Kate Belin

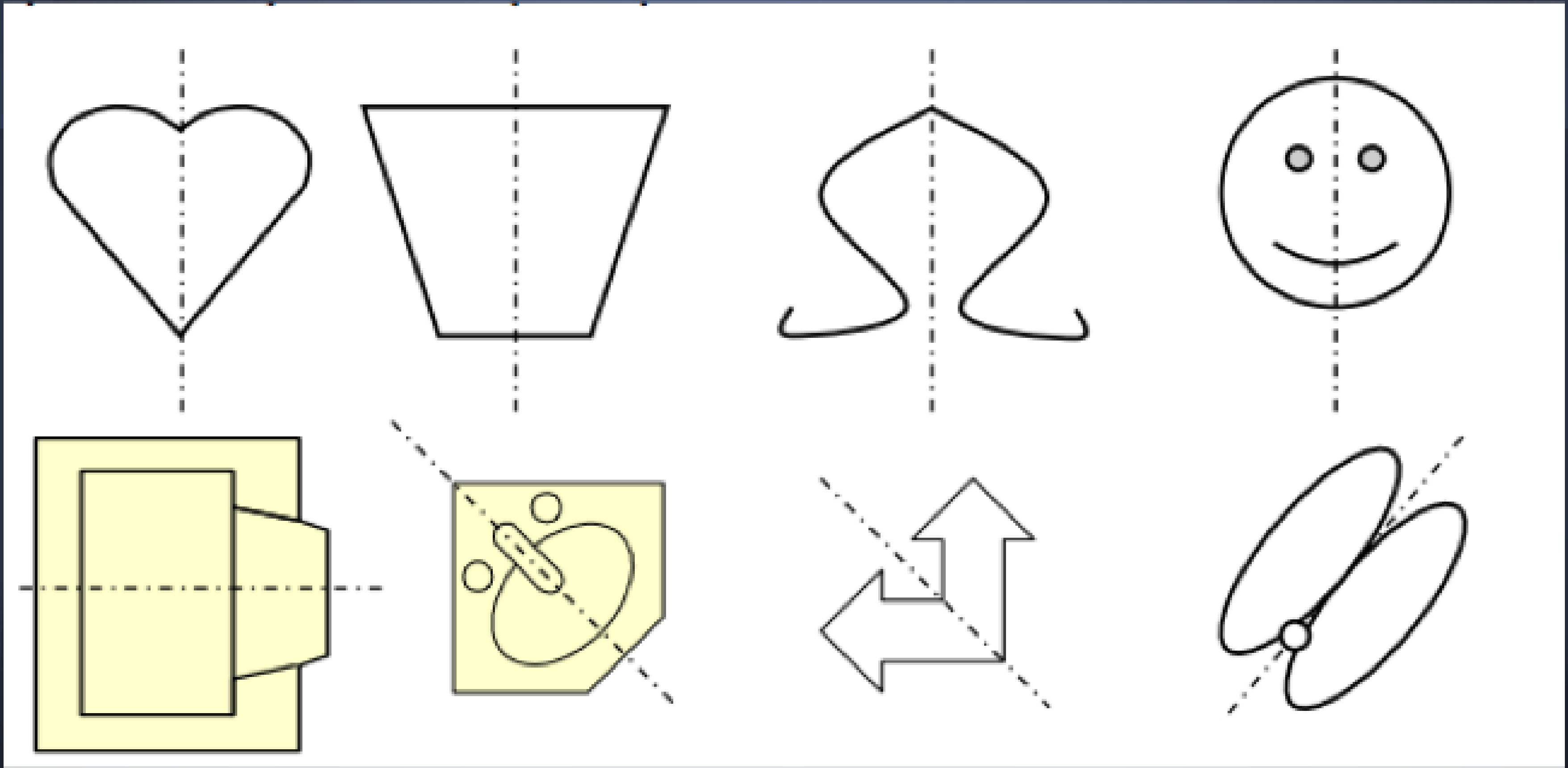
- Fannie Lou Hamer Freedom High School, Bronx, NY
- Math for America
- The Algebra Project

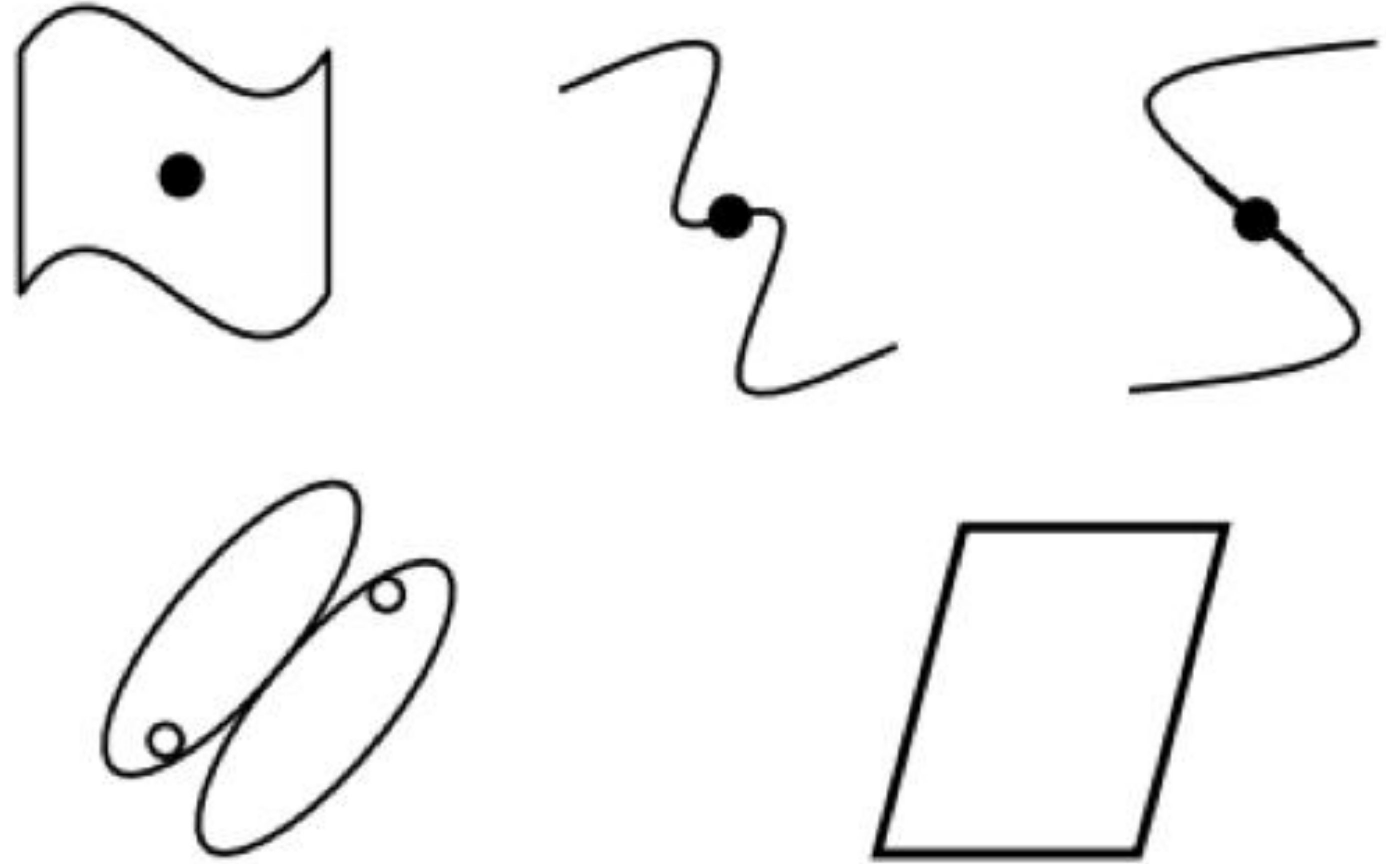
Fannie Lou Hamer Freedom High School

- Bronx, NY
- New York Performance Standards Consortium school
- 69% Hispanic, 28% Black
- 100% free lunch
- Under 500 students
- 51% graduates enroll in college
- 14% graduate with test scores high enough to enroll at CUNY without remedial help

