10/30 Carl Mueller

w/. R. Dalang Y. Xiao (S. Tindel).

$P(X_t \in A)$ hits $P(X_t \in A$ for some $t) > 0$.

Random String (polymer).

(1.1) $\partial_t u = \Delta u + \dot{W}(t,x)$ $x \in \mathbb{R}$ $u \in \mathbb{R}^d$.

Rigorous meaning:

$$u(t,x) = \int G_t(x-y)\, u_0(y)\, dy + \int_0^t \int G_{t-s}(x-y)\, W(dy\, ds)$$

<u>Hitting</u>: Potential theory

$X_t = B_t$ Brownian motion

A  x

$u(x) = P(B.$ hits $A)$

$u(x)$ harmonic $\Delta u = 0$ on A^c

$u|_A = 1$

g green function.

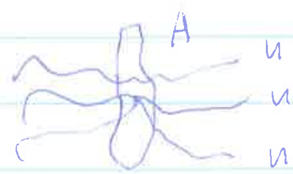
$\Delta u = -\delta$

$\mathrm{Cap}(A) = \max\{\mu(A) :$ μ supp. on A $\int g(x,y)\,\mu(dy) \le 1\}$

$P(X$ hits $A) = 0 \iff \mathrm{Cap}(A) = 0$.

u is Markov.

Potential theory is intractable.



u hits A if

$P(u(t,x) \in A$ for some t some $x) > 0$.

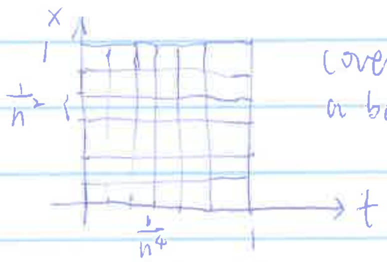
d_c is the critical dimension for u to hit points if

u hits A if $d < d_c$

not if $d \geq d_c$

If X_t is space filling, it should hit points.

$d_c = \dim(\text{range}(X))$.



cover by a ball of radius $\frac{1}{n}$.

$t \mapsto u(t,x)$ Hölder $\frac{1}{4} - \varepsilon$

$x \mapsto u(t,x)$ Hölder $\frac{1}{2} - \varepsilon$

need n^6 balls. $d_c = \dim(\text{range}(X)) = 6$ if $X = u$.

(M - Tribe)

Multiple points:

double $X_t = X_s$ $t \neq s$

$Z_{s,t} = X_t - X_s$.

X has double points iff Z hits 0 away from diagonal.

can replace white noise $\dot{W}$ by colored noise, or $\sigma(t,x)\dot{W}(t,x)$

Dalang - Khoshnevisan - (E.) Nualart et. al.

$P(u \text{ hits } A) \geq C \cdot \text{cap}(A)$ not Newtonian cap

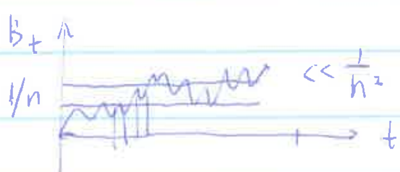
$\leq C\, \mathcal{H}_\alpha(A)$ Hausdorff dimension.

Doesn't cover the critical case.

strategy (based on Talagrand)

in case of 2-dim BM.

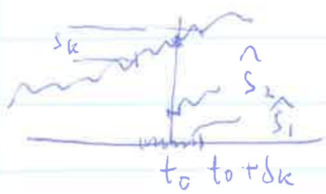
$t \mapsto B_t$ is Hölder $\frac{1}{2} - \varepsilon$.



Balls needed to cover $>> n^2$

$B[t, t+\delta] := \{B_s : t \leq s \leq t+\delta\} \subset \left(B_t - \sqrt{\delta}\left(\log\frac{1}{\delta}\right)^\beta, B_t + \sqrt{\delta}\left(\log\frac{1}{\delta}\right)^\beta\right)$

$\mathcal{H}_2(B[0,t]) > 0$.



typically $|S_k| >> \sqrt{\delta_k}$

$\hat{S}_k = [t_0, t_0+\delta_k]$

Let $S_k = \{ B_s : t_0 \le s \le t_0 + \delta_k \}$ $\quad \delta_k \downarrow 0$ quickly.

S_k are somewhat independent

w/. high probability $|\delta_k| << \sqrt{\delta_k}$ for some k

Based on this, we can achieve an economical cover of $B_{[0,1]}$, thus showing Lebesgue measure $m(B_{[0,1]}) = 0$.

Argument depends on B being a Gaussian process.

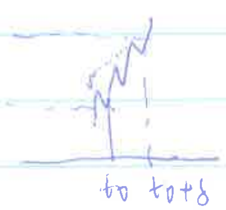
Sidak's inequality: For $X_1, \dots, X_n$ jointly Gaussian $\varepsilon_i > 0$

$$P(|X_i| < \varepsilon_i \quad i=1,\dots,n) \geq \prod_{i=1}^{n} P(|X_i| \le \varepsilon_i)$$

strategy in nonlinear case. $\partial_t u = \Delta u + f(u) \dot{W}(t,x)$.

Near a given (t_0, x_0), freeze coefficients so that the process is a small perturbation of a Gaussian process.

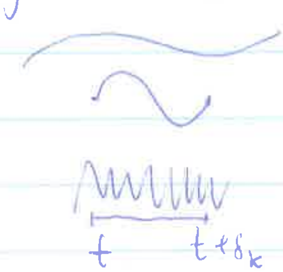
...... dot line : small perturbation.



why $\simeq$ independence for different k?

use a harmonizable representation. (Fourier expansion w/. ind. coefficients.)

what effect do different frequencies have on size $|S_k|$ of the range $B[t_0, t_0+\delta_k]$



not much

will affect

will affect but will be small.