Mathematics Sciences Research Institute • March 19, 2015

Critical Issues in Mathematics Education Workshop

Developmental Mathematics: For Whom? Toward What Ends?

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A VIEW FROM BELOW: (WHAT) COULD WORK ON ELEMENTARY TEACHING CONTRIBUTE TO THE CONVERSATION ABOUT DEVELOPMENTAL MATHEMATICS AT THE COLLEGE LEVEL?

SCHOOL OF EDUCATION
UNIVERSITY OF MICHIGAN

FROM THE MORNING

- Transformation in mathematics classrooms: expectations, culture, norms
- “risk factors”
- Coming to see the complexity of whether, why, and how students end up seeming to be “underprepared for college”
- Filling in potholes while advancing the road
- The importance of perseverance and developing students’ sense of purpose and efficacy

EDUCATION

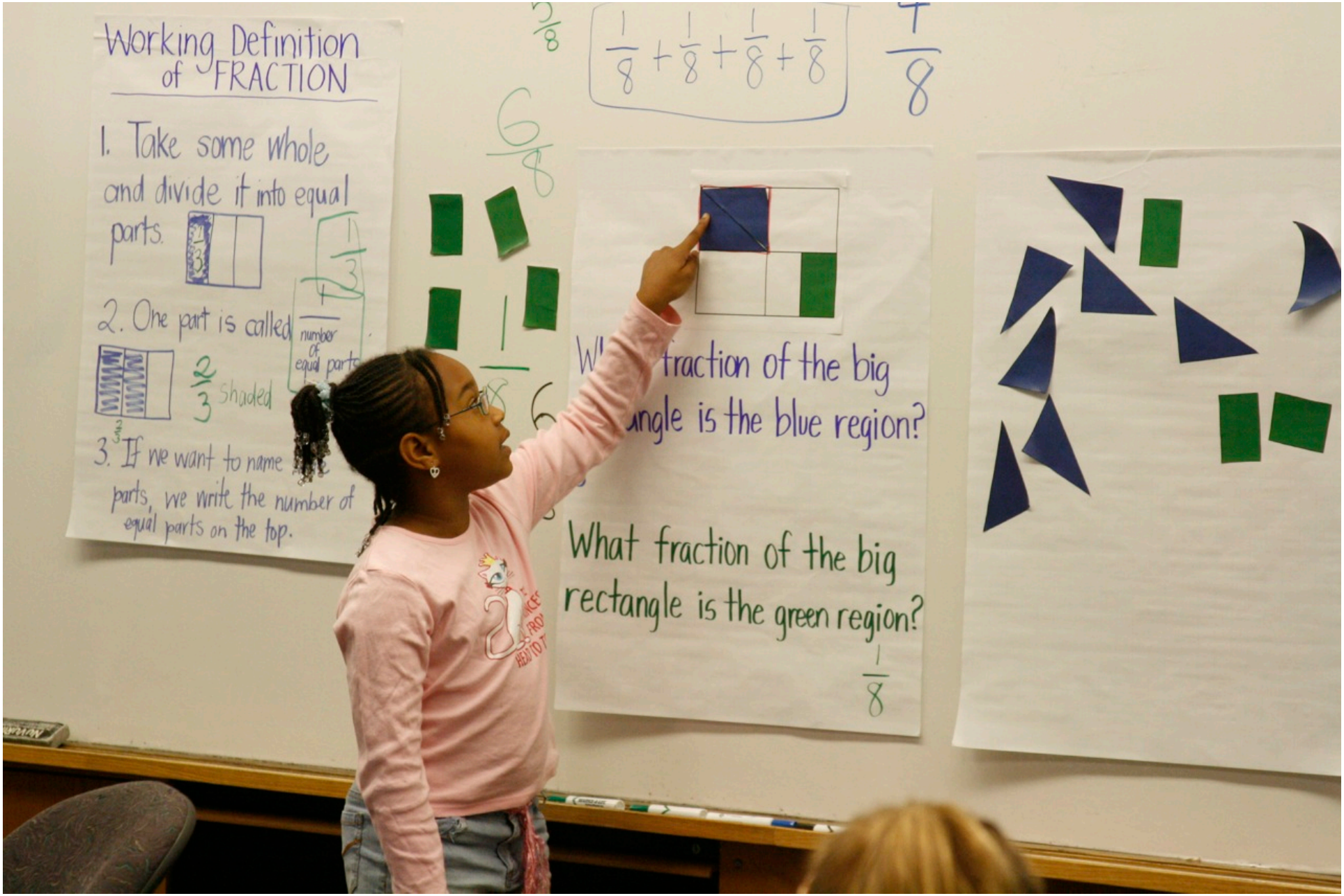
WHAT DO WE HAVE TO DO WITH IT?

