Yael Algom-Kfir

8/19/16
2:00 pm.

Fiberation of free-by-cyclic group.

Dowdall - I Kapovich Leininger.

DKL. Palm tree group.

$F_n = \langle x_1, \dots, x_n \rangle$.

$\Phi \in \mathrm{Aut}(F_n)$.

$G_\Phi = F_n \underset{\Phi}{\times} \mathbb{Z} = \langle x_1, \dots, x_n, t \mid t x_i t^{-1} = \varphi(x_i) \rangle$

$G_\Phi \overset{?}{=} G_\psi$.

If $w \in F_n$, $i_w \in \mathrm{Aut}(F_n)$.
conj by w.

$\Phi' = i_w \Phi$

$G_{\Phi'} = G_\Phi = G_\phi$, $\phi \in \mathrm{Out}(F_n)$, $\phi = [\Phi]$.

Given $F_n \rtimes_\phi \mathbb{Z}$: get $u \in \mathrm{Hom}(G, \mathbb{Z})$.
epimorphism.

$u(F_n) = 0$

$u(F_n) = 0$

$u(t) = 1$

Conversely, $1 \longrightarrow \ker(u) \longrightarrow G \overset{u}{\twoheadrightarrow} \mathbb{Z} \longrightarrow 1$
t

t defines $\phi \in \mathrm{Out}(\ker(u))$.

If $\ker(u)$ is finit gen. + free then

$G = \ker(u) \rtimes_\phi \mathbb{Z}$.

f.g $\Longrightarrow$ free.

Geoghan.

Mahalik - Sapir Wise

$\{$ normal subgroup of G with quotient $\mathbb{Z}\} \longleftrightarrow \underset{epi}{Hom}(G, \mathbb{Z})$

$$Hom(G, \mathbb{Z}) \subset Hom(G, \mathbb{R}) = Hom(G^{ab}, \mathbb{R}) \cong \mathbb{R}^b, \text{ where } b = rk(G^{ab}).$$

If $b=1$, then there is only one description of $F_n \rtimes_\phi \mathbb{Z}$ as free-by cyclic group.

(exe: what auto does -1 corresponds to?) 

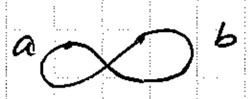
If $b>1$, then there are ∞ many such descriptions

Get a topological Picture:

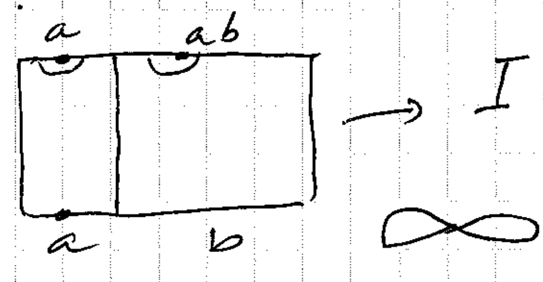
Let $\phi \in Out(F_n)$. Let Γ be a graph. Suppose $\pi_1(\Gamma, \ ) \cong F_n$. $f: \Gamma \to \Gamma$ be a linear map s.t $f_* \in \phi$

edges $\xrightarrow{f}$ edge paths.

$$M_f = \Gamma \times [0,1] / (x,1) \sim (f(x), 0).$$

$F_2 = \langle a, b \rangle$ 

$f: \begin{array}{l} a \to a \\ b \to ab \end{array}$



$\Gamma \times I$

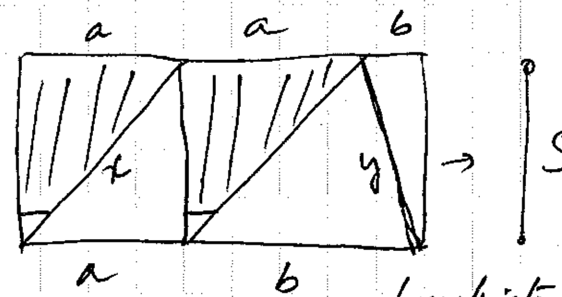
$M_f \xrightarrow{p} S'$ There is a flow,

fibration(?) $\Psi : M_f \times R_+ \longmapsto M_f$.

$$\Psi(\Psi(x,t), s) = \Psi(x, t+s).$$

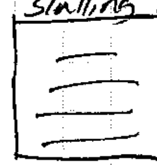
The interesting thing happens at $p^{-1}(0)$.

Ψ decends and also p.



This is called folded mapping torus.

