$= d_3' + d_4 \cdot$

3/19/16 Part-two — Moon @ 11:30°

Cannon: In σ-shape cones, $\exists\ N \geq 1$ s.t N-type determines ~~[illegible]~~ cone-type, thus there are finitely many cone types ($\leq \mu_N$).

Want: σ_n satisfy recursion; $S(x) = \sum \sigma_n x^n$ is a rational function

Try for F_2 $\quad \sigma_n = \# S_n$; let $\sigma_n^{(\tau_i)} = \#$ words of length n and cone-type τ_i

$$\sigma_n = \sum_{i=0}^{M} \sigma_n^{(\tau_i)}$$

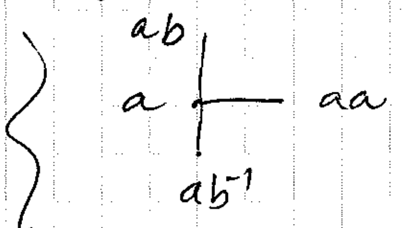
$\sigma_n^{(\tau_n)} =$

$\alpha_{ij} :=$ # words of type τ_j one ~~[illegible]~~ outbound edge from putting vertex of type τ_i

So, from fig

$\alpha_{ab} = 1$

$\alpha_{aa^{-1}} = 0$



$\mu_j = \#$ vertices s.t outbound edge will get me a disjunction of type τ_j.

$\mu_a = 1$

General Recursive Recipe.

$$\sigma_n^{(\tau_j)} = \frac{\sum_{i=0}^{M} \alpha_{ij}\, \sigma_{n-1}^{(\tau_i)}}{\mu_j}, \qquad \sigma_n^{(\tau_0)} = \begin{cases} 1, & n = 0 \\ 0, & \text{else.} \end{cases}$$

$\sigma_n^{(\tau_i)} = \{ 1 \quad i = n$

$$\sigma_n^{(\tau_n)} = \frac{1\,\sigma_{n-1}^{(\tau_a)} + 1\,\sigma_{n-1}^{(\tau_b)} + 1\,\sigma_{n-1}^{(\tau_{b^{-1}})}}{1},$$

$$\sigma_n = 3\,\sigma_{n-1}^{(\tau_n)} + 3\,\sigma_{n-1}^{(\tau_n)} + 3\,\sigma_{n-1}^{(\tau_n)} + 3\,\sigma_{n-1}^{(\tau_n)}.$$

$$= 3\,\sigma_{n-1}$$

However, in $\mathbb{Z}^2$,

$\alpha_{1,1} = 2$. $\alpha_{1,j} = 0$ for other j.

$\mu_1 = 2$

$\alpha_{5,5} = 1$, $\alpha_{5,1} = \alpha_{5,2} = 1$ else, $\alpha_{5,j} = 0$.

$\mu_5 = 1$,

$$\sigma_n^{(\tau_1)} = \frac{2\,\sigma_{n-1}^{(\tau_1)} + 1\cdot\sigma_{n-1}^{(\tau_5)} + 1\,\sigma_{n-1}^{(\tau_8)}}{2}$$

$$= \sigma_{n-1}^{(\tau_1)} + \sigma_{n-1}^{(\tau_5)}.$$

$$\sigma_n^{(\tau_5)} = \frac{1\,\sigma_{n-1}^{(\tau_5)}}{1} = \sigma_{n-1}^{(\tau_5)}.$$

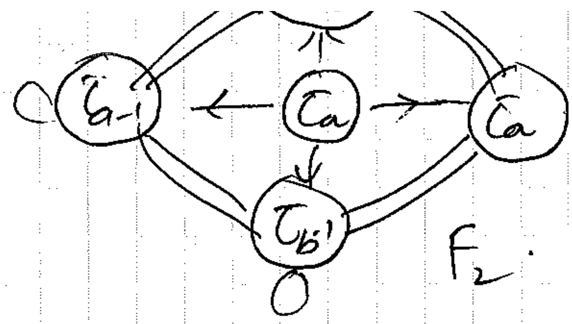
$$\sigma_n = 4\,\sigma_{n-1}^{(\tau_1)} + 8\,\sigma_{n-1}^{(\tau_5)}.$$

Quick Check

$\sigma_n = 4n$, $4n \stackrel{?}{=} 4(n-2) + 8$ ✓

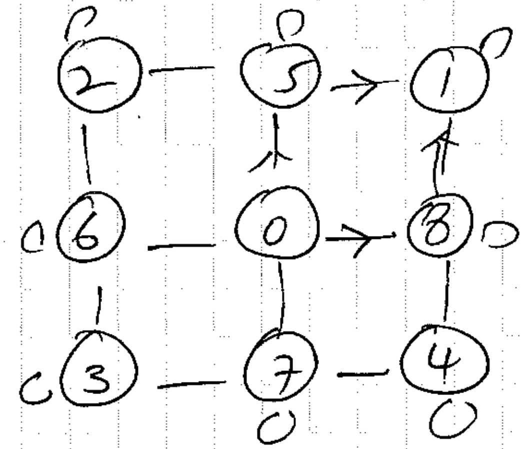
$\sigma^{(\tau_1)}$ $\sigma^{(\tau_5)}$

So, how to calculate geodesics of length n?



\# ~~[illegible]~~

This the end of cannon's proof.



Growth of groups — what's known.

observation: A finite growth ~~recursion~~ gives solutions to word problems.

So, groups of unsolvable word problems have non-rational growth in any geodesics.

Recursions $\sigma_n \leftrightarrow$ ~~[illegible]~~

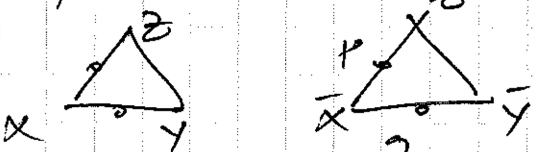
QI rigidity of lattices in soluable lie group

Exp. Bowden. $\partial(N \times_{\varepsilon} \mathbb{R}) \simeq S^n$.

Day's fixed point theorem.

X — proper geodesic metric space

CAT(0).



Euclidean E^2, hyperbolic space H^2, locally finite Trees. X has unique geodesics.

universal cover.

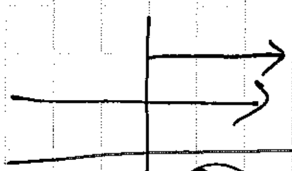
$X \times Y$ is CAT(0) iff X & Y are CAT(0).

Boundaries of Spaces –

Def^n: $\partial X = \{ c : \mathbb{R}^+ \to X \} / \sim$

$c \sim c'$ iff $k \geq 0$ s.t

$d(c(t), c(t')) \leq k, \quad \forall t \geq 0.$



Part-2

$G = \langle x, y, t \mid [x, y] = 1, t^{-1} x t = y \rangle.$

Isolated tits arcs.

X is hyperbolic iff $\partial_T X$ is discrete.

) Iff $\partial_T X \cong \partial_\infty X \iff$ X is a flat. "Bosche".

Iff $\partial_T X = A * B$.

Results:- G acts on E^2, there exists an element of infinite order such that G is virtually centered and $G/\langle g \rangle$ is 2 ended.

Theorem:- G must

Exercise:- $g \in G$,

$\min(g) = \{ \cdot \{ X\ A_g \cdot$