- How we see and hear students
- Who we are and how that shapes what we can see and hear
- What students hear and experience inside of classrooms
- The work of teaching in making access to challenging ("sophisticated"*) mathematics real

(*Wilkes, in preparation)

EDUCATION

EDUCATION


Working Definition of FRACTION
1. Take some whole and divide it into equal parts.
2. One part is called
number of equal parts
shaded
3. If we want to name parts, we write the number of equal parts on the top.
What fraction of the big rectangle is the green region?
1/8

EDUCATION

Why does the daily homework include “looking ahead” hard problems that we haven’t yet learned how to do?

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Can someone tell the class- What are the looking ahead problems supposed to be for?

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When your students don’t learn as expected, what do you find are typically the reasons?

EDUCATION

WHAT DO TEACHERS THINK WHEN STUDENTS STRUGGLE?

UNPRODUCTIVE

- Students aren't trying hard enough.
- Students lack mathematical ability.

"These kids are used to be[ing] spoon-fed and they'll sit there and say, 'I don't get it.'…[U]ntil you actually sit down and show them step [by] step how to do that problem, they don't get it. They don't know how to think."

PRODUCTIVE

- There might be something in the way this was taught.
- I could support the work better.

"I normally look first at me to see or is there something in the lesson that I didn't emphasize well enough or…I may talk to the teacher they had last year and say 'When you went over this was this something that they struggled with?