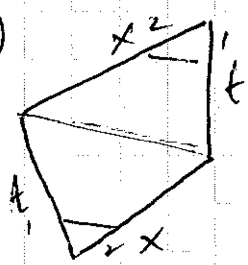
break into stalling blocks



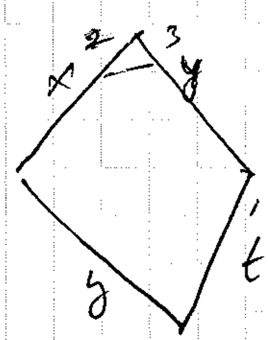
Let

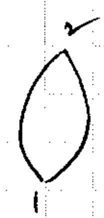
$u \in \mathrm{Hom}(G, \mathbb{R}) = H^1_b(X, \mathbb{R})$

Cell-Complex.



z_1, x, y, t





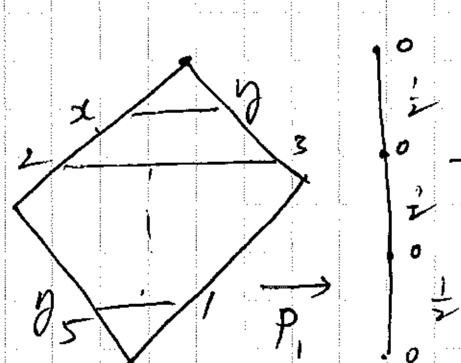
u_0 $x \to 1$
$y \to 1$
$t \to 1$

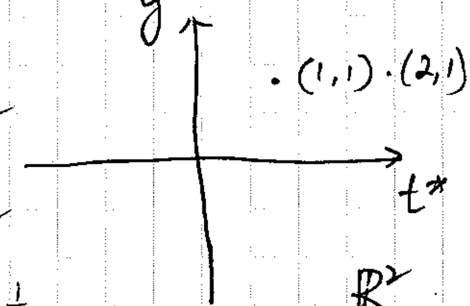
$B_1 : x = t$

$H_1(X, \mathbb{R}) = \langle t, y \rangle = \mathbb{R}^2$.

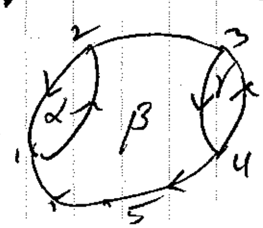
$u_1 (2,1)$.

$z_1 : t \mapsto 2 \mapsto x$.
$y \longmapsto 1$.





$(P_1)_* = u_1 \cdot G \to \mathbb{R}$.



$F_2 \underset{N}{\times} \mathbb{Z} \cong F_3 \underset{f_1}{\times} \mathbb{Z}$.

Let define $A_f = \left\{ u \in H(X, \mathbb{R}) \,\middle|\, \begin{array}{l} u = [z], z \in Z' \\ z(e) > 0 \end{array} \right\}$.
for all edges in X

If $u' \in A_f$, you a fiberation $p: X_f \to S^1$.

s.t. it is local diff on flow lines.

$(p')_* = u' : G \to S^1$.

no vertices. $(p')^{-1}(pt) = \Theta_{u'}$ a finite graph.

u' is primitive $\Leftrightarrow$ $\Theta_{u'}$ is connected.

and get a first return map: $f_{u'} : \Theta_{u'} \to \Theta_{u'}$

So, $G \cong \pi_1(\Theta_{u'}) \times_{f_{u'}} \mathbb{Z}$.

$H_1 = \langle t, g \rangle$

$\phi \in Out(F_n)$ There are many representatives $f: \Gamma \to \Gamma$.

Does $A_f = A_{f'}$, whenever $[f'_*] = [f_*]$?

$B_{R r_i}$ — Neuman. Streible defined an open subset $S \subset \mathbb{R}^b$.

In particular $u \in Hom(G, \mathbb{R})$
$ker(u)$ f.generated $\Leftrightarrow u \subset \Sigma \cap -\Sigma$.

Q: Is A_f a core over a component of $\Sigma \cap - \Sigma$?

If f_0 was irred. tt map $\Rightarrow$ all f_u are irredu. tt. map.

If ϕ_0 is atoroidal. $\Leftrightarrow$ (G_{ϕ_n} is Gromov-hyp.) and fully irreducible.

$\Rightarrow$ all ϕ_u are such.

Defn. $f: \Gamma \to \Gamma$ is a linear graph map is a t.t map if for each $e \in \Gamma$ and $k \in \mathbb{Z}$, $k > 0$ $f^k(e)$ is immersed. Irred. if $e, e' \in \Gamma \Rightarrow k$ s.t. $f^k(e)$ maps over e'.

$|f^k(\alpha)|$ grows exponentially the exp. is called the Dilitation.