

1. OPPENHEIM CONJECTURE AND RELATED PROBLEMS

Let Q be a real indefinite irrational quadratic form in n variables. Oppenheim conjectured in 1929 that if $n \geq 5$, $\forall \epsilon > 0$, $\exists x \in \mathbb{Z}^n - 0$ s.t. $|Q(x)| < \epsilon$. The “ $n \geq 5$ ” was inspired by Meyer’s theorem on rational quadratic forms and removed by Davenport. Raghunathan’s conjecture is also inspired by this. For $n \geq 5$, Q diagonal it was proved by Chowla, Birch, Davenport and Heilbronn, and for $n \geq 21$ proved by Davenport and Heilbronn. Before and after it was proved by Margulis, effective proofs for various cases are found by Iwaniec, Birch, Bentkars, Gotze, Margulis etc.

A related problem is counting lattice points in ellipsoids. Jarwik, Waffisz, Bentkus, Gotze etc.