1. Word metric asymptotics for actions of hyperbolic groups on Gromov hyperbolic spaces

Joint work with Taylor, Tiozzo.

Γ: word hyperbolic group, acting on X, a Gromov hyperbolic geodesic metric space, by isometry. The action is called CAP if it sends geodesics to uniform quasigeodesics (i.e. there is a uniform (K, C) s.t. $\frac{1}{K}(b-a) - C \leq d(\gamma(b), \gamma(a)) \leq K|b-a| + C$.

Theorem (G-T-T) If the action is non-elementary, CAP, and has at least 2 loxodromic elements, then:

$$\lim_{R\to\infty} \frac{\#\{g \in B_R^\Gamma(e), g \text{ lox.}\}}{|B_R^\Gamma(e)|} = 1$$

(i.e. a.e. element is loxodromy.)

Example: Γ hyperbolic, with finite generating set S, $H_1, \dots H_r$ quasiconvex subgroups, $X = Caylay(\Gamma, S \cup H_1 \cdots \cup H_r)$, then the action is CAP. In particular, if Γ is a lattice in $SL(2, \mathbb{R})$ and H_i are the cusp subgroups, X is the Farey graph, and the theorem implies that most elements in such a lattice are hyperbolic.

Example: Hyperbolic group action on trees with quasiconvex stablizers.

Example: Quasiconvex subgroups of RAAGs acting on the extension geaph.

Lemma 1: $\forall K > 0$, $\lim_{R\to\infty} \frac{\#\{g\in B_R^\Gamma(e), d(gx,x)\leq K\}}{|B_R^\Gamma(e)|} = 0$.

Lemma 2: $\forall \epsilon, \exists K$, $\lim_{R\to\infty} \frac{\#\{g\in B_R^\Gamma(e), (g^{-1}x, gx)_x > K\}}{|B_R^\Gamma(e)|} < \epsilon$.

The theorem follows from them and the fact that if g is not loxodromic, $(g^{-1}x, gx)_x \geq \frac{1}{3}$.

To prove the Lemmas, use the Patterson-Sullivan measure γ on $\partial\Gamma$, and:

Theorem (Connell-Muchnik) There is measure μ on Γ s.t. γ is μ-stationary.

For $\zeta \in \partial X$, $x \in X$, $N \in \mathbb{R}$, define the shadow $Sh_x(\zeta, N) = \{y \in X \cup \partial X : (y, x) > N\}$, $\overline{Sh_x(\zeta, N)} = \{g \in \Gamma : gx \in Sh_x\}$.

Now use random walk technique to show the shadow restricted to $\partial\Gamma$ has measure that decay uniformly.