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Cubical accessibility and bounds on curves on surfaces.

w/ Benjamin Beaker.

Surfaces

S_g = closed surface of genus g = 

Q: how many simple curves can be simultaneously disjointly embedded on S_g?

- Up to homotopy

A: $3g-3$ (pants decomposition)

Q: can we understand decompositions of other objects in the same way?

3-manifolds

(Primary decomposition)

Knesor, Milnor? : for M^3: closed 3-manifold, $\exists$ bound B w.r.t. M s.t. no more that B non-homotopic spheres may be disjointly embedded in M.

(idea: count tetrahedra in a triangulation)

no essential spheres

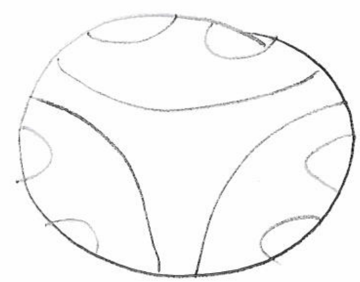
Haken: a closed irreducible 3-manifold has a bound b w.r.t. M s.t. any collection of of π_1-injective surfaces, $\mathcal{C}$ has size $|\mathcal{C}| \leq b$.

non-parallel

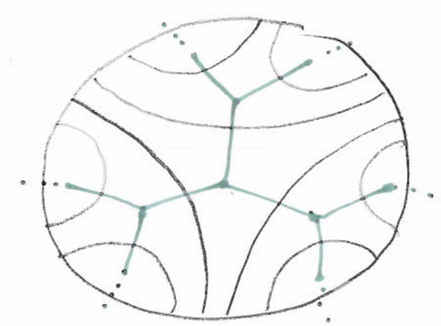
How is this related to CAT(0) geometry?

can lift curves to disjoint lifts in $\mathbb{H}^2$

$\mathbb{H}^2$



certainly $\pi_1 S_g$ acts on this "striped" $\mathbb{H}^2$ but also it acts on the dual tree:



V = region

E = adjacency

so want to concern ourselves with groups acting on CAT(0) spaces like the above tree.

Grushko: G : f.g acting on a tree $G \curvearrowright T$ with trivial edge stabilizers (equiv. $G = \ast_\alpha G_\alpha$)

then $|G \backslash T|$ is bounded by a constant depending only on G, really on $rk(G)$.

For generalisations on accessibility see work of Bestvina + Feighn, Dunwoody, Sela, and many more!

Dunwoody: G: f.p. $G \curvearrowright T$ w/ finite edge-stabilizers.

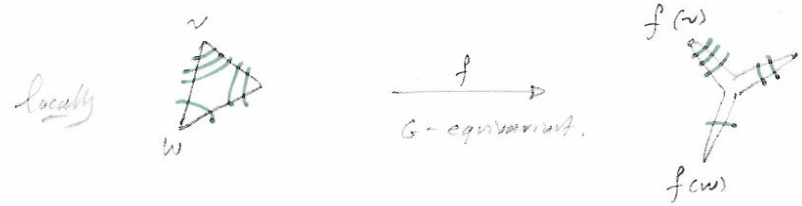
then $|G \backslash T| \leq$ constant

depends only on G

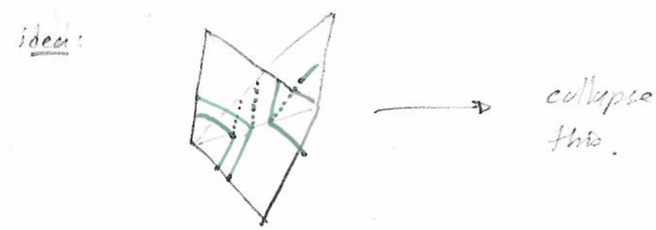
a RESOLUTION of $G \curvearrowright T$ is a tree obtained by taking.

K = standard 2-complex of G (compact $\pi_1 K = G$)

then $G \curvearrowright \tilde{K}$ and can collapse action:



$G \curvearrowright T'$ T' = dual tree to $\tilde{K}$



we call T' the resolution.

Properties

- $T' \longrightarrow T$.
- edge stabilizers of T' are all f.g. $|G \backslash T'|$ is bounded by constant depending only on G

Beaker + Lazarovich :

$\forall\, g, d \in \mathbb{N}$ with

$$\mathcal{C} = \left\{ \begin{array}{l} \text{curves on } S_g \text{ s.t.} \\ \text{at most } d \text{ lifts} \\ \text{pairwise intersect in } \tilde{S}_g = \mathbb{H}^2 \end{array} \right\} \Big/ \sim$$

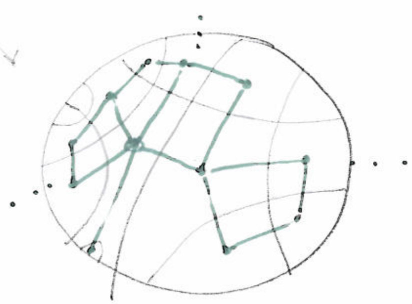
analogy

$\Rightarrow |\mathcal{C}| \leq D$ ← depending only on g, d.

$d = 1$ clear

$d = 2$ look at cube complex dual to intersection pattern of lifts.

higher-dimensions : use Sageev construction



Rk $S_g - \gamma$

↑ curve in $\mathcal{C}$

each region contributes ≤ 0 to Euler characteristic

Q : Can bound be improved ?

Beaker + Lazarovich :

K cpt 2-simplicial complex

P is a d-pattern

$\Rightarrow |P| \leq D \qquad D = D(d, K)$

- Analogously to Dunwoody's Thm.

have resolutions of

$G^{f.p.} \curvearrowright X^d$ — CAT(0) c.c. $\dim(X) = d$.

namely $\tilde{K} \longrightarrow X$

where hyperplanes can pull back to a d-pattern on $\tilde{K}$.

For $k > 0$, $\mathcal{H} = \{H < G\}$

a $(k, \mathcal{H})$-acylindrical action

has $\mathrm{stab}(\gamma) \in \mathcal{H}$

for $|\gamma| \geq k$.

<u>Sela</u>, <u>Delzant</u> : $G \curvearrowright T$

is $(k, \mathcal{H})$ - acylindrically s.t.

DNE $G \curvearrowright T'$ with edge stabilizers in $\mathcal{H}$

$\Rightarrow$ $\exists$ bound $D = D(G, k)$ s.t.

$|{}_G \backslash T| \leq D$.

<u>Beaker + Lazarovich</u> :

$G \curvearrowright X$ $(k, \mathcal{C})$ - acylindrical

action is essential and

DNE $G \curvearrowright Y$ essential w/

$\mathcal{H}$ - hyperbolic stabilizers.

$\Rightarrow$ $\left|\text{hyperplanes}/G\right|$ is bounded