Remedial Mathematics and the Common Core

William McCallum
The University of Arizona

MSRI CIME, 2015

How can we help struggling students?

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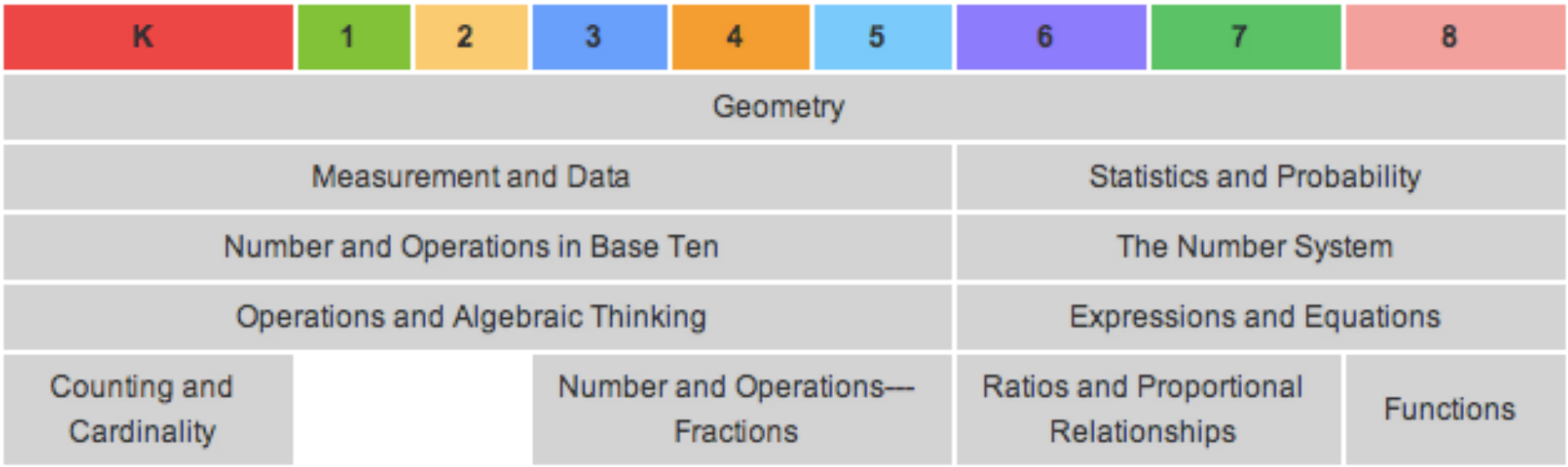
What does it mean to be college ready?