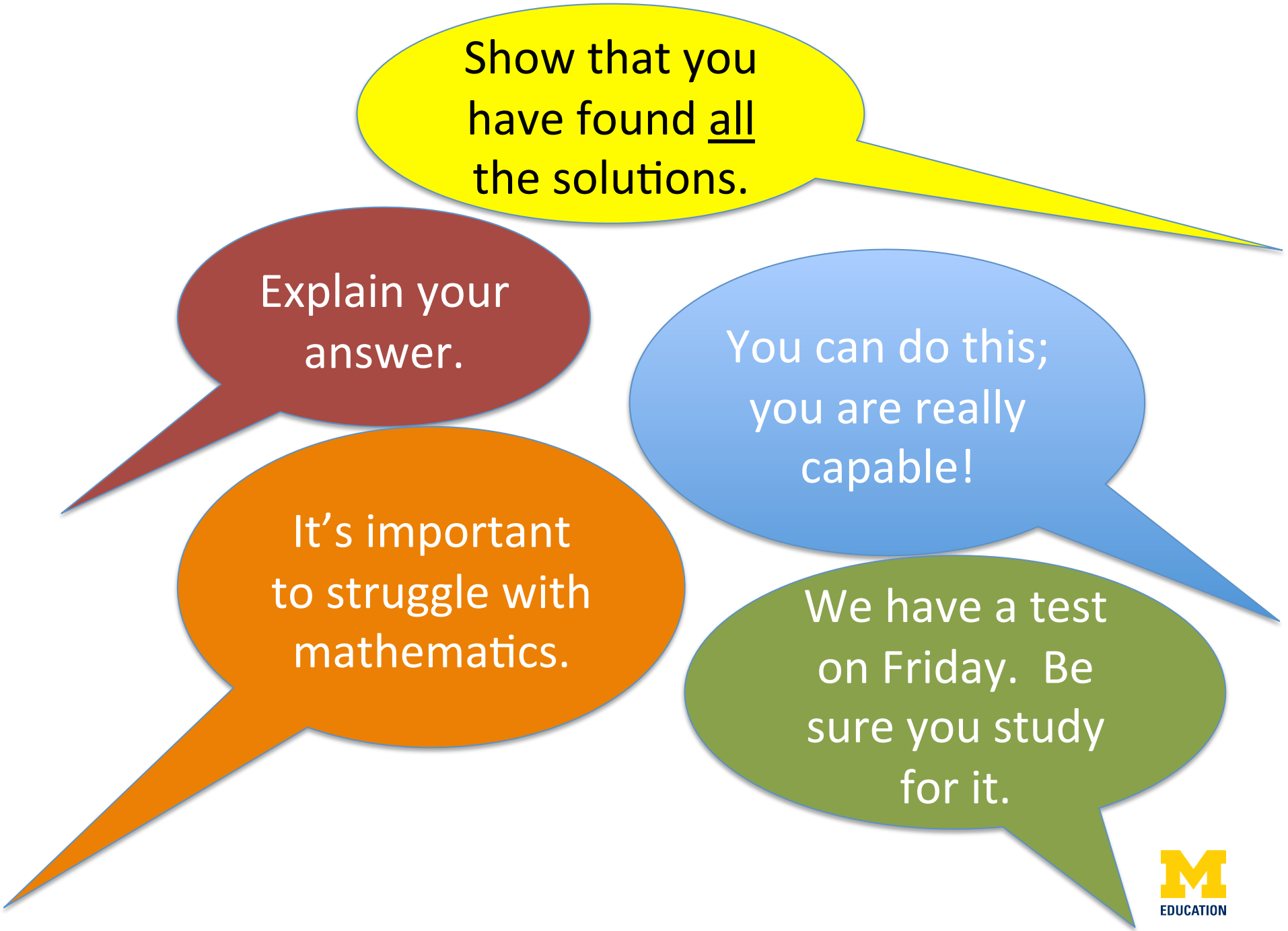
Jackson & Gibbons, 2014

EDUCATION

A CORE CHALLENGE OF TEACHING

- Teaching complex mathematical knowledge and skill
- Teaching all students

What does this involve?


Show that you have found all the solutions.
Explain your answer.
You can do this; you are really capable!
It's important to struggle with mathematics.
We have a test on Friday. Be sure you study for it.
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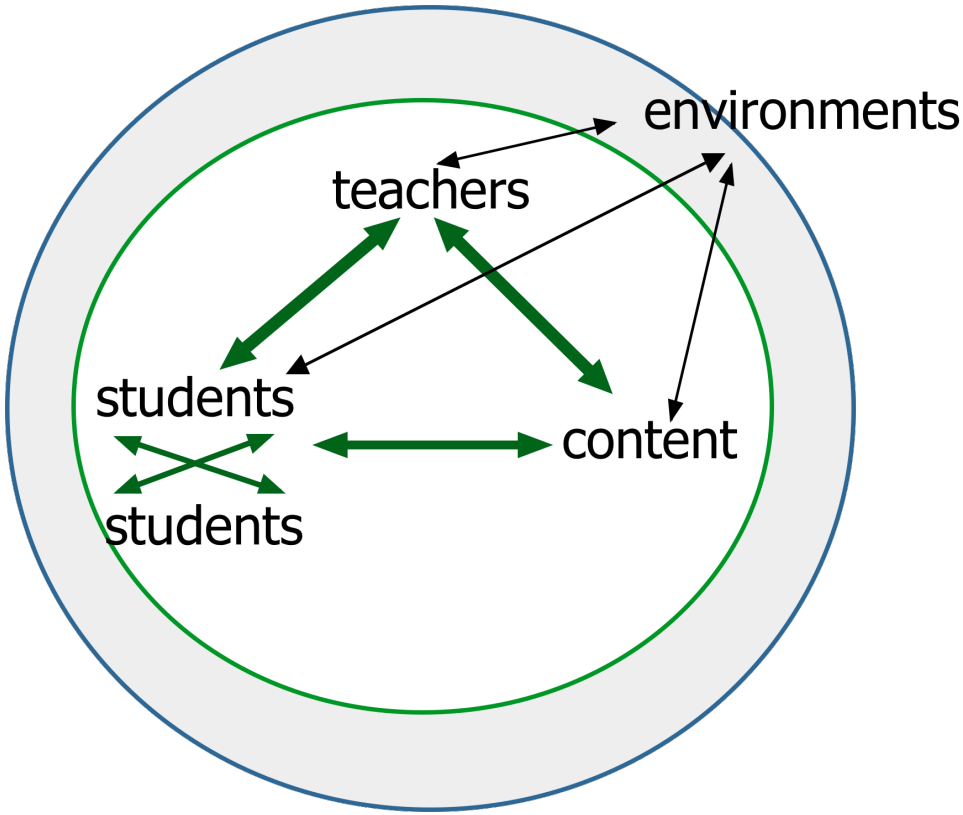
WHAT DOES IT TAKE TO HELP STUDENTS SUCCEED IN MATHEMATICS?
WHAT IS MATHEMATICS INSTRUCTION?



