

Remedial Mathematics and the Common Core

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How can we help struggling students?

If you are struggling, it makes a difference in what environment you are struggling.

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What does it mean to be college ready?

A physics professor says: “Of course, it is easy to see that

$$L_0 \sqrt{1 - \frac{v^2}{c^2}} = 0$$

when $v = c$.”

The Common Core was built on progressions

K	1	2	3	4	5	6	7	8
Geometry								
Measurement and Data					Statistics and Probability			
Number and Operations in Base Ten					The Number System			
Operations and Algebraic Thinking					Expressions and Equations			
Counting and Cardinality			Number and Operations— Fractions			Ratios and Proportional Relationships		Functions

The pathway to algebra

Operations and Algebraic Thinking



Expressions and Equations



Number and Operations—
Base Ten



Number and
Operations—
Fractions



The Number System



Algebra

K

1

2

3

4

5

6

7

8

High School



Kindergarten: Understanding place value

- Kindergartners arrange teen numbers into 10 ones and some more ones, in preparation for viewing 10 ones as a new unit called a ten in Grade 1.

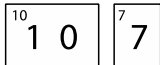
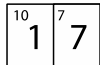


Children place small objects into 10-frames to show the ten as two rows of five and the extra ones within the next 10-frame.

layered

separated

front:



back:



Layered place value cards help children see the 10 "hiding" inside any teen number.

Rest stop: how to read the standards

Number and Operations in Base Ten

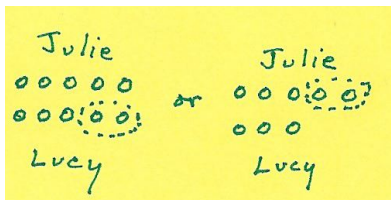
2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
 - a. 100 can be thought of as a bundle of ten tens — called a “hundred.”
 - b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.

Kindergarten to Grade 1: The connection between addition and subtraction

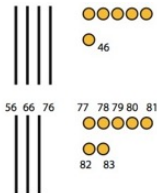
- Lucy has 3 apples. Julie has 5 apples. How many more apples does Julie have than Lucy?
- Lucy has 3 apples. Julie has 5 apples. How many fewer apples does Lucy have than Julie?
- If $x + 2 = 5$, then $x = 5 - 2$.



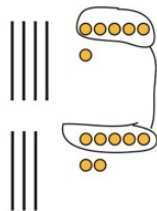
Grade 1–2: Addition using place value and the properties of operations

- Students might start by counting on by 10s, then by 1s.
- $(40 + 6) + (30 + 7) = (40 + 6 + 30) + 7$
- They move towards the standard algorithm by adding tens and ones separately.
- $40 + 6 + 30 + 7 = 40 + 30 + 6 + 7$.
- $4x + 6 + 3x + 7 = 4x + 3x + 6 + 7$

$$\begin{array}{r} 46 \\ +37 \\ \hline 83 \end{array}$$

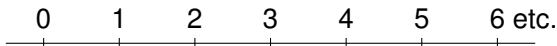


$$\begin{array}{r} 46 \\ +37 \\ \hline 83 \end{array}$$

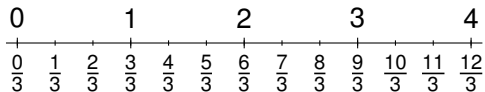


Grade 3: Extending from whole numbers to fractions

The number line



The number line marked off in thirds



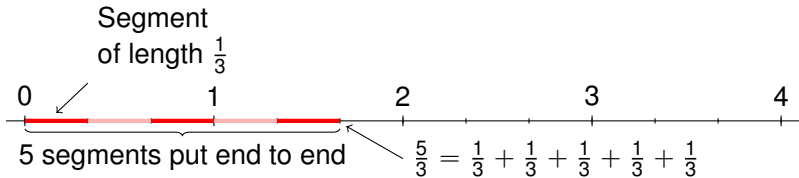
Rest stop: A note on the number line

When community college students were asked to mark the approximate locations of -0.7 and $1\frac{3}{8}$ on a number line, only 21% were able to locate both correctly.

Cathy Kessel, citing *What Community College Developmental Mathematics Students Understand About Mathematics* James W. Stigler, Karen B. Givvin, and Belinda J. Thompson University of California, Los Angeles

Grade 4: Extending operations with whole numbers to fractions

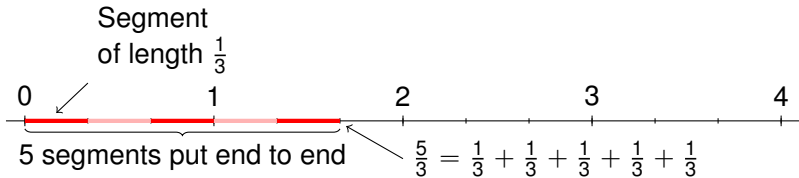
Using the number line to see that $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$



$$\frac{7}{5} + \frac{4}{5} =$$

Grade 4: Extending operations with whole numbers to fractions

Using the number line to see that $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$



$$\begin{aligned}\frac{7}{5} + \frac{4}{5} &= \overbrace{\frac{1}{5} + \cdots + \frac{1}{5}}^7 + \overbrace{\frac{1}{5} + \cdots + \frac{1}{5}}^4 \\ &= \frac{\overbrace{1 + 1 + \cdots + 1}^{7+4}}{5} \\ &= \frac{7+4}{5}.\end{aligned}$$