Symmetry

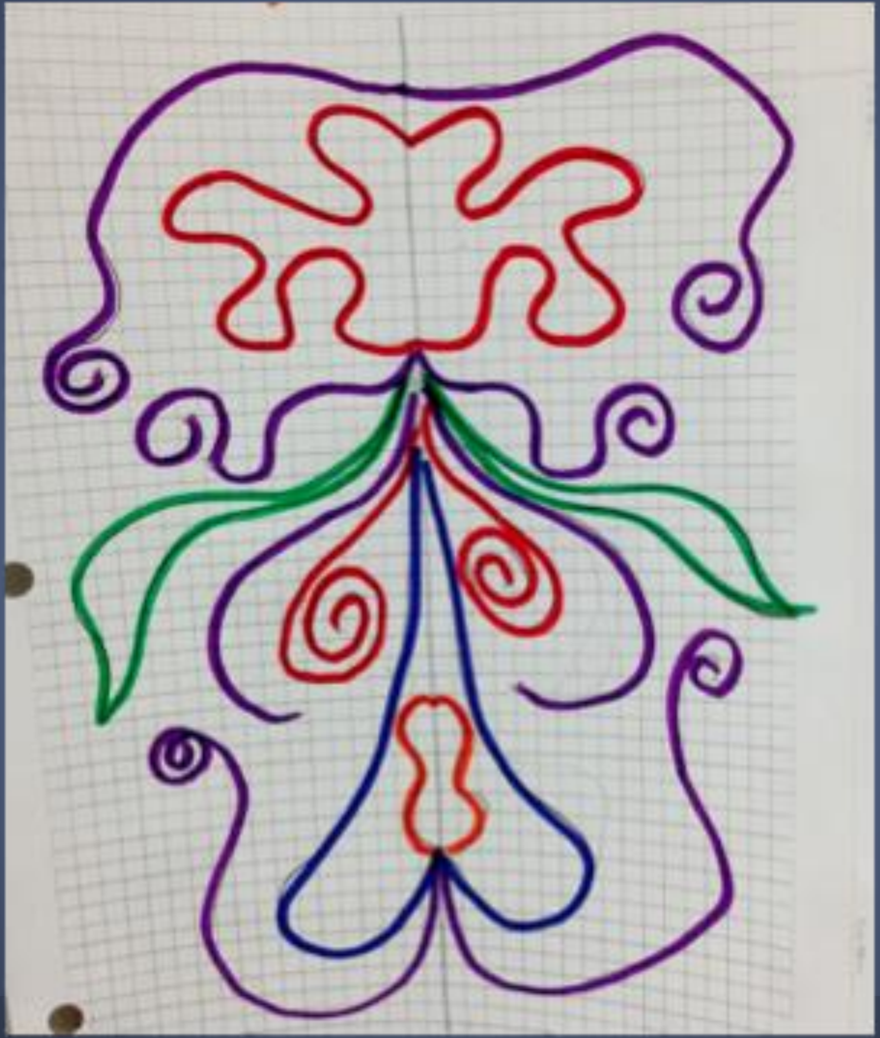
Identifying symmetry

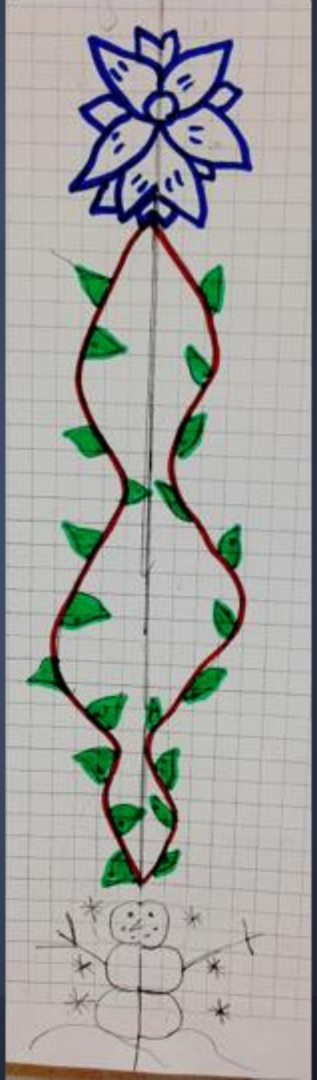





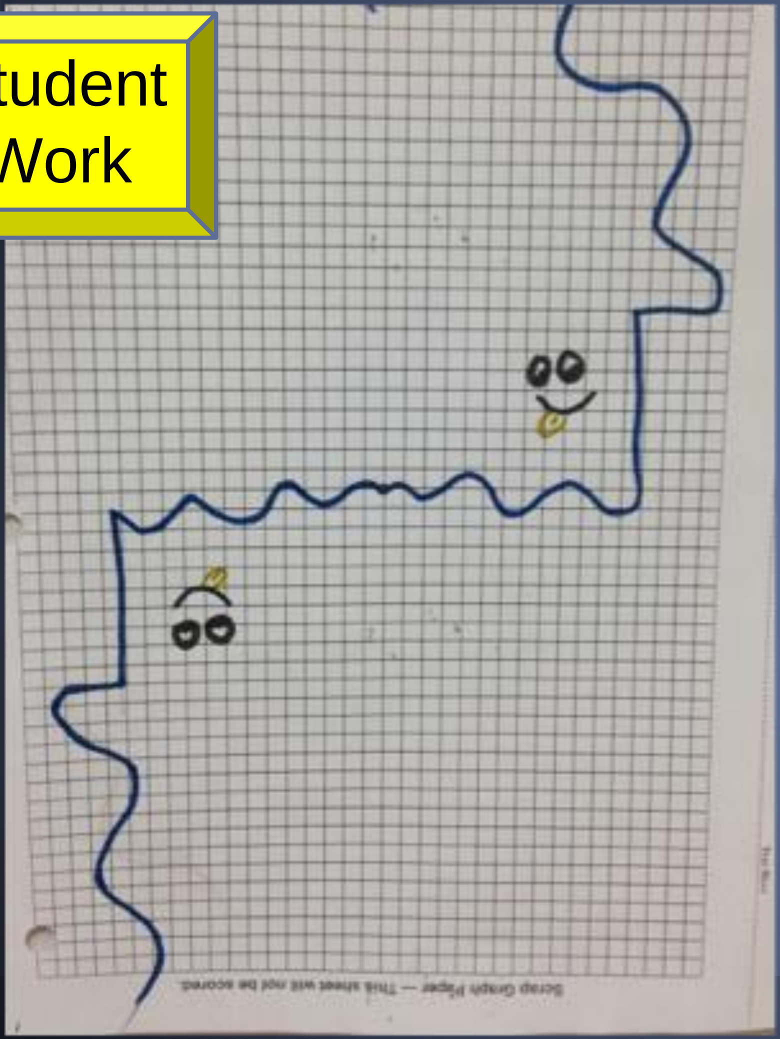
Creating symmetric figures

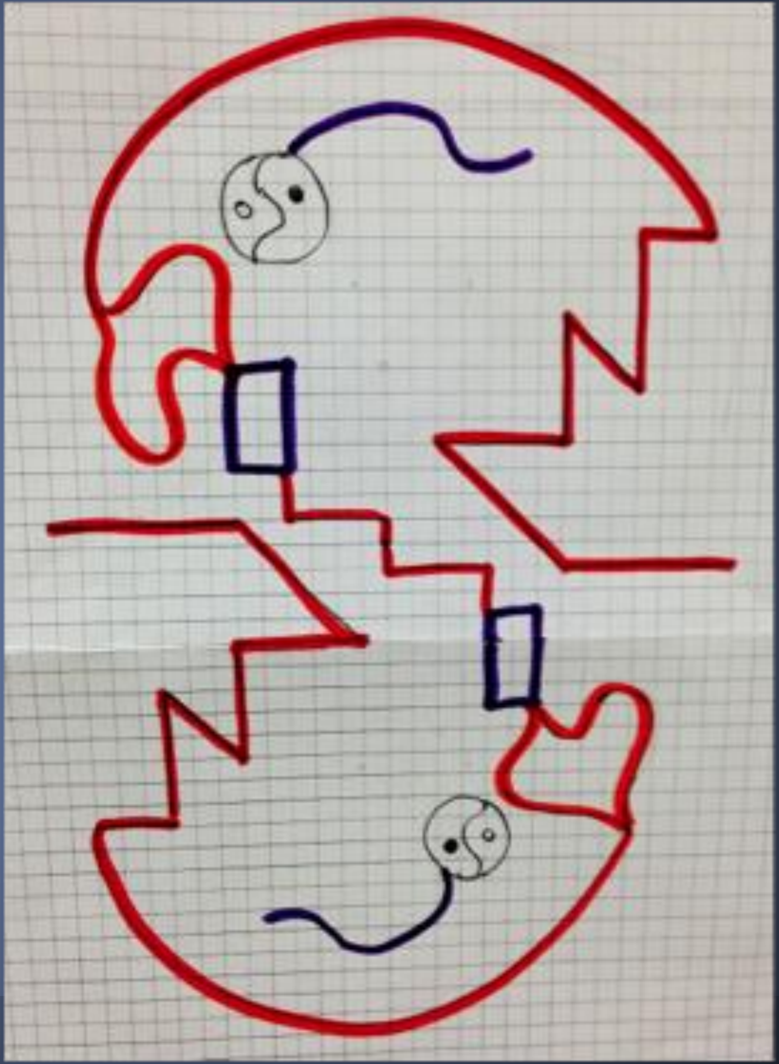
Student Work



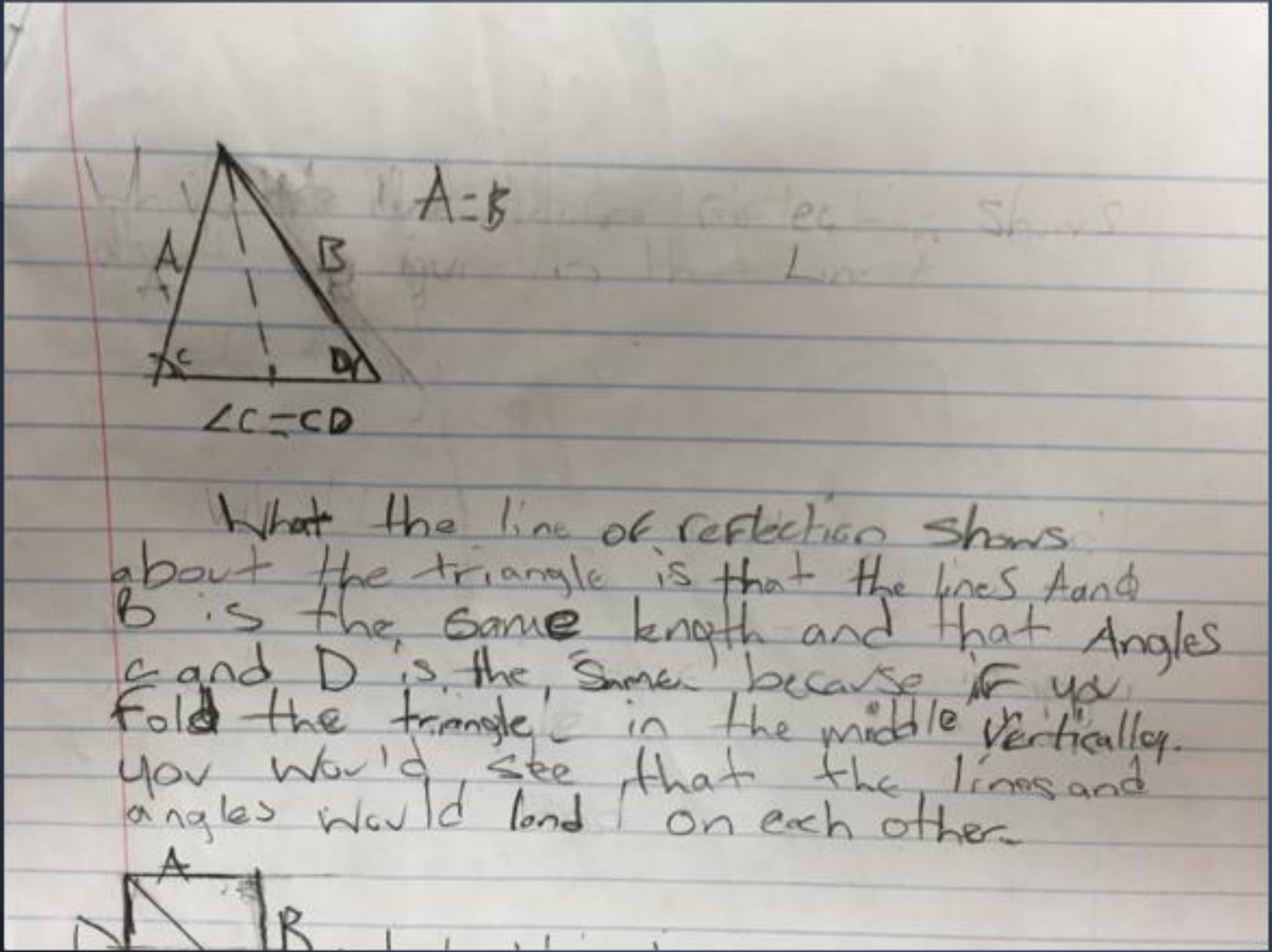


Student Work



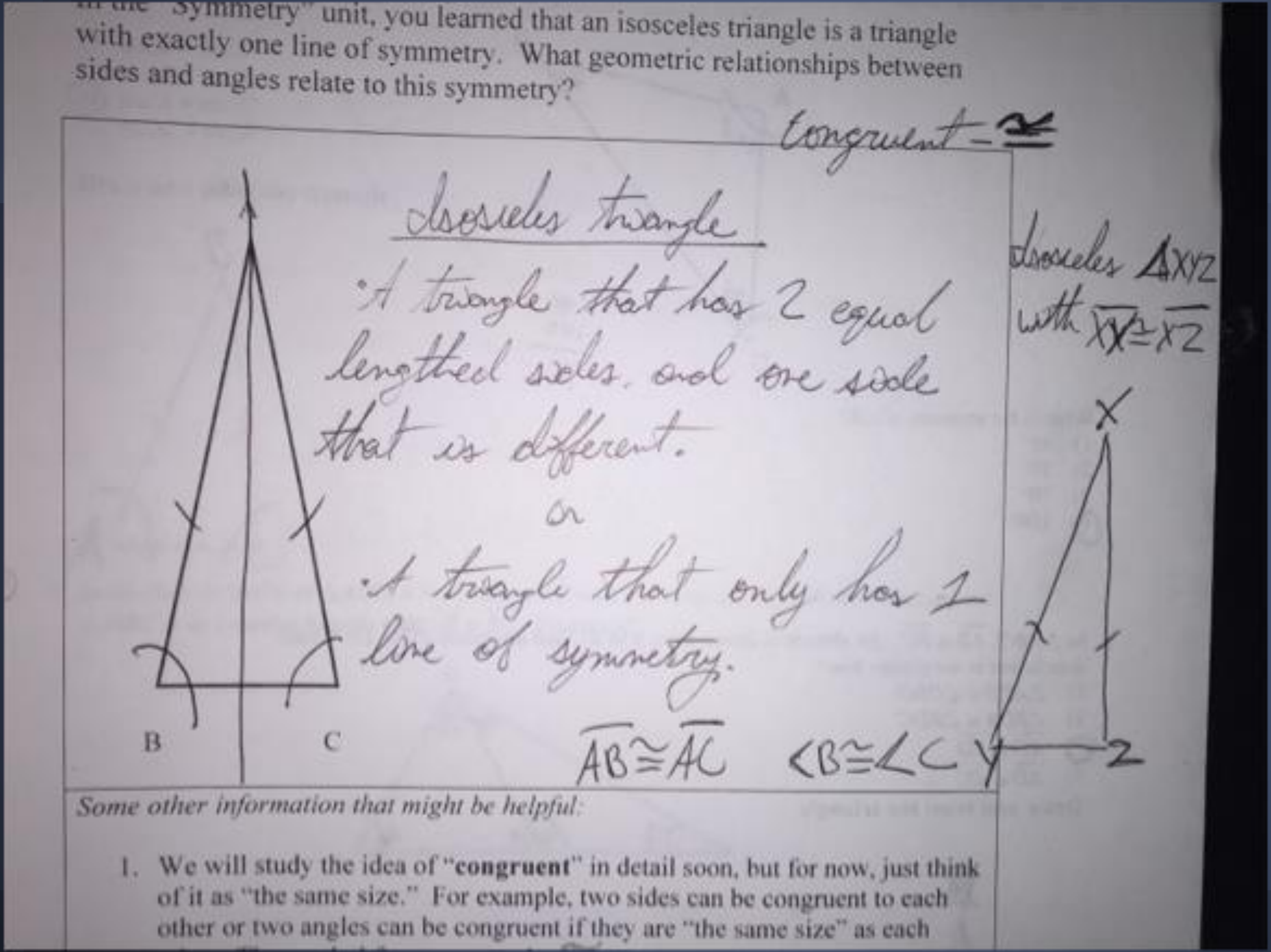


What symmetries tell us



Student Work

Student Work