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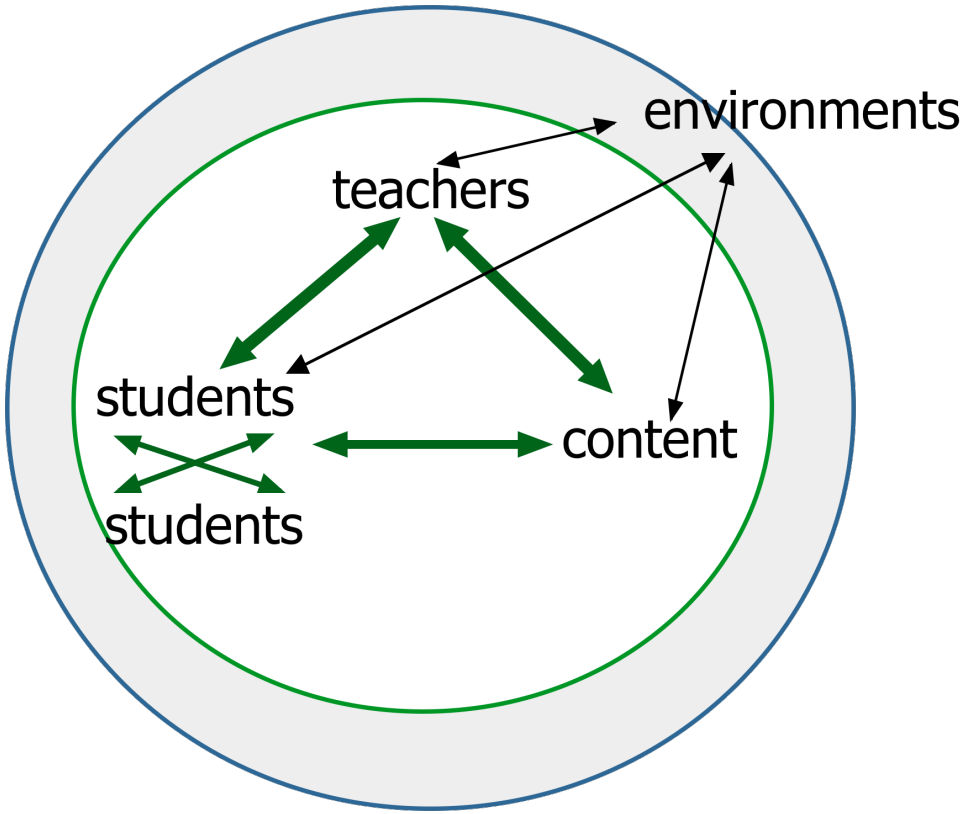
WHAT IS "TEACHING"?



Takes responsibility for:

- deliberately maximizing the quality of the interactions . . .
- . . . in ways that maximize the probability that students learn
- . . . worthwhile content and skills

WARM-UP PROBLEM

How many different three-digit numbers can you make using the digits 1, 2, and 3, using each digit exactly once?

Show all the three-digit numbers you found.

How do you know that you found them all?