A physics professor says: “Of course, it is easy to see that

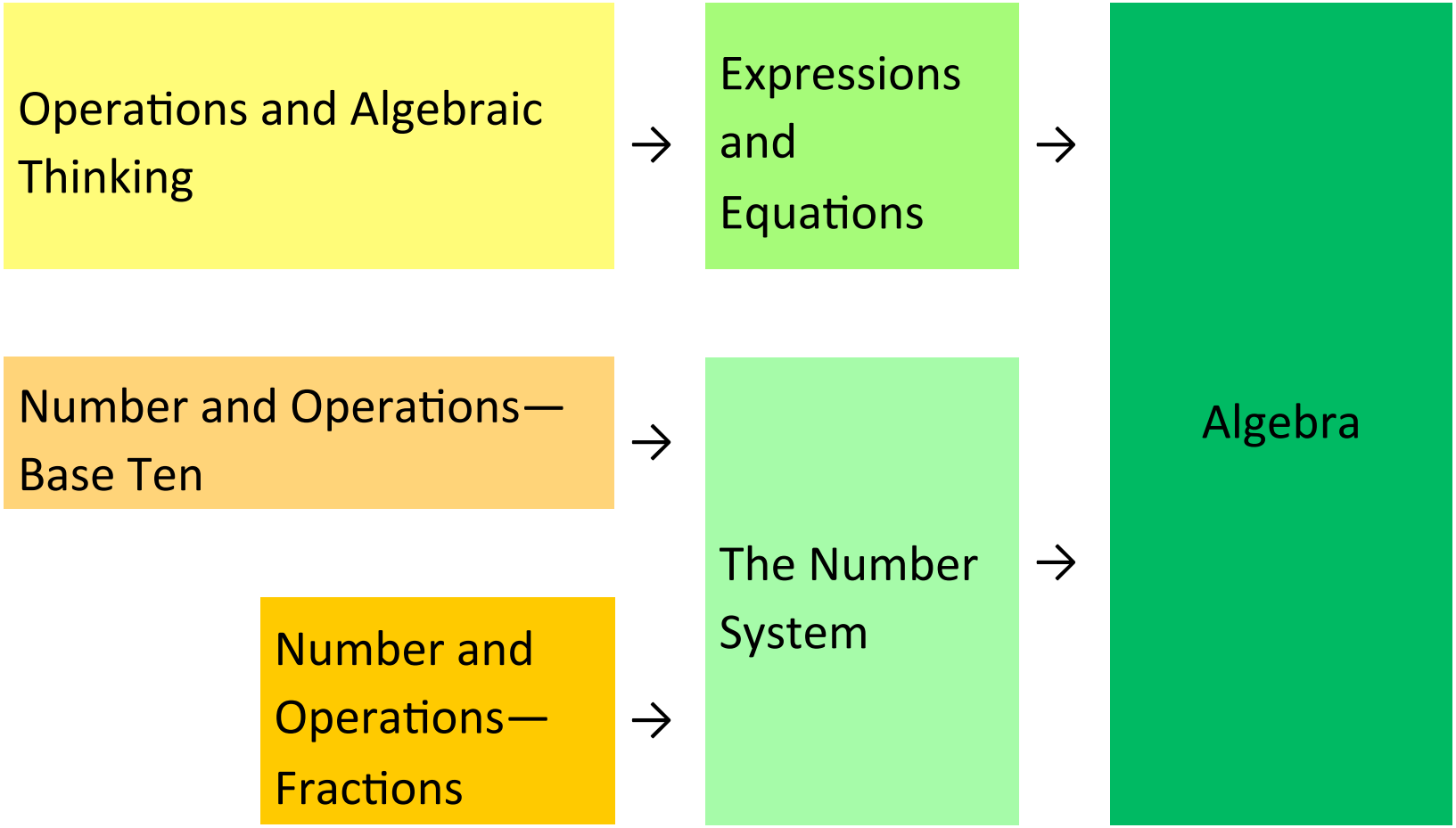
$$L_0 \sqrt{1 - \frac{v^2}{c^2}} = 0$$

when $v = c$.”

The Common Core was built on progressions



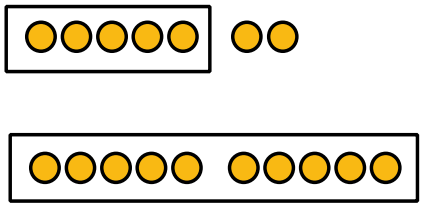
The pathway to algebra



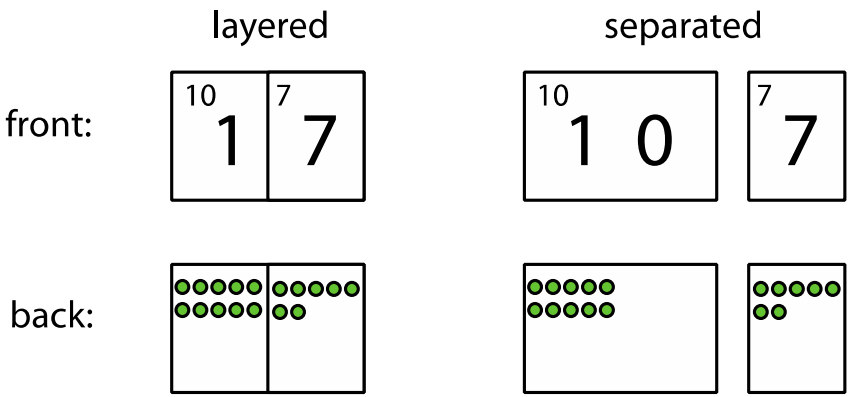
K 1 2 3 4 5 6 7 8 High School

Kindergarten: Understanding place value

- Kindergartners arrange teen numbers into 10 ones and some more ones, in preparation for viewing 10 ones as a new unit called a ten in Grade 1.



Children place small objects into 10-frames to show the ten as two rows of five and the extra ones within the next 10-frame.



Layered place value cards help children see the 10 ``hiding'' inside any teen number.

Rest stop: how to read the standards

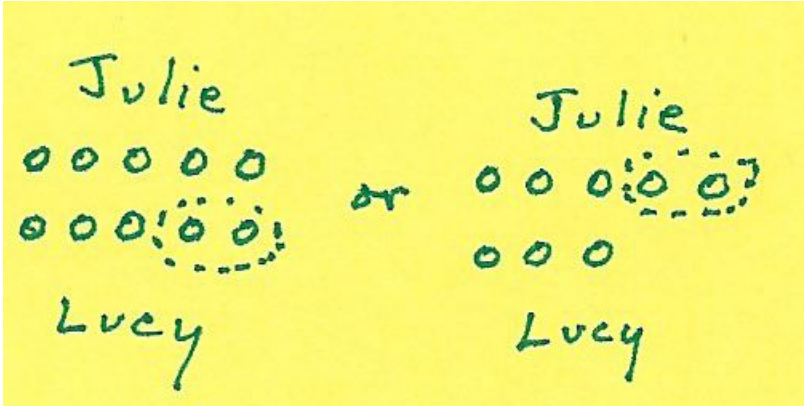
Number and Operations in Base Ten **2.NBT**