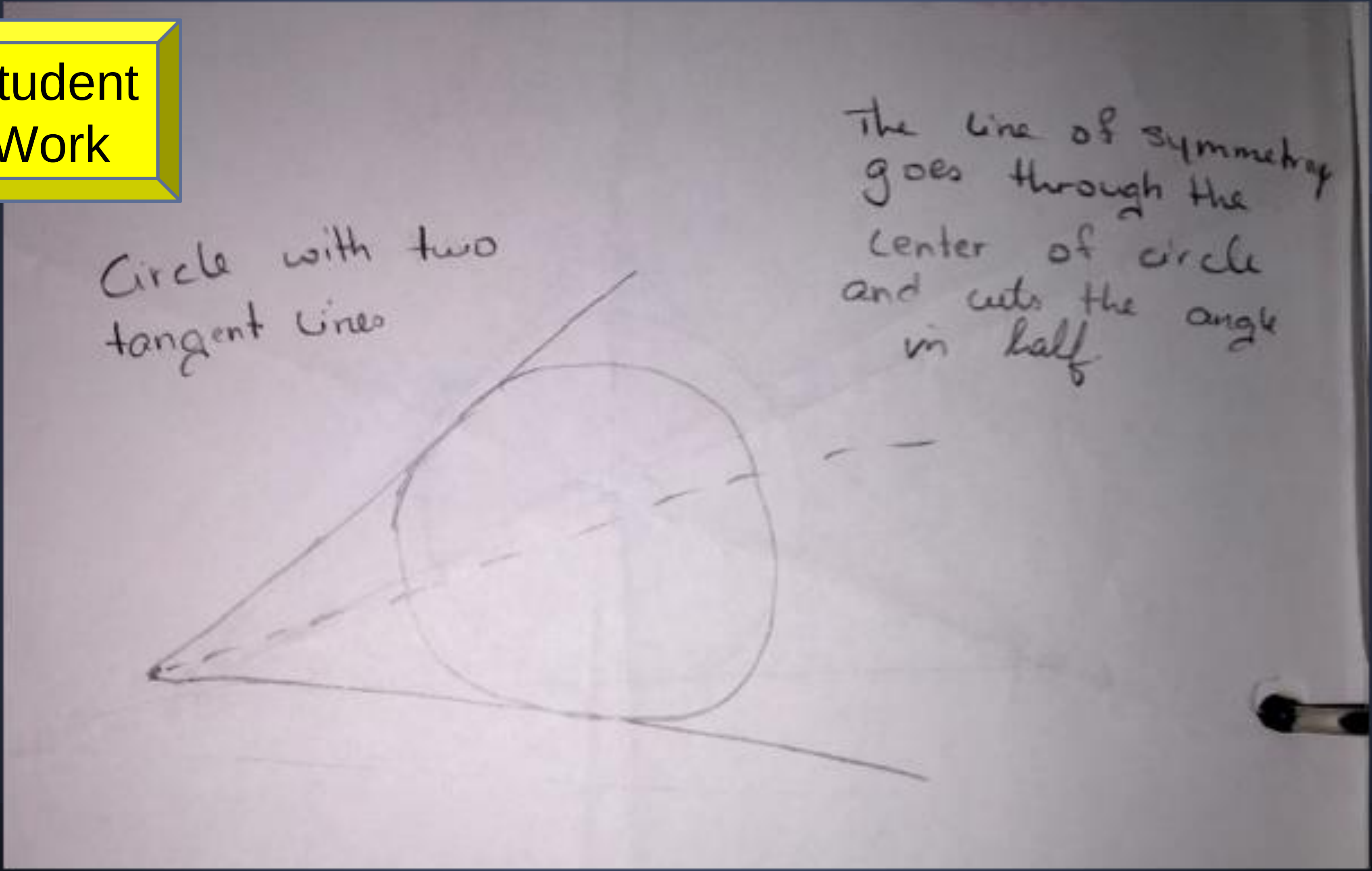


In the "Symmetry" unit, you learned that an isosceles triangle is a triangle with exactly one line of symmetry. What geometric relationships between sides and angles relate to this symmetry?

Some other information that might be helpful:

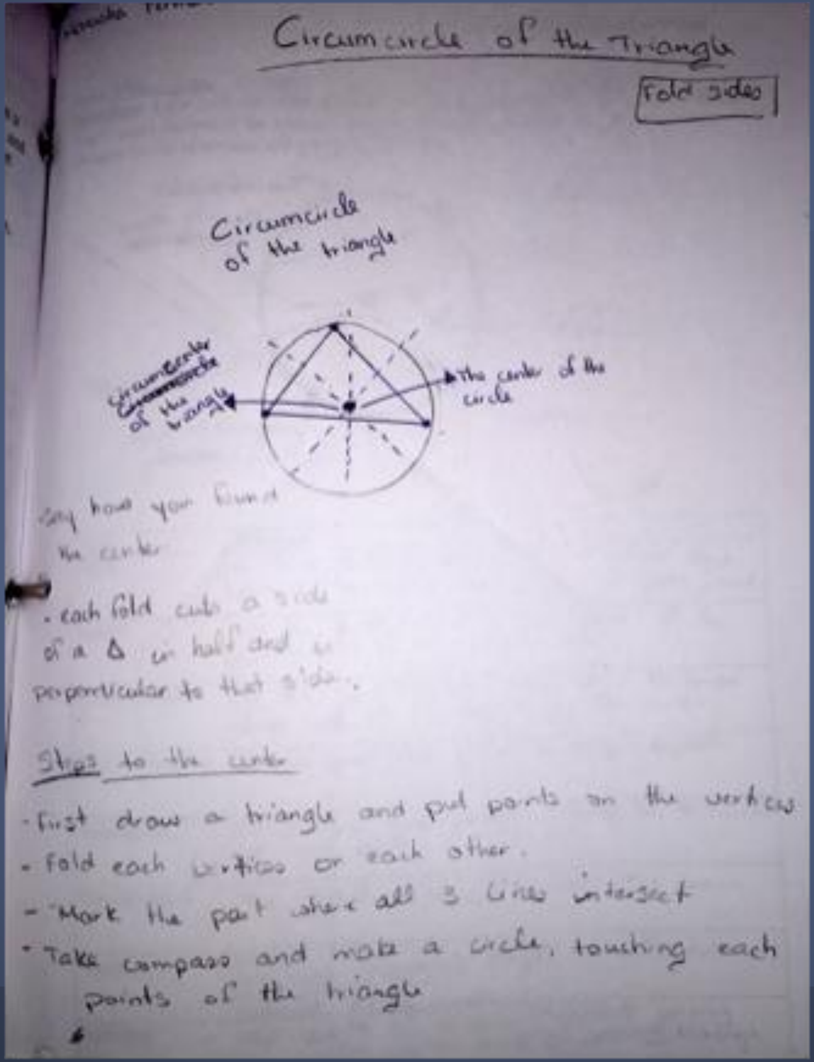
1. We will study the idea of "**congruent**" in detail soon, but for now, just think of it as "the same size." For example, two sides can be congruent to each other or two angles can be congruent if they are "the same size" as each

Student Work



Symmetry in geometric proof

Student Work



Circumcircle of the Triangle

Fold sides

Circumcircle of the triangle

Circumcenter of the triangle

The center of the circle

Say how you found the center

- each fold cuts a side of a Δ in half and is perpendicular to that side.