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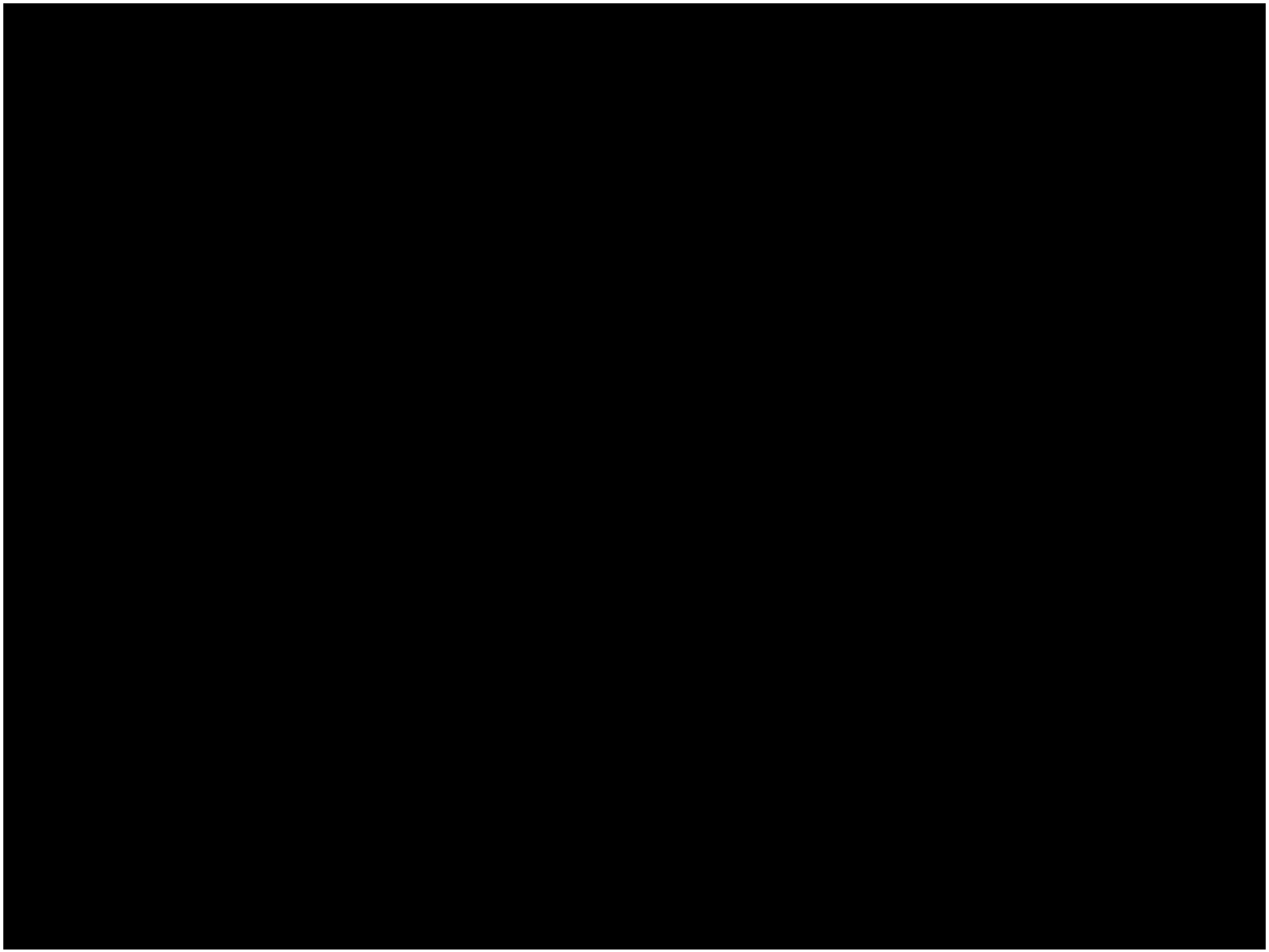

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DISCUSS

What did you notice about the teacher moves to:

- Unpack and make the content visible
- Provide language and tools for explaining
- Support explanatory talk and writing

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COMMON CORE: MATHEMATICAL PRACTICES

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

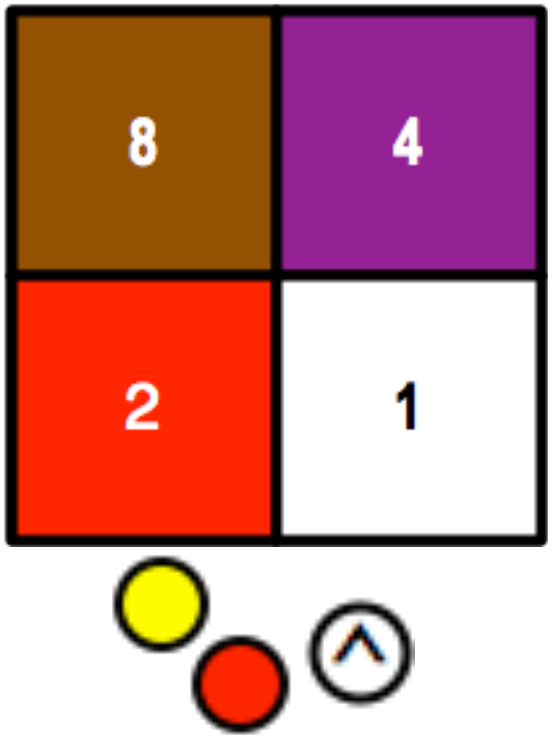
MP8. Look for and express regularity in repeated reasoning.