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

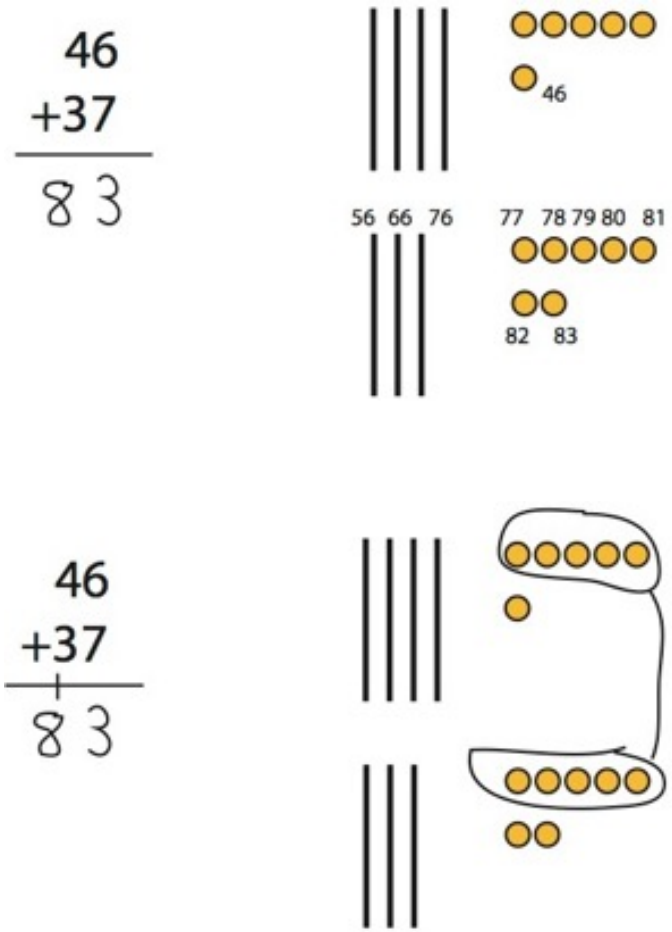
Kindergarten to Grade 1: The connection between addition and subtraction

- Lucy has 3 apples. Julie has 5 apples. How many more apples does Julie have than Lucy?
- Lucy has 3 apples. Julie has 5 apples. How many fewer apples does Lucy have than Julie?
- If x + 2 = 5, then x = 5 - 2.



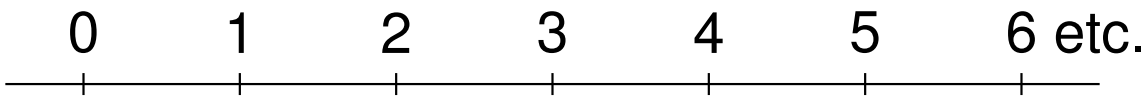
Grade 1–2: Addition using place value and the properties of operations

- Students might start by counting on by 10s, then by 1s.
- (40 + 6) + (30 + 7) = (40 + 6 + 30) + 7
- They move towards the standard algorithm by adding tens and ones separately.
- 40 + 6 + 30 + 7 = 40 + 30 + 6 + 7.
- 4x + 6 + 3x + 7 = 4x + 3x + 6 + 7