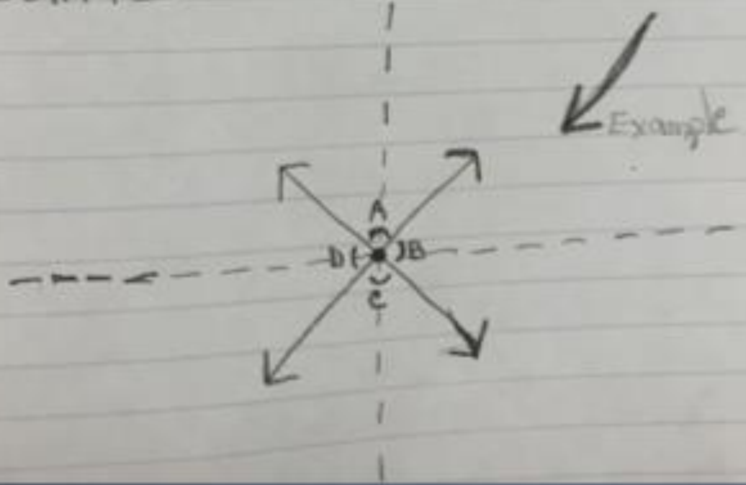
Steps to the center

- First draw a triangle and put points on the vertices
- Fold each vertices on each other.
- Mark the point where all 3 lines intersect
- Take compass and make a circle, touching each points of the triangle

Student Work

6. How can you use symmetries to prove geometric ideas?

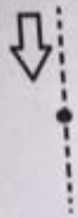
You can use symmetry to prove whether 2 angles are congruent so you can know which angles measure the same

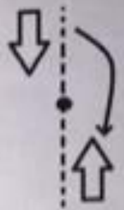


Student Work

t happens when you try to draw a figure that has half-turn & exactly one line of reflection symmetry?

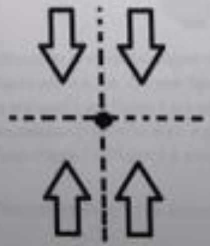
- Theorem: If a figure has half turn symmetry and reflection symmetry, then it must also have at least one more reflection.
- Proof: ***step 1:*** draw a dotted line with a dot in the center with any figure you like. ***Step 2:*** make a half turn.





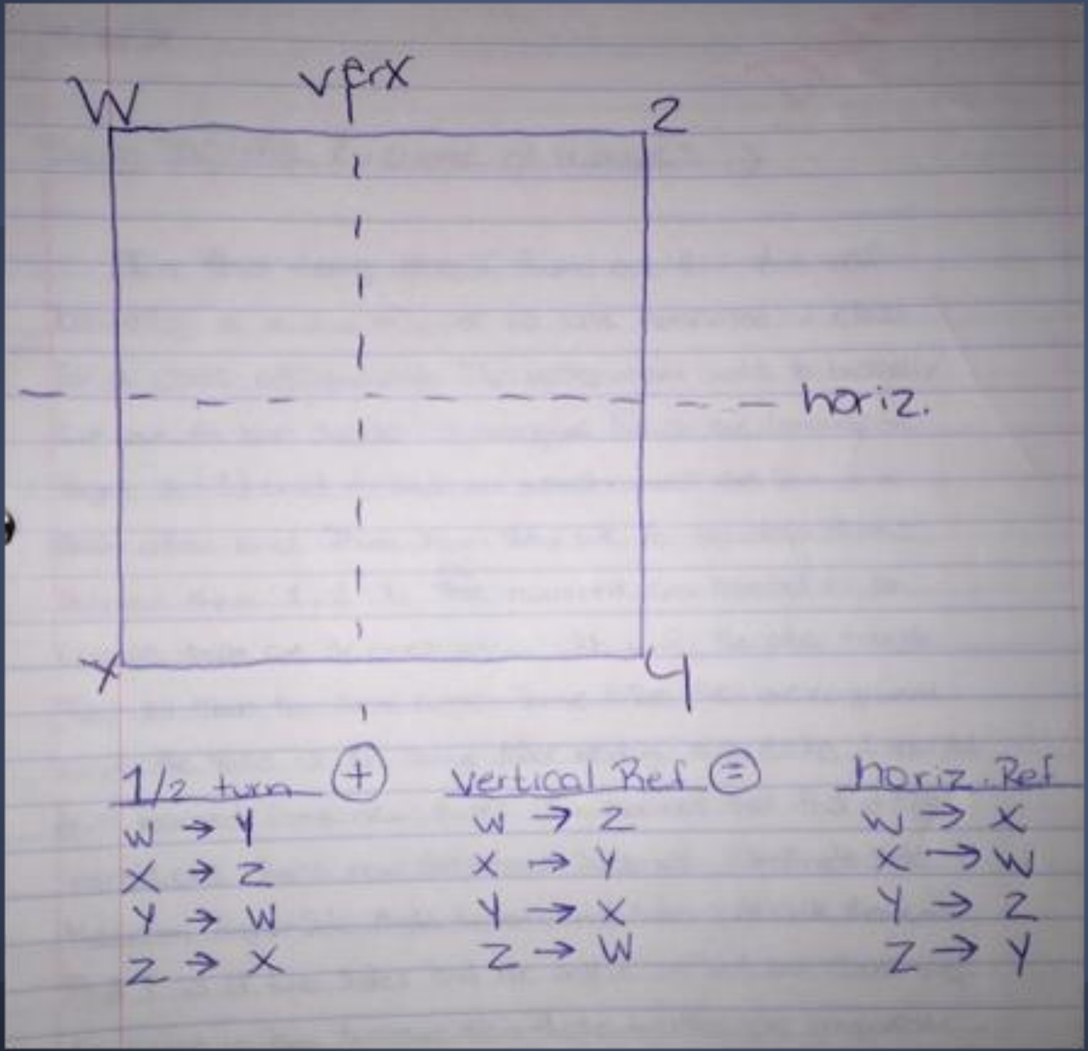
Step 3: Do reflection symmetry on the figure just drawn.

Step 4: do a half turn on the figure reflected.



<-you end up with a second line of symmetry

Groups: a binary operation



Student Work

Student Work

1st (dashed line) – vertical and 2nd (½ turn)

A → D → B => [A → B]

↑ ↑

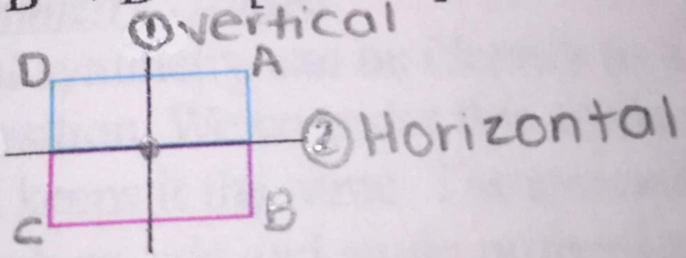
Vertical ½ turn

B → C → A => [B → A]

↳ ↳ ↳ ↳ ↑

Vertical Composition

C → B → D => [C → D]

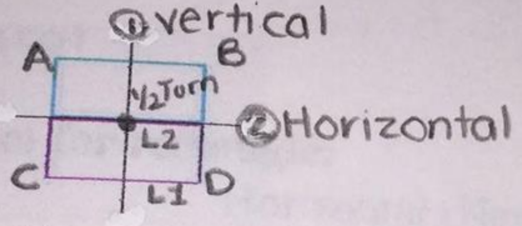


D → A → C => [D → C]

↑ **(Rectangle):**

by the ½ turn about the dot, takes A → C, B → D, C → A, and D → B (L2 – new symmetr horizontal).

Rectangle: ↓



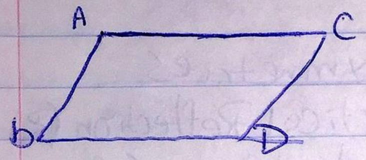
Student Work

L1 (vertical):	½ turn (midpoint center):	L2 (horizontal ref.)
A → B	A → D	A → C
B → A	B → C	B → D
C → D	C → B	C → A
D → C	D → A	D → B
—	—	*(new ref.)* ↑

***Conclusion:** Therefore, this theorem is true because if you have one reflection on a f contains a ½ turn, then you will always find a new reflection on the figure. **End of proof**

Symmetry Group - Parallelogram

Parallelogram



Symmetries

1/2 turn (S 1/2)
Identity (I)

Arrow Diagram

1/2 turn (S 1/2)
A - B
B - A
C - D

Identity (I)
A - A
B - B
C - C
D - D

Multiplication Table

x	S 1/2	I
I	S 1/2	I
S 1/2	I	S 1/2

Student Work

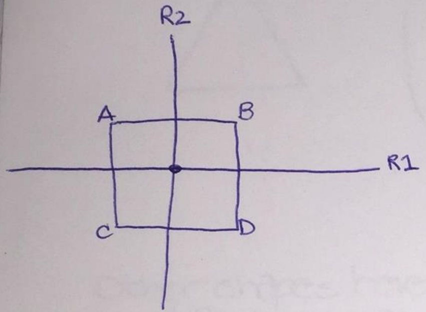
Symmetry Group - Rectangle

Student Work

DO 1st ↓ DO 2nd →	rH	RV	R 1/2	I
RH	I	R 1/2	RV	RH
RV	R 1/2	I	RH	RV
R 1/2	RV	RH	I	R 1/2
I	RH	RV	R 1/2	I

Student Work

Explain: Symmetry groups can be displayed through letters and how many different symmetries can be found in the letters. The way you find the different symmetries is by using a multiplication table.



