1:20 pm. Kati - Jus. 8/18/16. 1

Introduction:- Sofic Group.

Theorem:- (Harald Hagget - Kati).

Hix group is sofic iff

$\forall \varepsilon > 0 \ \exists$ p prime $f: \mathbb{Z}/p\mathbb{Z} \to \mathbb{Z}/p\mathbb{Z}$.

a) $f(f(f(f(x)))) = x, \quad \forall x \in \mathbb{Z}/p\mathbb{Z}$.

b). $f(x+1) = \cancel{2} f(x), \quad \forall x \in V, \quad \forall V \subseteq \mathbb{Z}/p\mathbb{Z}$.

"$f(x) = x$ almost everywhere".

Def^n Sofic Group.

G is Sofic group if $\forall \varepsilon > 0, \forall$
$F \subseteq G$, $\underset{id.}{e} \in F \ \exists \ \varphi : F \to S(n)$.

- $d(\varphi(g)\varphi(h), \varphi(g, h)) < \varepsilon \quad , \forall g, h, gh \in F$

$\varphi(g)$ does not have fixed pt. $\varphi(e) = 1$.

$d(\sigma, \tau) = |\{i : \sigma(i) \neq \tau(i)\}|$

Ex:- finite groups.

Def^n G is Sofic if its Cayley graph is identity subamenable.

Graph $G = (V, E)$ is subamenable if $\forall \varepsilon > 0$
edge-colored with f in set of colors. $\forall$ ball of radius r
in G

$\exists$ finite graph (V', E') and $W \subseteq V'$ s.t.

- $|W| > (1-\varepsilon)|V'|$
- $\forall x \in W$, $\underset{\in (V',E')}{B_r(x)} \sim \underset{\in G}{B_r}$.

Ex Show that both def.ⁿ are equivalent.

<u>Properties of Sofic Group</u>:-

* G is sofic iff every finitely generated subgroup is sofic.
* $H \leq G$, G is sofic $\Rightarrow$ H is sofic.
* direct product of sofic gps are sofic.
* direct limits.

<u>OPEN</u>: non-sofic Group.

* <u>amenable</u> are sofic.

G is amenable group if $\forall \varepsilon > 0$

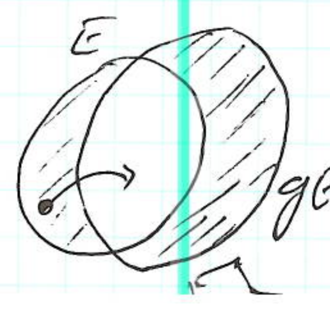
$\forall F \underset{fin}{\subset} G$ $\exists E \underset{fin}{\subset} G$ s.t.

$|gE \triangle E| < \varepsilon \cdot |E|$ $\forall g \in F$.

amenable $\Rightarrow$ sofic

$\forall \varepsilon > 0$, $\exists F$

$\bigcup_{g \in F} gE$



$\varphi(g) \in S(X)$.

<u>Sofic</u>. <u>Amenable Groups</u>.

- Groups of subexponential ~~growth~~ growth are amenable.

$\limsup |B_r(S)|^{1/r} = 1$, $\langle S \rangle = G$.

$\forall \varepsilon > 0 \ \exists k \in \mathbb{N}$ s.t. $|B_{k+1}| < (1+\varepsilon)|B_k|$

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$g \in S$. This implies that all abelian groups are amenable.

<u>Ex</u> $G = \langle S \rangle$ is amenable if $\forall \varepsilon \ \exists E$ s.t.

$|SE \triangle E| < \varepsilon |E|$, $\forall s \in S$.

<u>More examples</u>.

- locally embeddable into finite groups.

$\forall F \subset_{fin} G \ \exists H$ - finite group.

$\varphi : F \to H : \varphi(gh) = \varphi(g)\varphi(h)$, $\forall g, h, gh \in F$.

Obs. if G is finitely presented, then res is finite

Initially subamenable groups — H — but H is amenable.

Sofic-by-amenable. $e \to H \to G \to K \to e \Rightarrow G$ sofic

H sofic. K amenable

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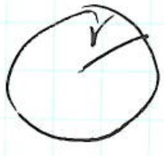
$G_1 \underset{H}{*} G_2$ - Sofic provided that G_1, G_2 are Sofic and H is ameanable.

<u>Gromove</u> Is every Sofic group is a limit of ameanable group of the space of marked group.

G_n , S_n $|S_n| = |G_n| = n'$

 $\forall r \; \exists k_0$ s.t G_{k_n} , $\forall k > k_0$

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Q:- There exist an ameanable isolated sofic group?

Abelis — p is prime $G_L(\mathbb{Z}[\frac{1}{p}])$

$$\begin{pmatrix} 1 & a_{12} & \cdots & a_{1n} \\ & \ddots & & a_{2n} \\ 0 & & \ddots & 1 \end{pmatrix}$$ * powers $1/p$. $a_{ij} \in \mathbb{Z}[1/p]$.

$M = \begin{pmatrix} 1 & & a m_1 \\ & 1 & m_2 \\ & & 1 \end{pmatrix}$ $m_1, m_2 \in \mathbb{Z}$. G/M —

<u>Conjecture</u>:- If G is fin represented non ameanable, simple $\Rightarrow$ G is non-Sofic.

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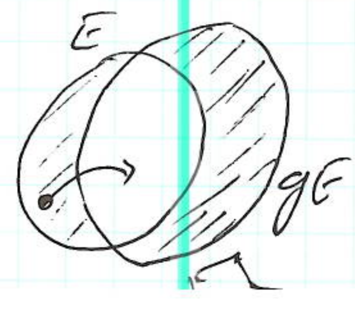

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