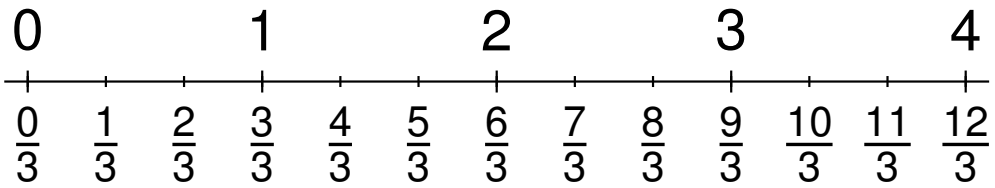


Grade 3: Extending from whole numbers to fractions

The number line



The number line marked off in thirds



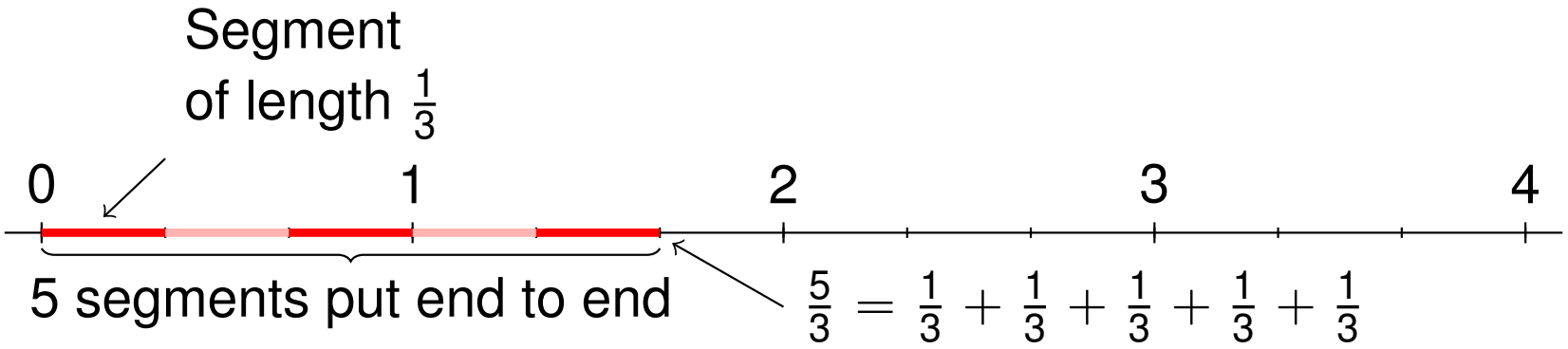
Rest stop: A note on the number line

> *When community college students were asked to mark the approximate locations of -0.7 and $1\frac{3}{8}$ on a number line, only 21% were able to locate both correctly.*

Cathy Kessel, citing *What Community College Developmental Mathematics Students Understand About Mathematics* James W. Stigler, Karen B. Givvin, and Belinda J. Thompson University of California, Los Angeles

Grade 4: Extending operations with whole numbers to fractions

Using the number line to see that $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$



$$\frac{7}{5} + \frac{4}{5} =$$

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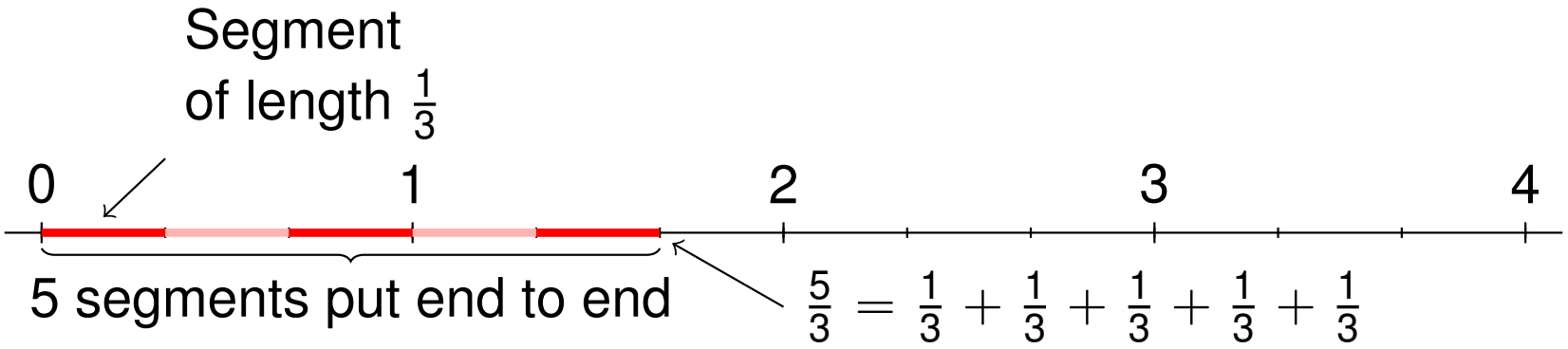

$$\begin{aligned}
\frac{7}{5} + \frac{4}{5} &= \overbrace{\frac{1}{5} + \cdots \frac{1}{5}}^{7} + \overbrace{\frac{1}{5} + \cdots \frac{1}{5}}^{4} \\
&= \frac{\overbrace{1 + 1 + \cdots + 1}^{7+4}}{5} \\
&= \frac{7+4}{5}.
\end{aligned}$$