Identity - The letters stay the same

A → A
B → B
C → C
D → D

•	R1	R2	S1/2	I
R1	I	S1/2	R2	R1
R2	S1/2	I	R1	R2
S1/2	R2	R1	I	S1/2
I	R1	R2	S1/2	I

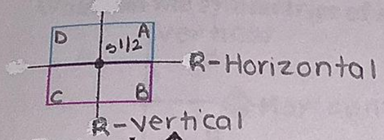
R1 Reflection

A → C
B → D
C → A
D → B

R2 Reflection

A → C
B → D
C → A
D → B

Student Work



Rectangle: ↑

Arrow diagram for rectangle:

Vertical	Horizontal (New)	Half-turn	Identity
<u>R_1</u> A → B B → A C → D D → C	<u>R_2</u> A → C B → D C → A D → B	<u>S½</u> A → D B → C C → B D → A	<u>I</u> A → A B → B C → C D → D

Multiplication table for rectangle:

x	R_1	R_2	S½	I
R_1	I	S½	R_2	R_1
R_2	S½	I	R_1	R_2
S½	R_2	R_1	I	S½
I	R_1	R_2	S½	I

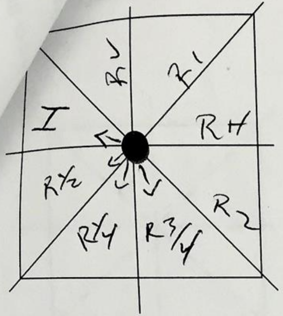
Symmetries of a rectangle:

The table above is an example of a **commutative property of multiplication**! **Commutative property of multiplication** means that when we multiply rational numbers, we assume that we can change the order of the factors without changing the product. For instance, formula: axb = bxa and with numbers this will be like: 5x4 = 4x5.

½ turn symmetry
Identity (trivial – full-turn symmetry)
Line of reflection, vertically
Line of reflection, horizontally → Total of 4 symmetries, but specifically 2 for now.
Line of reflection diagonally (2)

Symmetry Group - Square

Student Work



Rv > vertical reflection

Rh > horizontal reflection

I > full-turn symmetry

R1/2 > half-turn symmetry

R1 > 1 reflection

R2 > 2 reflection

R1/4 > 1/4-turn symmetry

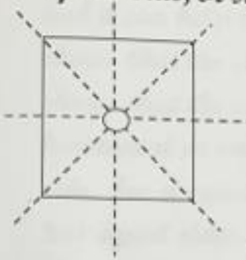
R3/4 > 3/4-turn symmetry

1st ↓ / 2nd →	Rv	Rh	R1	R2	R1/4	R1/2	R3/4	I
Rv	I	R1/2	R1/4	R3/4	R1	RH	R2	Rv
Rh	R1/2	I	R3/4	R1/4	R2	R1	Rv	RH
R1	R1/4	R1	I	R1/2	RH	Rv	R2	R1
R2	R3/4	R2	R1/2	I	Rv	RH	R1	R2
R1/4	RH	R1	R2	Rv	I	R3/4	R3/4	R1/4
R1/2	R2	R3/4	Rv	RH	R1	I	R1/4	R3/4
R3/4	R1/2	Rv	RH	R1	R2	R1/4	I	R3/4
I	Rv	RH	R1	R2	R1/4	R1/2	R3/4	I

This graph is to show the relationship Between the reflections and the ~~turn~~ turns (symmetry) on a square.

Student Work

6. Use a multiplication table to describe the symmetry group for an equilateral, or square.



Half Turn = $R_{1/2}$ {◯}

Vertical Reflection = R_v

Horizontal Reflection = R_h

Diagonal = D_1

Diagonal = D_2

1 Quarter Turn = ¼

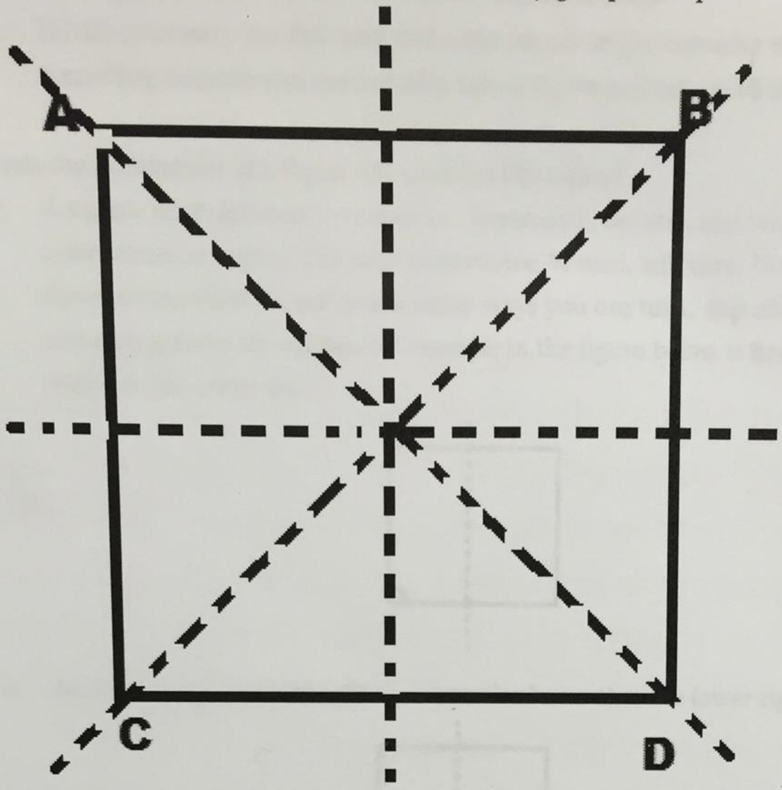
3rd Quarter Turn = ¾

Full Turn = $\mathcal{L}$

	$R_{1/2}$	R_v	R_h	D_1	D_2	¼	¾	$\mathcal{L}$
$R_{1/2}$	$\mathcal{L}$	R_h	R_v	D_2	D_1	¾	¼	$R_{1/2}$
R_v	R_h	$\mathcal{L}$	$R_{1/2}$	¼	¾	D_1	D_2	R_v
R_h	R_v	$R_{1/2}$	$\mathcal{L}$	¾	¼	D_2	D_1	R_h
D_1	D_2	¾	¼	$\mathcal{L}$	R_h	R_v	$R_{1/2}$	D_1
D_2	D_1	¼	¾	R_v	$\mathcal{L}$	$R_{1/2}$	R_v	D_2
¼	¾	D_1	D_2	R_h	$R_{1/2}$	$R_{1/2}$	R	¼
¾	¼	D_2	D_1	$R_{1/2}$	R_v	R_h	$R_{1/2}$	¾
$\mathcal{L}$	$R_{1/2}$	R_v	R_h	D_1	D_2	¼	¾	$\mathcal{L}$

Student Work

❖ Using a multiplication table to describe the symmetry group of a square.



Rv:	*R2:*	*R3/4:*	*I:*
A-B	A-A	A-B	A-A
B-A	B-C	B-D	B-B
C-D	C-B	C-A	C-C
D-C	D-D	D-C	D-D

KEY :

R_H–Horizontal Reflection.

R_v –*Verticle Reflection.*

R_1-*Diagonal Reflection (A-D).*

R_2-*Diagonal Reflection (B-C).*

$R_{1/4}$-*1/4 turn.*

$R_{2/4}$-*Half turn.*

$R_{3/4}$- *¾ Turn.*

I : full turn.

