflecting on Practice: Design Principles

- Grounded in research
- Use artifacts of practice around important mathematics (videos, articles, student work, ..)
- Build from participants' knowledge and backgrounds
- Focus on key math concepts
- Create situations that promote discussion
- Anticipate questions and responses
- Plan for records of work

What is an equation?

7:28 (min:sec)

7:42 (min:sec)

Nick Branca, Lesson **Lab**

Reflecting on Practice Topics

- Discussions that Enhance Learning
- Questions that Count
- Strategies for Formative Assessment
- Worthwhile Tasks

Implementation

- Three rooms of about 20 participants, rotated weekly
- Work in small groups formed by various configurations
- Two teacher leaders in each room facilitate sessions and discussions
- Highly structured interactive discussions with targeted learning objective for each day

Working Groups: Design Principles

- Create an activity to help colleagues better understand and adapt to the CCSSM using the umbrella of Teachers as Professionals (C-TaP).
- Topics related to needs of teachers with respect to the CCSSM
- Detailed template for activity includes objective, teacher questions and anticipated responses, ways to record work, …
- Structured feedback provided by sharing ideas across teams, across grade levels, and by mathematicians and math educators who review midcourse and end product

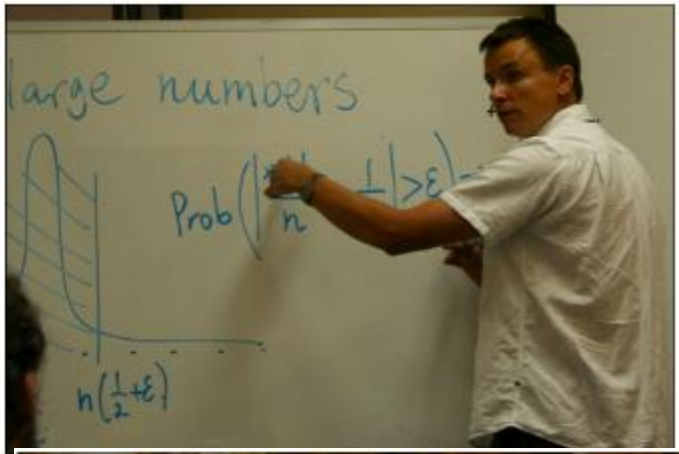
Working Groups: Implementation

- Teacher leaders with SSTP leadership identify areas of need related to the work of Illustrative Mathematics Project associated with CCSSM.
- Participants choose topic and then grouped into teams of three
- Structured feedback provided by sharing ideas across teams, across grade levels, and by reviewers
- Products become part of the Illustrative Mathematics professional development offerings and are used by participants in local and regional settings

Cross Program

- Cross Program talks
- Research mathematicians discussion groups
- Pizza and Problem Solving
- Clay Institute Lectures
- SSTP Working Group takes undergraduate math faculty course
- Assigned seating at lunch
- Fourth of July Parade

Andrei Okounkov (Clay Senior Scholar and Fields Medalist)



“Free” time

- 5 minute shorts
- Sharing nights
- 4:30 sessions on topics of interest
  - fractions in the CCSSM, technology and teaching mathematics, teaching mathematics in a foreign country, how to become involved in NCTM
- Building Nights (Zome tools, origami, ...)
- Karioke

Monitoring SSTP

- Formative
  - Breakfast Club
  - Weekly staff/ table leader meetings
- Summative
  - Exit surveys
  - Academic year surveys
  - Interviews with sample of participants

Transformative Experiences

"PCMI is simply the best professional development experience I have ever had." (Participant/Exit Survey)

"The focus on being intentional really hit home for me. I kept thinking about how I did things on the fly so much in my class and how that may have affected their learning…"(Participant Report)

"*The problem solving sessions helped me structure problem solving in my own classrooms in a more engaging way. The pedagogy sessions, with the focus on the mathematical practice standards, assessment, and feedback, really sparked some growth in my department; we all looked at the practice standards in a new light, and began to implement the Common Core in a way we hadn't even considered before my PCMI experience."* (Returning Participant Application)

Transforming Approach to Math

"The structure—the logical scaffolding structure of the problem sets themselves were so beautifully done that it helped me understand the scaffolding of concepts in my teaching. Everything is tightly connected, and when it is tightly connected, everyone understands better."
(Participant, Evaluator Interviews)