Grade 5: Connection between division and fractions

Why is

$$\frac{5}{3} = 5 \div 3?$$

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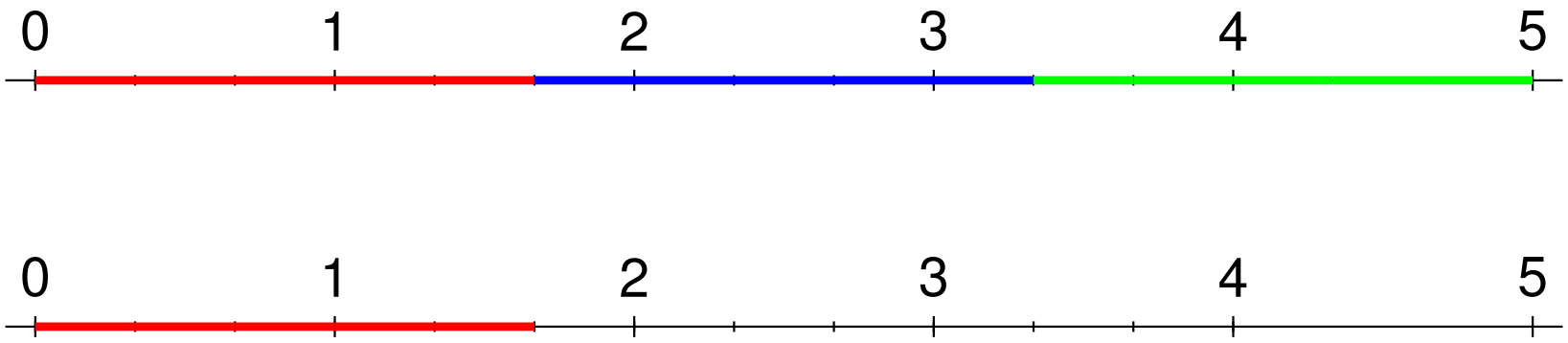

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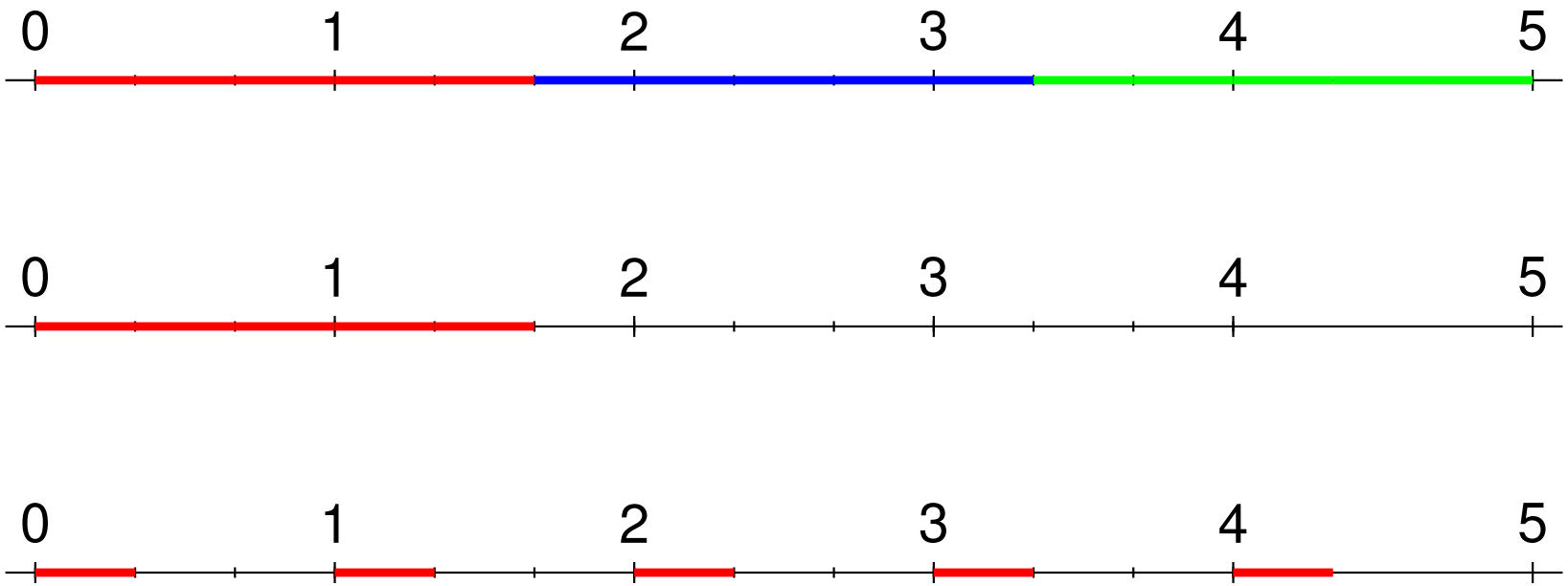