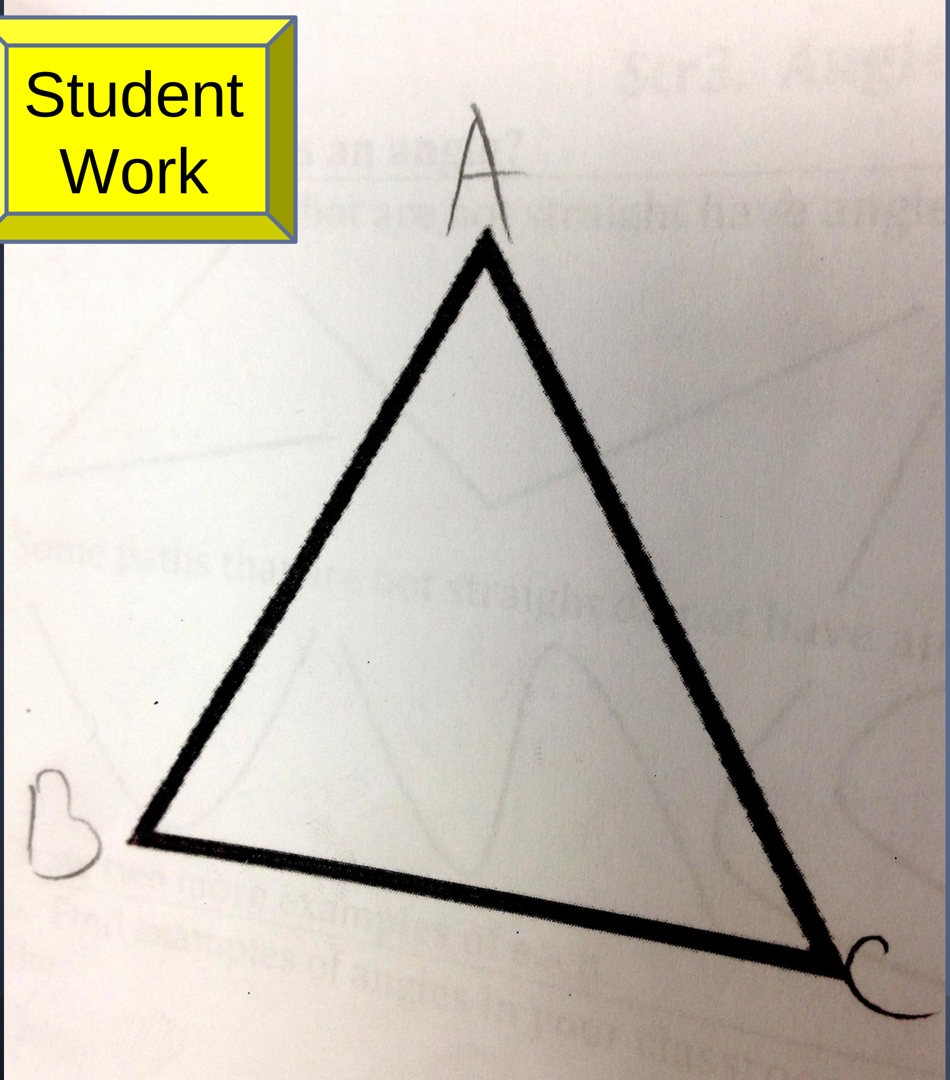
R_H:	R_1:	$R_{1/4}$:	$R_{2/4}$:
A-C	A-D	A-C	A-D
B-D	B-B	B-A	B-C
C-A	C-C	C-D	C-B
D-B	D-A	D-B	D-A

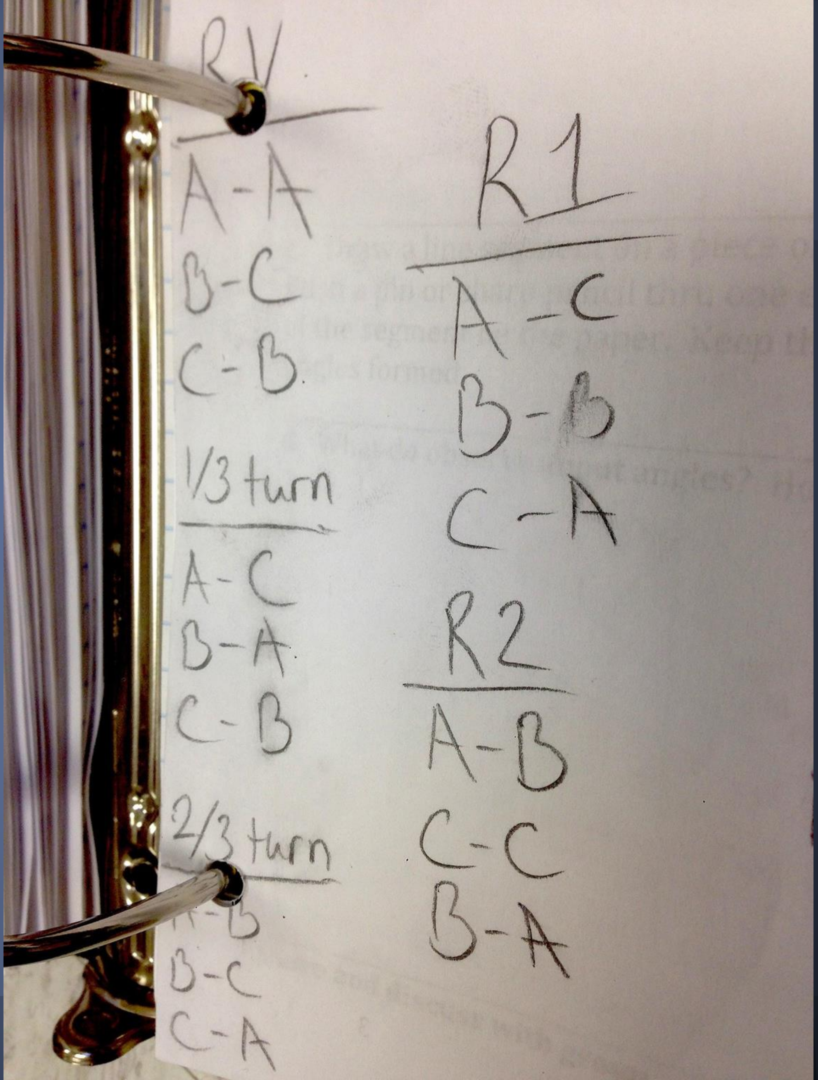
X	R_H	R_v	R_1	R_2	$R_{1/4}$	$R_{2/4}$	$R_{3/4}$	I
R_H	I	$R_{2/4}$	$R_{3/4}$	$R_{1/4}$	R_2	R_V	R_1	R_H
R_v	$R_{2/4}$	I	$R_{1/4}$	$R_{3/4}$	R_1	R_H	R_2	R_V
R_1	$R_{1/4}$	$R_{3/4}$	I	$R_{2/4}$	R_H	R_2	R_V	R_1
R_2	$R_{3/4}$	$R_{1/4}$	$R_{2/4}$	I	R_V	R_1	R_H	R_2
$R_{1/4}$	R_1	R_2	R_V	R_H	$R_{2/4}$	$R_{3/4}$	I	$R_{1/4}$
$R_{2/4}$	R_V	R_H	R_2	R_1	$R_{3/4}$	I	$R_{1/4}$	$R_{2/4}$
$R_{3/4}$	R_2	R_1	R_H	R_V	I	$R_{1/4}$	$R_{2/4}$	$R_{3/4}$
I	R_H	R_V	R_1	R_2	$R_{1/4}$	$R_{2/4}$	$R_{3/4}$	I