Grade 5: Connection between division and fractions

Why is

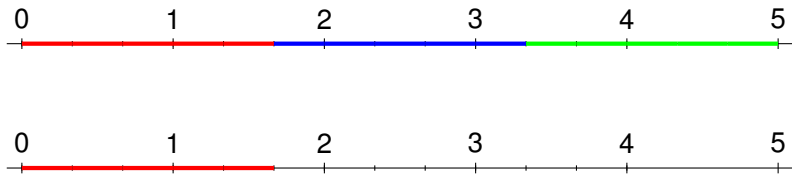
$$\frac{5}{3} = 5 \div 3?$$



Grade 5: Connection between division and fractions

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Grade 5: Connection between division and fractions

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Grade 5: Connection between division and fractions

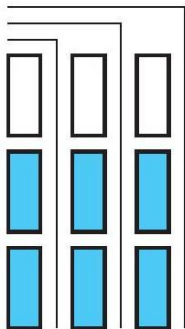
Why is

$$\frac{5}{3} = 5 \div 3?$$



Grade 6: Ratios and Equivalent Ratios

Equivalent ratios



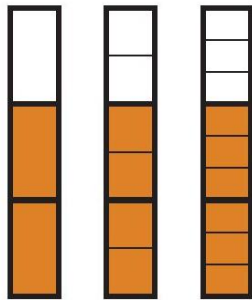
cups blue	2	4	6
total cups	3	6	9

more parts, same size parts



more total paint
more blue pigment

Equivalent fractions



$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$$

more parts, smaller parts



same whole amount
same portion

Grade 7: From Ratios to Proportional Relationships

	cups grape	cups peach
	5	2
+5	10	4
+5	15	6
+5	20	8
+5	25	10

Diagram illustrating a table of values for cups of grape and peach. The table shows a constant rate of change. Red arrows on the left indicate an increase of +5 in cups of grape, and blue arrows on the right indicate an increase of +2 in cups of peach.

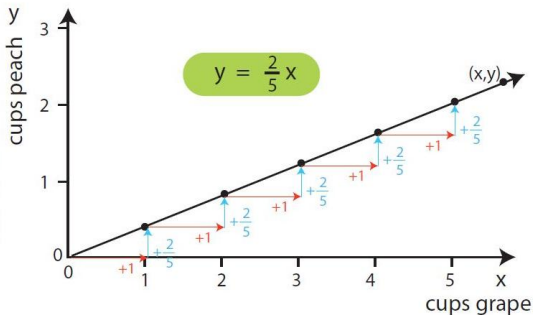
	cups grape	cups peach
	5	2
	10	4
	15	6
	20	8
	100	200

Diagram illustrating a table of values for cups of grape and peach, showing a constant rate of change. Green arrows indicate the relationships between the rows: a multiplier of $\cdot 20$ from the first row to the fifth row, and a multiplier of $\cdot 3$ from the second row to the fourth row. Small arrows also show a multiplier of $\cdot 2$ from the first row to the second row.

Grade 7–8: From Proportional Relationships to Linear Functions

x cups grape	y cups peach
(0)	(0)
5	2
1	$\frac{2}{5}$
2	$2 \cdot \frac{2}{5}$
3	$3 \cdot \frac{2}{5}$
4	$4 \cdot \frac{2}{5}$
x	$x \cdot \frac{2}{5}$

Red arrows on the left indicate a constant increase of +1 in the x-values. Blue arrows on the right indicate a constant increase of $+\frac{2}{5}$ in the y-values.



MP8: Look for and express regularity in repeated reasoning

Moving from the table and the graph to the equation

for each 1 unit you move to the right, move up $\frac{2}{5}$ of a unit.

when you go 2 units to the right, you go up $2 \cdot \frac{2}{5}$ units.

when you go 3 units to the right, you go up $3 \cdot \frac{2}{5}$ units.

when you go 4 units to the right, you go up $4 \cdot \frac{2}{5}$ units.

when you go x units to the right, you go up $x \cdot \frac{2}{5}$ units.

starting from $(0, 0)$, to get to a point (x, y) on the graph, go x units to the right, so go up $x \cdot \frac{2}{5}$ units.

therefore $y = x \cdot \frac{2}{5}$

High School: Seeing structure in expressions

Which of the following could be an expression for the function whose graph is shown below? Explain.

(a) $(x + 12)^2 + 4$

(b) $-(x - 2)^2 - 1$

(c) $(x + 18)^2 - 40$

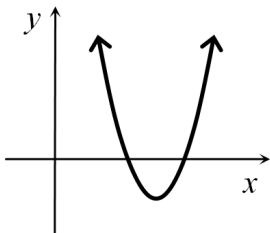
(d) $(x - 10)^2 - 15$

(e) $-4(x + 2)(x + 3)$

(f) $(x + 4)(x - 6)$

(g) $(x - 12)(-x + 18)$

(h) $(20 - x)(30 - x)$



Task from Illustrative Mathematics. For solutions and discussion, see illustrativemathematics.org/illustrations/640.

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Better to be behind but on the pathway rather than lost in an alien universe.