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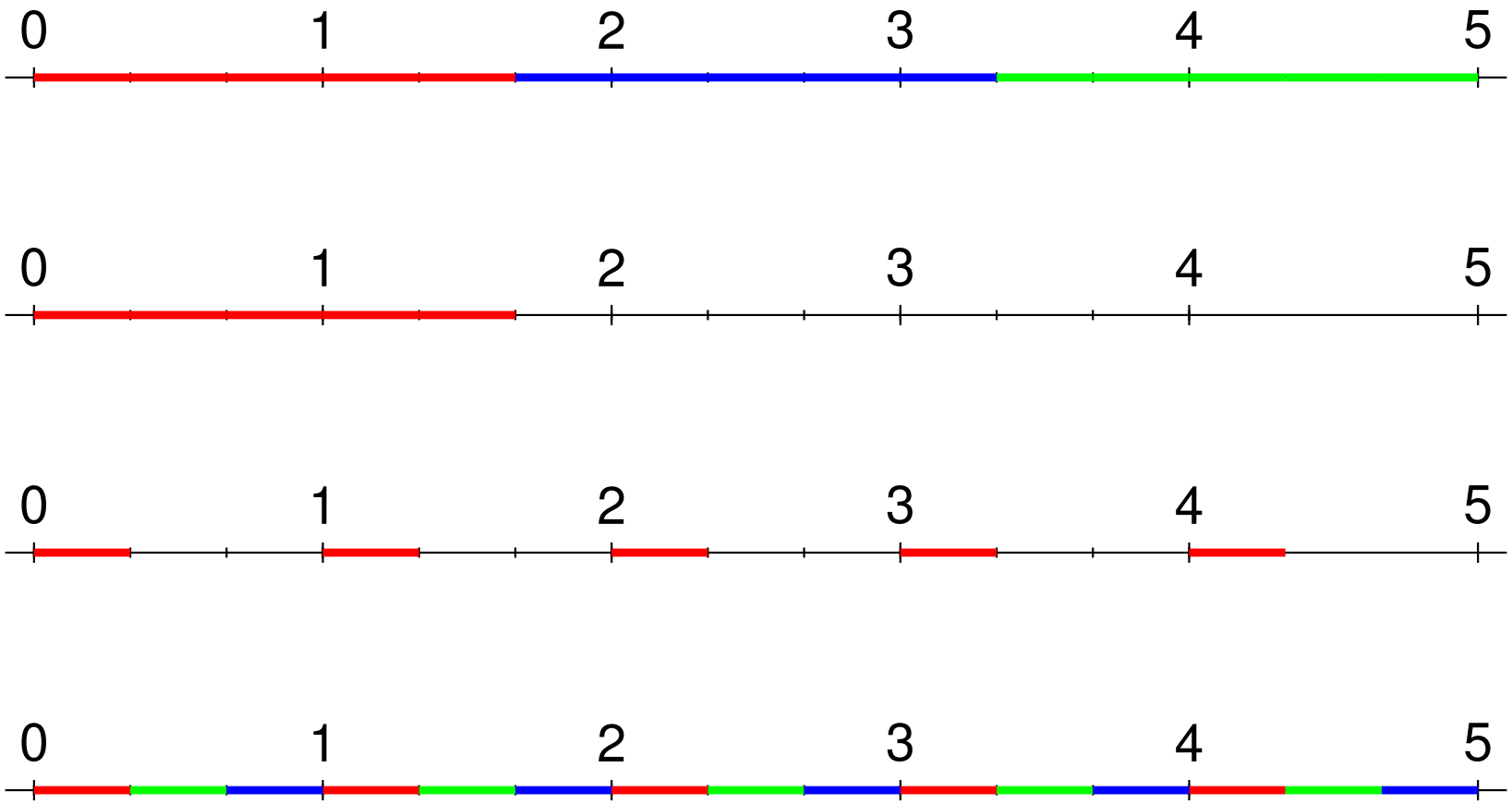
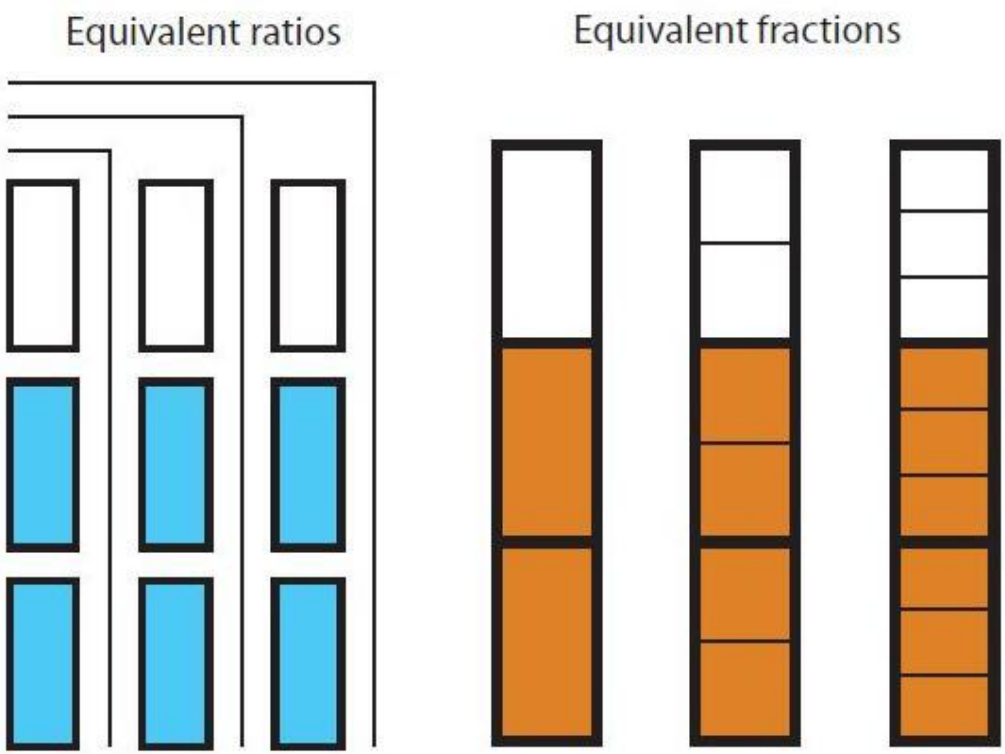