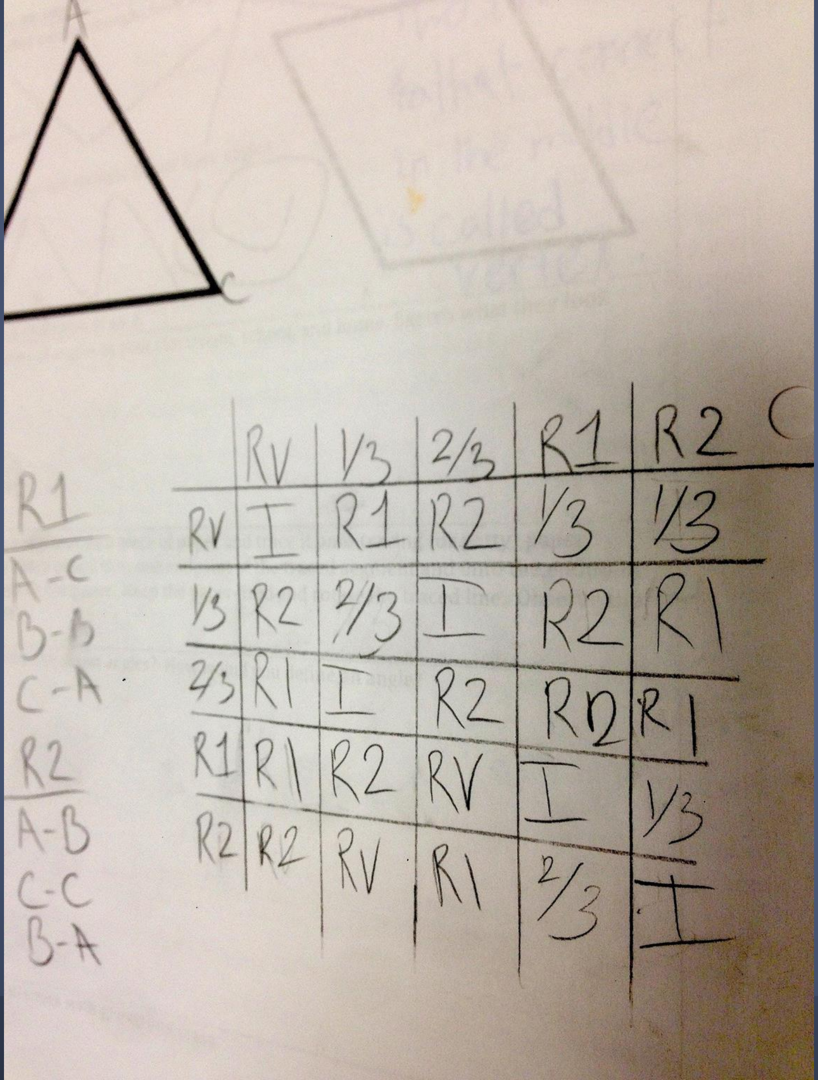
Student Work

Symmetry Group - Equilateral Triangle

Student Work





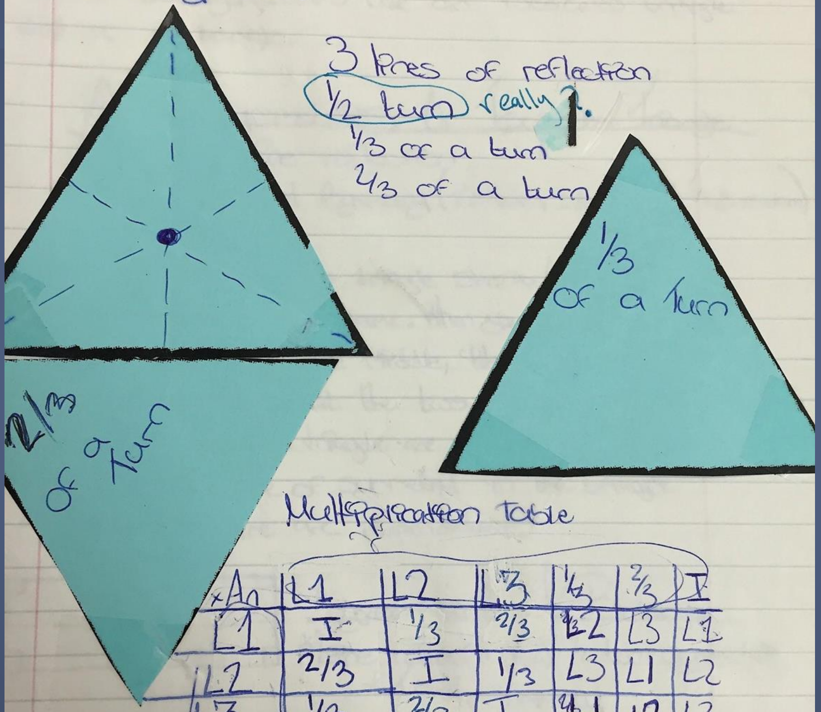


Student Work

Student Work

Symmetrical group is the number of symmetries in a group. An equilateral triangle has 6 symmetries

3 lines of reflection
1/2 turn really?!
1/3 of a turn
2/3 of a turn



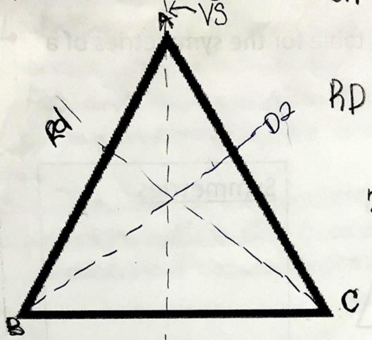
Multiplication Table

×An	L1	L2	L3	1/3	2/3	I
L1	I	1/3	2/3	L2	L3	L1
L2	2/3	I	1/3	L3	L1	L2
L3	1/3	2/3	I	L1	L2	L3
1/3	L3	L1	L2	2/3	I	1/3
2/3	L2	L3	2/3	I	1/3	2/3
I	L1	L2	L3	1/3	2/3	I

Student Work

Classwork (Symmetry)

Equilateral Triangle



Full turn = A=A
B=B
C–C

1/3 = A-C
B-A
C-B

RD – A-B
B–A
C–C

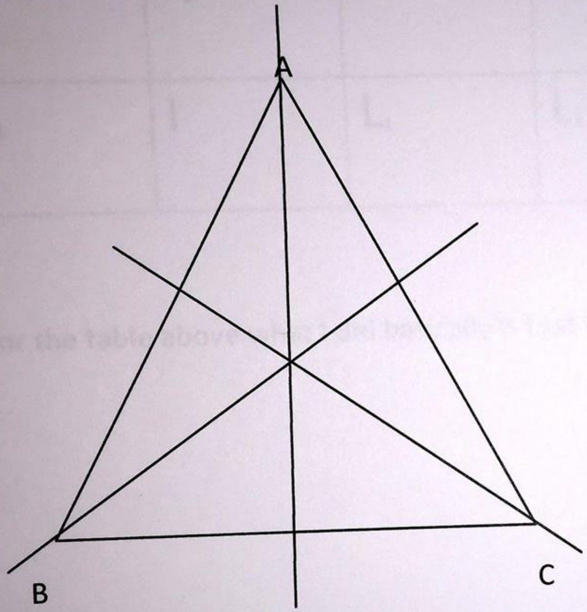
VS = A-A
B–C
C–B

2/3 = A-B
B-C
C-A

D^2 = A–C
B–B
C–A

	1/3	Vertical symmetry	Full turn	Reflection Diagonal	D2	2/3
1/3	2/3	D^2	1/3	VS	RD	Full turn
Verticle symmetry	D^2	Full turn	VS	1/3	1/3	RD
Full turn	1/3	VS	Full turn	RD	D^2	2/3
Reflection Diagonal	VS	2/3	Rd	Full turn	1/3	D2
D^2	Rd	1/3	D^2	2/3	Full turn	VS
2/3	Full turn	Rd	2/3	D^2	VS	Full turn

symmetry of something which is much more abstract. This type of symmetry is actually very useful to many people.



Symmetries:

$R_{1/3}$ – 1/3 turn clockwise

$R_{2/3}$ – 2/3 turn clockwise

I – Nothing (same as 3/3 turn)

L_1 – point A going straight down the middle

L_2 – point b going across/up to the right

L_3 – point c going across/up to the left

Student Work

Student Work

	$R_{1/3}$	$R_{2/3}$	I	L_1	L_2	L_3
$R_{1/3}$	$R_{2/3}$	I	$R_{1/3}$	L_3	L_1	L_2
$R_{2/3}$	I	$R_{1/3}$	$R_{2/3}$	L_2	L_3	L_1
I	$R_{1/3}$	$R_{2/3}$	I	L_1	L_2	L_3
L_1	L_2	L_3	L_1	I	$R_{1/3}$	$R_{2/3}$
L_2	L_3	I	L_2	$R_{2/3}$	I	$R_{1/3}$
L_3	I	L_1	L_3	$R_{1/3}$	$R_{2/3}$	I

For the table above what I did basically is that I added $R_{1/3}$ to $R_{2/3}$ and I got $R_{3/3}$.

Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Are my students ready for college mathematics?