They were really good in creating these problem sets—very intentional—and they built on each other. Concepts were very tight and there was a lot of interconnecting. I've tried to be more like that. I try to think how the concepts link together, how I will teach to link to other concepts later on. I'm also creating problem sets that are accessible to kids at all levels. (Participant, Evaluator Interviews)

Transforming Teaching

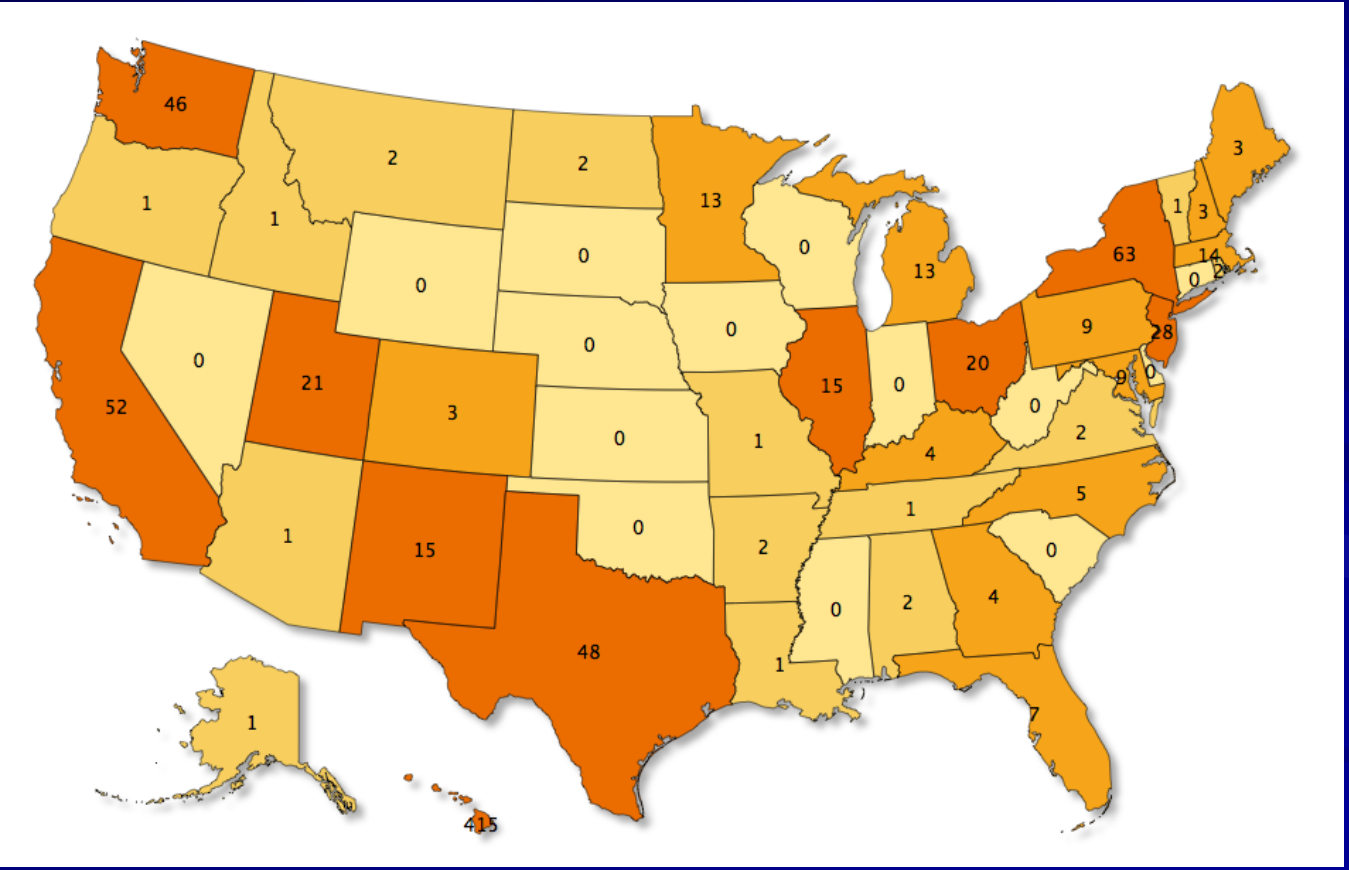
"In my own classroom I have overhauled my classroom questioning following our 2012 RoP about pushing/probing questions. In a recent 15-minute observation my Principal informed me he NEVER heard me make a statement ONLY questions while students were involved in their group work activity. Also with the suggestions and support of colleagues from PCMI I constructed an applied math college course Math in Society, taught in Fall 2013." (Returning Participant Application)