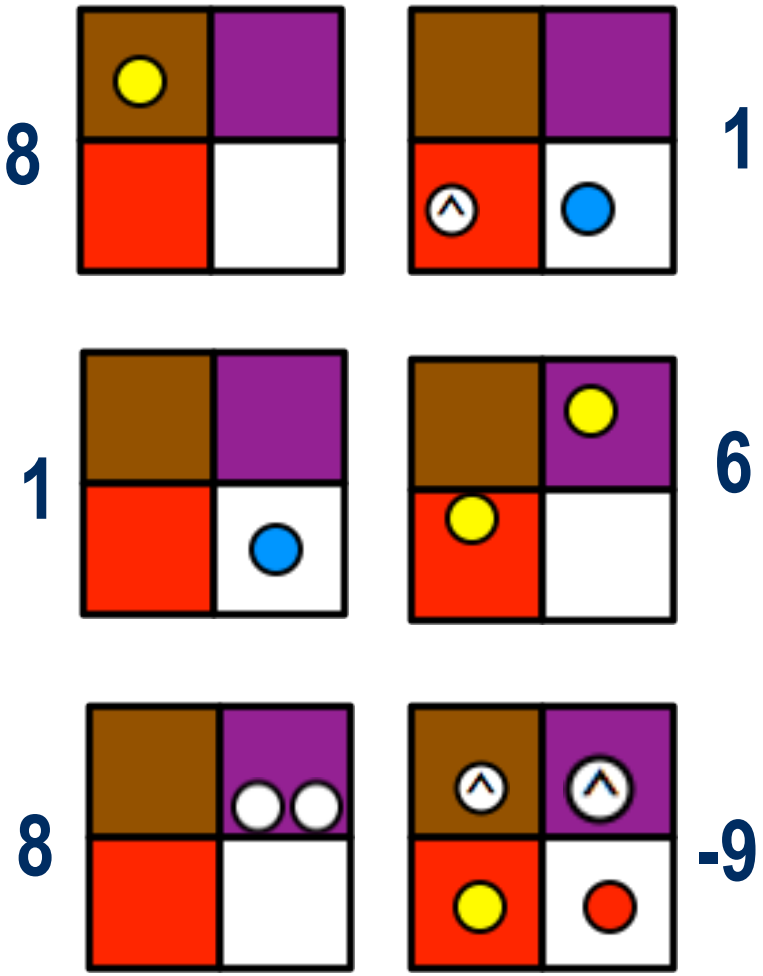
EXPLICIT TEACHING OF MATHEMATICS

- Naming, labeling, writing about important aspects of mathematical ideas, concepts, and procedures
- Naming, highlighting, scaffolding specific mathematical practices
- Naming and supporting qualities of productive mathematical habits and mindset

(Mann, Owens, & Ball, 2013; Selling, 2014)

THE MINICOMPUTER





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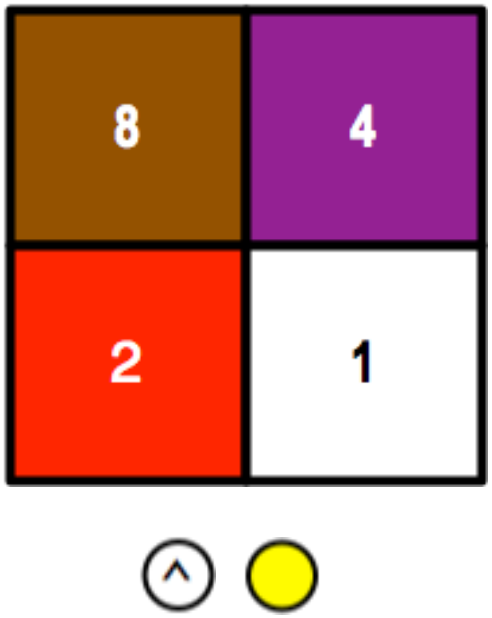
WHAT IS THIS PROBLEM ASKING, AND WHAT IS INVOLVED IN TRYING TO SOLVE IT?



