1. Diophantine approximation for algebraic numbers

A number θ is called well approximable (WA) if $\forall\epsilon$, $\exists p, q$ s.t. $|\theta - p/q| < \epsilon/q^2$. It is equivalent to that in continuous fraction $\theta = [a_0, a_1, \dots]$, $\{a_i\}$ are unbounded. Almost all real numbers are WA, all quadratic irrationals are not WA. This can be shown by continuous fraction or by the fact that if θ is quadratic, irrational integer, $|q\theta - p||q\bar{\theta} - p| \geq 1$. How about cubic irrationals?

"Ergodic principle": whatever can happen will happen eventually & approximately, most of the time.

Relationship with dynamics: example: $\{n^2\alpha\}$ is dense in $[0, 1]$ for irrational α, follows from $T : \mathbb{T}^2 \to \mathbb{T}^2$: $(\theta, \phi) \mapsto (\theta + \alpha, \phi + 2\theta + \alpha)$ is minimal. Continuous fraction correspond to $x \mapsto \{1/x\}$, but this can not encode the "cubic" condition.

A dynamical system that may be useful: Let Ω^* be the set of 2 dimensional lattices in $\mathbb{R}^3$ up to scaling, there is a depth function $\mu(< v, w >) = \inf_{x,y\in\mathbb{Z}} ||xv + yw||^2/|v \wedge w|$. Let A be a 3-by-3 rational matrix with irrational eigenvalues, let $H = \{h \in GL_3 | hA = Ah\}$, take $u_0 \in \mathbb{Q}^3 - 0$, consider H orbit on the lattice spanned by u_0, Au_0. Is there similar result as Raghunathan's theorem?