Anne Parreau

Σ a compact oriented surface $\partial\Sigma \neq \phi$.

$\Gamma = \pi_1(\Sigma)$

$\rho : \Gamma \to G = PGL(3, \mathbb{K})$

$\mathbb{K}$ is a field with ultranatiee absolute value $|x+y| \leq \max\{|x|\, |y|\}$

Motivation : degeneration of representations in $PGL_3(\mathbb{R})$.

ex. of convex real projective structures on Σ

(refined) length function :
($\mathfrak{C}$-length)

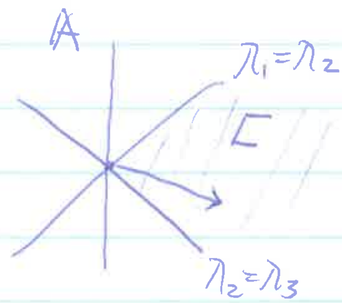
Cartan projection

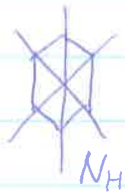
$g \in G \qquad \ell^{\mathfrak{C}}(g) = (\lambda_1 \geq \lambda_2 \geq \lambda_3) \in \overline{\mathfrak{C}}$ $\quad \lambda_i = \log|a_i|$, a_i eigenvalue of g

Model flat of type A_2

$A = \{\lambda = (\lambda_1, \lambda_2, \lambda_3) \in \mathbb{R}^3 \mid \Sigma \lambda_i = 0\}$

$\mathfrak{C} = \{\lambda_1 > \lambda_2 > \lambda_3\}$





$\ell(g) = \|\ell^{\mathfrak{C}}(g)\|$

$\ell_H(g) = \lambda_1 - \lambda_3 = N_H(\ell^{\mathfrak{C}}(g))$.

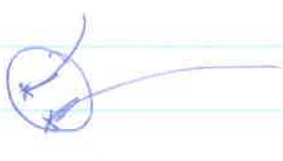
G acts on X the associated Euclidean building

- X is a CAT(0) space
- marked flats $f : A \to X$.

transition maps are in $W \curvearrowright A$ (permutation group).

$= W \ltimes$ Translations (Affine).

$\forall \gamma_1\, \gamma_2$ geodesic $\exists$ flats contains initial points of γ_1 and γ_2



• The Γ-distance $d^\Gamma : X \times X \longrightarrow \Gamma$

Fock – Goncharov's parameters (any field $\mathbb{K}$)

Farey set $\widetilde{F}_\infty(\Sigma) = \{$ boundary components of $\widetilde{\Sigma}\}$

Let τ be an ideal triangulation of Σ

lift τ to $\widetilde{\tau}$ on $\widetilde{\Sigma}$ $\rightarrow$ vertices $(\widetilde{\tau}) = \widetilde{F}_\infty(\Sigma)$

Take $z_\tau \in \mathbb{K} - \{0, -1\}$ for each τ in $T = \{$triangle of $\tau\}$

$s_e \in \mathbb{K} - \{0\}$ for each oriented edge of τ $e \in \vec{E}$

Then there exists a unique (ρ, F).

$\rho : \Gamma \longrightarrow PGL_3 \mathbb{K}$.

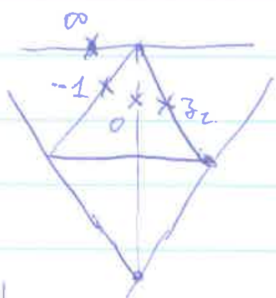
flag map. $F : F_\infty(\Sigma) \longrightarrow \mathrm{Flags}(\mathbb{P}^2\mathbb{K})$) ρ – equivariant

$i \longmapsto F_i = (p_i, D_i)$

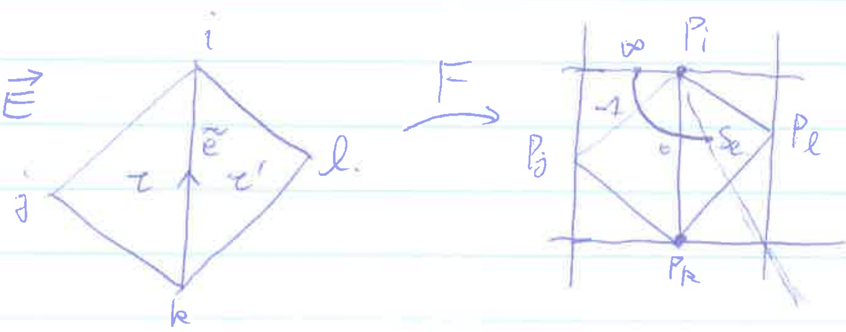
$\forall\ \widetilde{\tau}$ triangle of $\widetilde{\tau}$ lifting $\tau \in \mathcal{T}$

(i, j, k)

the triple ratio of (F_i, F_j, F_k) is z_τ



$\forall\ e \in \vec{E}$



$s_e = b(D_i, P_iP_j, P_iP_k, P_i(D_k \cap D_l))$

b is the cross ratio $b(\infty, -1, 0, a) = a$.

$\mathfrak{z}_\tau = \log|Z_\tau|$
$\mathfrak{s}_e = \log|S_e|$

Parameter: $(Z,S) = ((Z_\tau)_\tau, (S_e)_e)$.
$\rho_{Z,S}$ is the associated represantation.

