

Title: A quantitative version of the commutator theorem for zero trace matrices

Abstract: As is well known, a complex $m \times m$ matrix A is a commutator (i.e., there are matrices B and C of the same dimensions as A such that $A = [B, C] = BC - CB$) if and only if A has zero trace. If $\|\cdot\|$ is the operator norm from ℓ_2^m to itself and $|\cdot|$ any ideal norm on $m \times m$ matrices then clearly for any A, B, C as above $|A| \leq 2\|B\|\|C\|$. Does the converse hold? That is, if A has zero trace are there $m \times m$ matrices B and C such that $A = [B, C]$ and $\|B\|\|C\| \leq K|A|$ for some absolute constant K ? If not, what is the behavior of the best K as a function of m ?

I'll talk mostly on a couple of years old result of Johnson, Ozawa and myself which gives some partial answers to this problem for the most interesting case of $|\cdot| = \|\cdot\|$. The solution is closely related in both directions to the best possible estimates in the Kadison–Singer problem. Time permitting I'll also speak on a more recent result for $|\cdot| =$ the Hilbert–Schmidt norm.