Grade 6: Ratios and Equivalent Ratios



cups blue	2	4	6
total cups	3	6	9

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$$

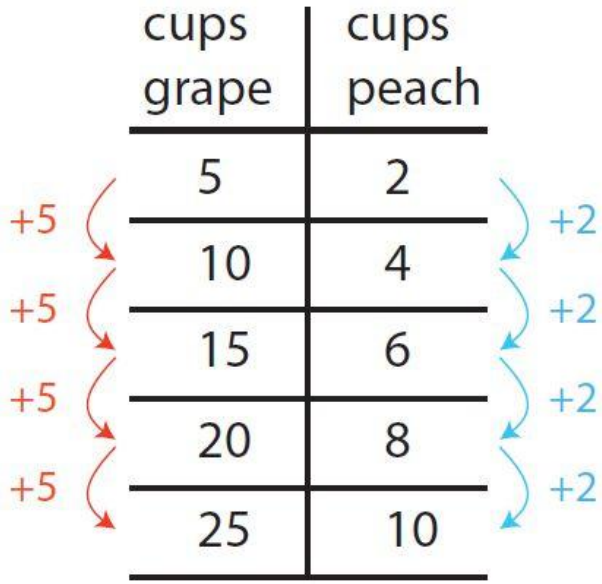
more parts, same size parts

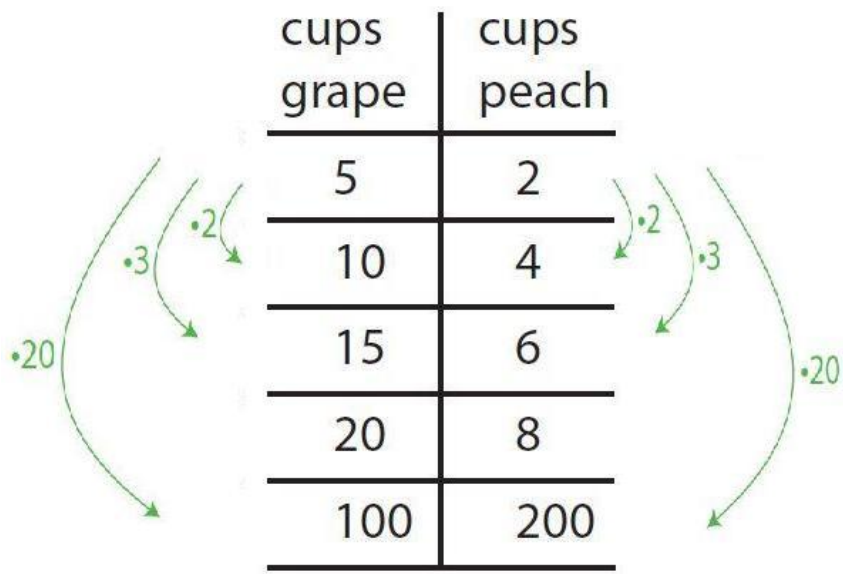
more total paint
more blue pigment

more parts, smaller parts

same whole amount
same portion

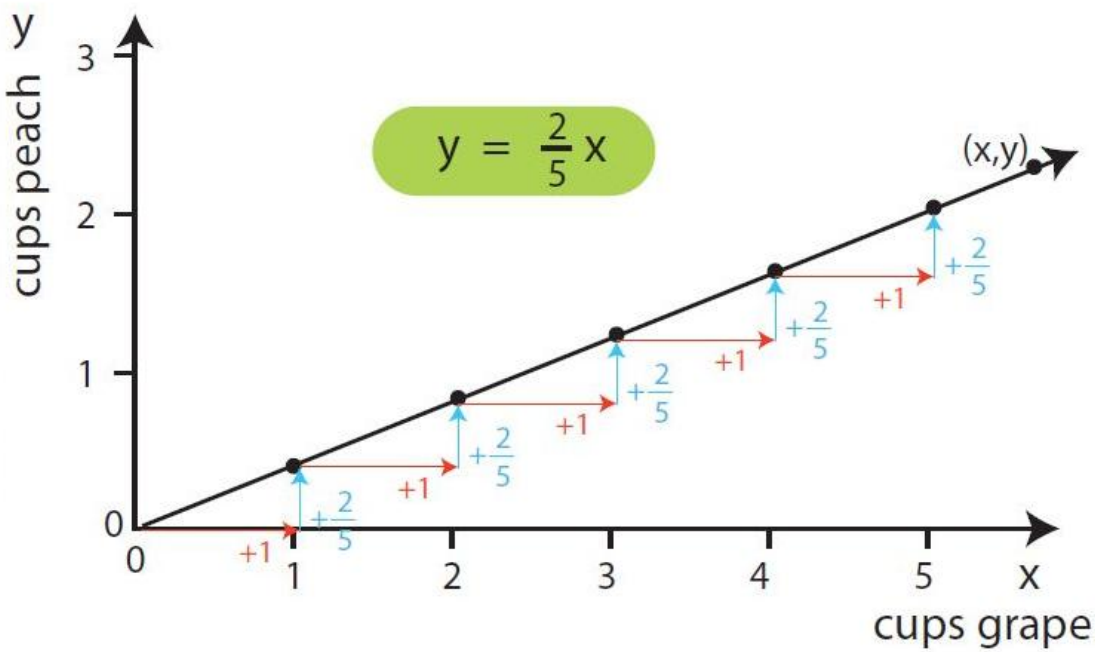
Grade 7: From Ratios to Proportional Relationships





Grade 7–8: From Proportional Relationships to Linear Functions

x cups grape	y cups peach
(0)	(0)
5	2
1	$\frac{2}{5}$
2	$2 \cdot \frac{2}{5}$
3	$3 \cdot \frac{2}{5}$
4	$4 \cdot \frac{2}{5}$
x	$x \cdot \frac{2}{5}$



MP8: Look for and express regularity in repeated reasoning

Moving from the table and the graph to the equation

for each 1 unit you move to the right, move up $\frac{2}{5}$ of a unit.

when you go 2 units to the right, you go up $2 \cdot \frac{2}{5}$ units.