"I have received so many rich, valuable morsels from the SSTP program that I have integrated over the past few years and continue to find new ways to be a better teacher. One way is having classroom discourse and how to scaffold it to make it safe for students to share, to be "wrong", to learn from each other and learn from their mistakes and others." (Participant Exit Surveys)

Broader Impact

- Involvement of research mathematicians in education
- Network of SSTP teachers who have common vision of teaching and learning mathematics
- Online resources (http://mathforum.org/pcmi/)
- Past participants facilitate online sessions for those unable to be at PCMI
- E-CMI NSF grant to explore scaleability

Being around people with such diverse math experiences who approach problems in very different ways was valuable.



The relationships I made and network connections have really opened up my feeling as both a member of the math educator community and as someone with an exciting career path!

Park City Mathematics Institute

Mathematics Education Program

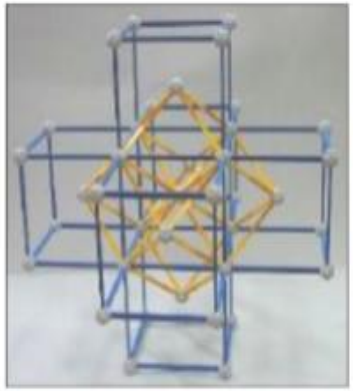
Projects, Papers, Activities, Policy Briefs

- **Japanese Lesson Study Research Lesson**
  - Studying Dimensionality Through a Stair-Step Fractal
  - Don't Fence Me In
  - A Lesson in Student Learning: Rate of Change
- **Geometry**
  - An Amazing, Space Filling, Non-regular Tetrahedron
  - Paper Cup Mathematics
  - Circle Packing, A Directed Investigation of Descartes' Circle Theorem
  - Rhombic Dodecahedron - Hidden Within or Surrounding the Cube?

http://mathforum.org/pcmi/

An Amazing, Space Filling, Non-regular Tetrahedron

by Joyce Frost and Peg Cagle



WHAT IS IT?
A background paper and a related classroom activity about tessellating space using the rhombic dodecahedron.

GRADE LEVEL/STRAND
Secondary geometry

CLASS TIME
2 class periods (this would allow enough time for the construction of a group version for each of the three puzzles)

Go to: background paper and activity
Download: MS Word file

MATERIALS
8.5 by 11 inch paper to fold nets, glue sticks, scissors, file folders, packing tape

IM Workshops
http://www.illustrativemathematics.org

- Recognizing Mathematical Practice in Student Work in Geometry
- Modeling Algebra
- Using Low Threshold, High Ceiling CCSSM Tasks
- Technology in the CCSSM Classroom
- An Introduction to the Congruence in Transformational Geometry
- Using Probability to Make Informed Decisions
- Exploring Conditional Probability
- Identifying Evidence of Mathematical Practices in Student Work
- Systems of Equations
- Modeling Geometry
- Parents Overview of the Common Core
- Implementing Group Tasks in the CCSSM Classroom
- Introducing Quadratics
- Rate of Change

Conjectures: What makes SSTP work

- Implicit connection between delivery of content and reflecting on teaching practice
- Honor the context of teaching
- Diversity of participants
- Environment – everyone, literally, is about mathematics
- Time to share during working groups
- Leadership shared among mathematicians, teacher educators and teachers
- Residential/long term fosters small groups who share and learn from each other

References

- Gersten, R., Taylor, M. J., Keys, T. D., Rolfhus, E., & Newman-Gonchar, R. (2014). *Summary of research on the effectiveness of math professional development approaches.* (REL 2014–010). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southeast. Retrieved from http://ies.ed.gov/ncee/edlabs.
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