1. Effective estimates in reduction of quadratic forms

Joint work with Margulis.

A, B are $n \times n$ symmetric non-singular integer matrices, they are integral equivalent iff $\exists \gamma \in GL(n, \mathbb{Z})$ s.t. $A = \gamma^T B \gamma$. The norm of a matrix int. equiv. to B is bounded below by $c_n |detB|^{1/n})$, how about an upper bound?

In positive definite case this is done by Hermidt, Minkovsky. The upper bound is $C_n|detB|$ and is sharp due to $diag(1, 1, \ldots d)$. For indefinite forms:

Theorem (L-M) $\xi \in (-1, 1)$, B a 3×3 primitive indefinite integer matrix, then $\exists A \sim^{\mathbb{Z}} B$ s.t.:
(1) $||A|| = O(|detB|^{1/3})$
(2) $|\frac{a_{11}}{(detB)^{1/3}} - \xi| < c_n |detB|^{-1/2400}$.

Proof of (1):

Theorem (Dani, Margulis) $\exists X \subset SL(3, \mathbb{R})/SL(3, \mathbb{Z})$ compact, s.t. $\forall g \in SL(3, \mathbb{R}), Hg\Gamma \cap X \neq \emptyset$.

Let $H = SO(2, 1)$, $g \in SL(3, \mathbb{R})$ s.t. $B = |detB|^{1/3} g^T (diag(1, 1, -1))g$, the theorem above impies that $\exists g_1 = hg\gamma$ s.t. $||g_1|| < C$. Let $A = \gamma^T B \gamma$.

Proof of is based on the "folding" of H orbit in $SL(3, \mathbb{R})/SL(3, \mathbb{Z})$.