Using exactly one positive and one negative checker, find all the numbers that can be represented on this minicomputer board.

Prove your answer and explain why.

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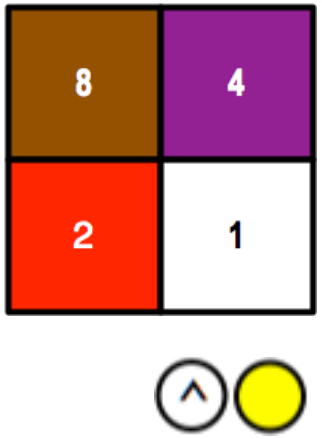
IDENTIFYING AND USING PROBLEM "CONDITIONS"



Using exactly one positive and one negative checker, find <u>all</u> the numbers that can be represented on this minicomputer board. Prove your answer and explain why.

CONDITIONS OF THE PROBLEM

Use exactly one positive and one negative checker.
Must be a number that can be made on the Minicomputer.

PROPOSED SOLUTIONS

3 *YES, because it can be made with a positive on the 4 and a negative on the 1. This (1) uses exactly one positive and one negative checker and (2) it can be made on the Minicomputer.*

9 *NO, because it cannot be made on the Minicomputer with exactly one positive and one negative checker on the Minicomputer.*

EDUCATION

THE ROLE OF CONDITIONS OF A PROBLEM

- Identifying conditions can help in making sense of and interpreting a problem (MP.1)
- Using the conditions can help in persevering in solving a difficult problem (MP.1)
- Conditions are useful in constructing a mathematical argument (MP.3)
- Referring to the conditions is useful in critiquing an argument (MP.3)

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EXPLICIT TEACHING ≠ DIRECT INSTRUCTION: INVESTIGATING SIMILARITIES AND DIFFERENCES

DIRECT INSTRUCTION

1. Breaks down practice or knowledge into small constituent parts
   - Presentation of information, rules, and examples
   - Tasks only address information from the presentation and the task is uncomplicated
   - Lessons are composed of 4 – 10 exercises (presentations with task series) with only 10% new material
2. Teacher's role to demonstrate, students follow, shift to independent practice

EXPLICIT TEACHING

1. Unpacks practice or knowledge to make it open to learners, but does not do the work for students
2. Teacher's role to make elements visible, provide language, supports

EDUCATION

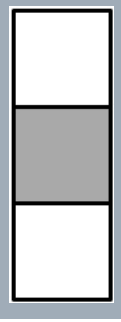
HELPING STUDENTS LEARN TO EXPLAIN MATHEMATICS

- Unpacking and making the content visible
- Providing language and tools
- Supporting explanatory talk and writing

(Consider Arie's explanation after several days of class.)

EXPLAINING FRACTIONS

What fraction of the rectangle below is shaded gray?



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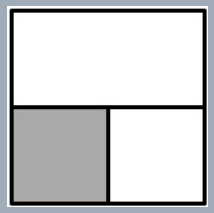

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1. UNPACKING AND MAKING CONTENT VISIBLE

- Using a task that highlights key concept of **equal** parts
  - The fractions chosen and the particular diagram (provokes key stumble)
  - Focus on the idea, not the complexity of the numbers
  - Uses student familiarity with concept image of fractions
- Displaying problem on large poster to coordinate student talk with the idea of equal parts

2. PROVIDING LANGUAGE AND TOOLS FOR EXPLAINING

Supplying language to support shift from everyday informal language to mathematical language (equal)

Providing the “removable line”

3. SUPPORTING EXPLANATORY TALK AND WRITING

Displaying large poster with problem, matches problem students have in their notebooks

Orienting students to one another

Valuing the following of others' arguments

Naming specific talk moves (e.g., talk to class, follow, repeat, agree, disagree, comment)