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~~[illegible]~~

Mod action on the space of geodesic currents

S_g: γ is a closed curve in $\pi_1(S_g)$.

$T'X$ $\sum a_i \gamma_i \in \pi_1(S_g)$

Mod-γ: set of all closed curve of topological type of γ.

Thm $X \in \mathrm{Teich}(S_g)$, γ closed curve.

$\# \{ \gamma_1 \in \mathrm{mod}\, \gamma \mid \ell_\gamma(X) \leq L \} \sim L^{6g-6} \cdot \underset{\underset{\mathbb{Q}}{\uparrow}}{C_\gamma} \cdot B(X)$

Question: Y_1, Y_2 arbitrary geodesic currents ← Bonahon

$L \to \infty$: $\{ Z \in \mathrm{Mod}\, Y_1 \mid i(Z, Y_2) \leq L \}$

$\sim L^{6g-6} \times B(Y_1) \times B(Y_2)$

$i: CS \times CS \to \mathbb{R}_+$

- generalize the geometric intersection between closed curves.
- $X \in CS$
- PCS compact.

⊛ Negatively curved metric (Otal)

⊛ Flat structure (Duchin–Leinger, Rafi).

⊛ Cone singularity (Hersonsky–Paulin).

⊛ (Huber–Margulis. X hyp

$\# \{ \gamma \mid \ell_\gamma(X) \leq L \} \sim e^L / L$ $L \to +\infty$

Question: Why polynomial behavior?

Simple closed current on S (Dehn–Thurston) grow at $O(L^{6g})$

Birman – Series.

Case of γ simple $\to$ M.

$\to$ Rivin – McShane $g = n = 1$

γ one-self intersection Rivin

~~Groth~~ Growth K-self intersection — Vireka Erlandson

Juan Souto.

Jenya Sapir:

"Thm": $\#\{\gamma \mid \ell(\gamma) \leq L \;\; i(\gamma,\gamma) = K\} \leq C e^{\sqrt{K}} \cdot L^{6g-6}$.

random closed geodesic of length ℓ of ℓ^2 self intersection

Lalley – Chas.

Case of γ simple

Growth of Mod. γ is a "topological statement"

$\rightarrow$ ~~#~~

$$\#\{\gamma_1 = g\gamma \mid \ell_X(\gamma_1) \leq L\} \sim L^{6g-6} \cdot C_\gamma \cdot B(X)$$

Holds for any geodesic current X

$B(X)$ is Thurston volume of $\{\lambda \mid i(X,\lambda) \leq 1\}$ $\subset \mathcal{ML}$.

$C_\gamma = \mathrm{Vol}_{\mathrm{Thu}}\left(i(\gamma,\lambda) \leq 1 / \mathrm{Stab}(\gamma) \right)$, $\lambda \in \mathcal{ML}$

"Question

~~Thm 2~~

$X \in \mathrm{Teich}(S_g)$. P pants decomposition on S_g.

$\mathrm{Mod}(P)$.

$\{g.P \mid \ell_{g.P}(X) \leq L\}$

$\downarrow$

$\{(\ell_{\alpha_1}(X) \ldots \ell_{\alpha_{3g-3}}(X))\} \hookrightarrow \{(X_1, \ldots, X_{3g-3}) \mid \Sigma X_i = 1\}$

What is the asymptotic length distributions a "random" path decomposition.

"Thm 2" limiting distibution

$$V(A) = \int_{\text{cone}} x_1 - x_n \, dx_1 dx_i \quad + n$$

<u>Idea</u>: Instead of $\overset{\gamma}{X}$, $\ell_{g.\gamma}(X) = \ell_\gamma(g^{-1}X)$

Teich(S_g)

~~$B_\gamma(L)$~~

$B(\gamma, L) = \{X \mid \ell_\gamma(X) \le L\}$

if γ is filling, then $B(\gamma, L)$ is cpt Teich(S)

<u>Prop</u>: $\mathrm{Vol}_{WP}(B(\gamma, L)/_{\mathrm{Stab}(\gamma)}) \sim L^{6g-6} \cdot C_\gamma \cdot C$

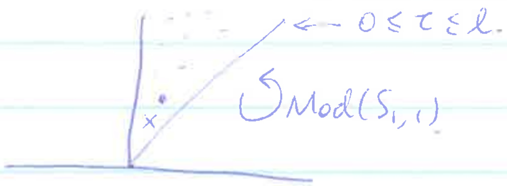
"Lemma": γ is filling, $\ell_\gamma : \mathrm{Teich}(S_g) \to \mathbb{R}_+$ is "asymptotically" piecewise ~~linear~~ in cpt cone. (away from finitely many linear subspaces)

Fix pants decomposition FN coordinates (ℓ, τ) on $\mathcal{T}(S_g)$.
(idea: Trace formulas!

$\mathbb{R}^2 = \{(x, y)\}$ example $\log(e^{x-y} + e^{2y-x})$

$L_1 \dots L_n$ $f - L(x_1 \dots x_n) \to C$ as $L_1 \dots L_n \to \infty$

$g = n = 1$.
$S_{1,1}$



Counting Mod.X in $B_\gamma(L) \rightsquigarrow$ linear $F(\ell, \tau) \le L$.

Assume $F(\ell, \tau) = \ell$

Orbit counting for $F \rightsquigarrow$ orbit count for a cone $\ell_\gamma(X) \le L$

Mod(S) orbit in $\underset{\ell_X < L}{\mathrm{Teich}(S)}$ $\sim$ distribution of $\ell_X \le L$ in $M_{1,1}$ as $L \to \infty$

Distribution result $\rightsquigarrow$ Ergodicity of the earthquake
⊕ Counting integral pts in ML
Minsky – Weiss non-divergence of EF