when you go 3 units to the right, you go up $3 \cdot \frac{2}{5}$ units.
when you go 4 units to the right, you go up $4 \cdot \frac{2}{5}$ units.
when you go x units to the right, you go up $x \cdot \frac{2}{5}$ units.

starting from $(0, 0)$, to get to a point (x, y) on the graph, go x units to the right, so go up $x \cdot \frac{2}{5}$ units.

therefore $y = x \cdot \frac{2}{5}$

High School: Seeing structure in expressions

Which of the following could be an expression for the function whose graph is shown below? Explain.

(a) $(x+12)^2+4$

(b) $-(x-2)^2-1$

(c) $(x+18)^2-40$

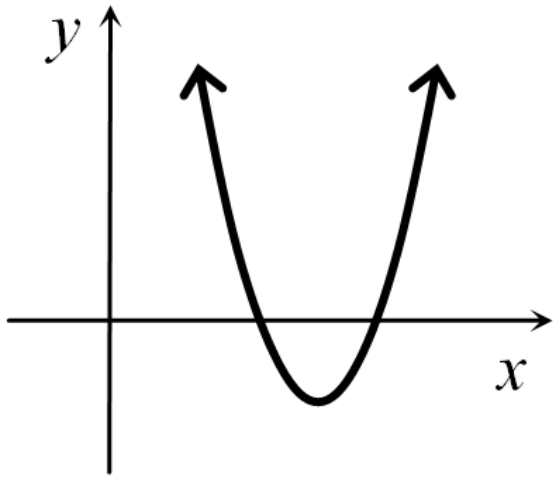
(d) $(x-10)^2-15$

(e) $-4(x+2)(x+3)$

(f) $(x+4)(x-6)$

(g) $(x-12)(-x+18)$

(h) $(20-x)(30-x)$



Task from Illustrative Mathematics. For solutions and discussion, see illustrativemathematics.org/illustrations/640.

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Better to be behind but on the pathway rather than lost in an alien universe.