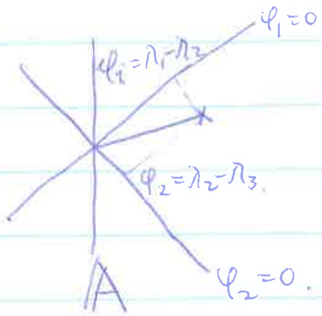
The $\mathbb{A}_2$ complex $K_{\mathfrak{z},\mathfrak{s}}$ associated to a geometric FG-parameter $(\mathfrak{z},\mathfrak{s})$

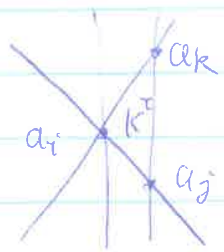
$\mathfrak{z} = (\mathfrak{z}_\tau)_\tau \in \mathbb{R}^T$

$\mathfrak{s} = (\mathfrak{s}_e)_e \in \mathbb{R}^{\vec{E}}$

A_2-complex = Euclidean polyhedral complex with cells = convex polygone in A_2, gluings by elts in Wall.

• For each triangle τ in T take a singular triangle K^τ in $\mathbb{A}$ of side length $\mathfrak{z}_\tau$

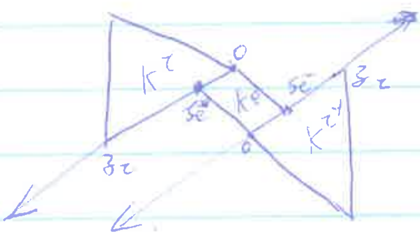




$a_i = 0$

$a_j = (\mathfrak{z}_\tau, 0)$

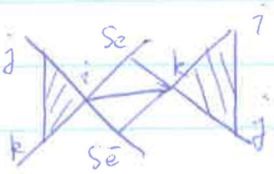
$\mathfrak{z}_\tau > 0$.



$K^e = [0, \mathfrak{s}_{\bar{e}}] \times [0, \mathfrak{s}_e]$

$S_e < 0$

$S_e > 0$.



$S_e > 0 \quad S_{\bar{e}} > 0$

hyp: $(\mathfrak{z}, \mathfrak{s})$ is left-shifting.

i.e. $\forall e \in \vec{E}$ $\quad s_e > \max(-\mathfrak{z}_\tau^-, -\mathfrak{z}_{\tau'}^+)$ where $t^+ = \max(t, 0)$

$t^- = \max(-t, 0)$

$\Leftrightarrow s_e > 0$ or $\frac{-\mathfrak{z}_{\tau'}}{\mathfrak{z}_\tau} \leq s_e \leq 0$. (then $s_{\bar{e}} > 0$).

Γ-distance on $\tilde{K}$: Γ-length of a geodesic from x to y

$(\sum d^\Gamma(x_i, x_{i+1}) \quad [x_i, x_{i+1}] \subset$ cell of $\tilde{K})$

$$d^\Sigma : \tilde{K} \times \tilde{K} \to \bar{\Gamma}$$

$$\gamma \longmapsto \ell^\Gamma(\gamma, K)$$

Thm 1: Let (Z, S) be a FG parameters (in $\mathbb{K}$) and $\rho = \rho_{\mathfrak{z},s}$

$T \to PGL_3 \mathbb{K}$. Denote $\mathfrak{z}_\tau = \log|Z_\tau|$ $\quad s_e = \log|S_e|$ $\quad (\mathfrak{z}, s)$

Suppose (1) For all triangle

$$|Z_\tau + 1| \geq 1$$

(2) $\forall e \quad |S_e + 1| \geq 1$

(3) $(\mathfrak{z}, s)$ is left-shifting.

(4) $(\mathfrak{z}, s)$ is edge separating

i.e. $\forall e_1 e_2$ edges of τ $\quad \begin{cases} -s_{e_1} - s_{e_2} < \mathfrak{z}_\tau^- \\ -s_{\bar{e}_1} - s_{\bar{e}_2} < \mathfrak{z}_\tau^+ \end{cases}$

$\forall \tau$

Let $K = K_{\mathfrak{z},s}$

Then: 1) $\ell^\Gamma \circ \rho$ is the Γ-length spectra of K

2) There exist an equivariant map

$\Psi : \tilde{K} \to X$ preserve the d^Γ

NB: $(2) \Rightarrow (1)$

Thm 2.

Let $\rho_n : \Gamma \to PGL_3(\mathbb{R})$ with FG-parameter

$$\mathcal{Z}_\tau^n = \exp(z_\tau^n)$$

$$S_e^n = \exp(z_e^n).$$

$$\lambda_n = \max |z_\tau^n|, |s_e^n|$$

$$\left.\begin{array}{l} \frac{1}{\lambda_n} z_\tau^n \xrightarrow[n\to\infty]{} z_\tau \\ \frac{1}{\lambda_n} s_\tau^n \xrightarrow[n\to\infty]{} s_e \end{array}\right\} \quad (z, s) \text{ geometric FG parameters.}$$

Suppose (z, s) satisfies (3) and (4).

then $\frac{1}{\lambda_n} \ell^\Sigma \circ \rho_n \xrightarrow[n\to\infty]{} \ell^\Sigma(\cdot, K_{z,s})$