

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Neelesh Tiruvilumala Email/Phone: tiruvilu@usc.edu

Speaker's Name: Anna Sfard

Talk Title: When Words Get in Your Eyes

Date: 2 / 10 / 16 Time: 4 : 15 am pm (circle one)

List 6-12 key words for the talk: Words as tools, vocabulary as observers, mathematical discourse, observing mathematics

Please summarize the lecture in 5 or fewer sentences: The talk explores the importance of language as it pertains to observing math teaching. Observers must decide whether the mathematics that is being taught is the mathematics that they would like to see. The limitations of language make it difficult to achieve this goal. As such, the speaker proposes methods by which to change our language to more effectively achieve this goal.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
 - **Overhead:** Obtain a copy or use the originals and scan them
 - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
 - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

**When words get in your eyes:
On challenges of investigating
mathematics-in-teaching
and on the importance of paying**

attention to words
Anna Stará, The University of Haifa

CIME - MSRI Workshop Series, Berkeley, CA

10 February 2016

**Remember the two classes learning
about equations and inequalities?**

**Remember the two classes learning
about equations and inequalities?**

**7th grade class in
Montreal,
Canada**

Remember the two classes learning about equations and inequalities?

**7th grade class in
Montreal,
Canada**

**11th grade class in
Johannesburg,
South Africa**

**Remember the two classes learning
about equations and inequalities?**

**7th grade class in
Montreal,
Canada**

**11th grade class in
Johannesburg,
South Africa**

You were asked:

**Remember the two classes learning
about equations and inequalities?**

**7th grade class in
Montreal,
Canada**

**11th grade class in
Johannesburg,
South Africa**

**You were asked:
What can you say about teaching
in each of these classes?**

February 16

**Remember the two classes learning
about equations and inequalities?**

**7th grade class in
Montreal,
Canada**

**11th grade class in
Johannesburg,
South Africa**

**You were asked:
What can you say about teaching
in each of these classes?**

Remember that

**Confused? You cannot proceed
without knowing what questions
to ask about teaching?**

grade class in
Johannesburg,
South Africa

You were asked:

**What can you say about teaching
in each of these classes?**

February 16

**Remember the two classes learning
about equations and inequalities?**

**7th grade class in
Montreal,
Canada**

**11th grade class in
Johannesburg,
South Africa**

**You were asked:
What can you say about teaching
in each of these classes?**

February 16

Remember those 11th grade classes in Johannesburg, South Africa?

What were the requests?

Let me change the requests!

7th grade in
Canada
11th grade class in
Johannesburg,
South Africa

You were asked:

What can you say about teaching

in each of these classes?

Remember that...ng

...qualities?

Let me change the request:

7th grade class in
Trenton, NJ
11th grade class in
Johannesburg,
Canada
South Africa

**What questions should be asked
about teaching in each of these
classes? or these classes:**

February 16

**As observers of mathematics
teaching, we may wish to tell two
stories:**

**As observers of mathematics
teaching, we may wish to tell two
stories:**

**a story about how
the teacher
manages
classroom
interactions**

**As observers of mathematics
teaching, we may wish to tell two
stories:**

**a story about how
the teacher
manages
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**a story about
mathematics
that is being taught**

**As observers of mathematics
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teaching, we may wish to tell two
stories:**

**a story about the
teacher manages
classroom
interactions**

**a story about
mathematics
that is being taught**

As observers of math

test

In this talk, let us focus at the question:

**As an observer, how
do I decide whether
the mathematics that is
being taught
is
the mathematics I would like
to see?**

in

to

Journal

Plan of this talk

Plan of this talk

1. **Our task as observers of teaching**
What is there to be observed?

Plan of this talk

1. **Our task as observers of teaching**
What is there to be observed?
2. **What are the challenges?**
Our words as (possibly imperfect) tools

Plan of this talk

- 1. Our task as observers of teaching**
What is there to be observed?
- 2. What are the challenges?**
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- 3. Getting what we need**
Changing the way we talk

Plan of this talk

- 1. Our task as observers of teaching**
What is there to be observed?
- 2. What are the challenges?**
Our words as (possibly imperfect) tools
- 3. Getting what we need**
Changing the way we talk
- 4. Testing the new way of talking**
Can we see more?

Plan of this talk

- 1. Our task as observers of teaching**
What is there to be observed?
- 2. What are the challenges?**
Our words as (possibly imperfect) tools
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Changing the way we talk
- 4. Testing the new way of talking**
Can we see more?
- 5. Conclusions**
What have we done and was it worth doing?

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3. **Getting what we need**
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4. **Testing the new way of talking**
Can we see more?
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What have we done and was it worth doing?

February 13, 2016

**What are
the challenges
of the
question about
mathematics
in teaching?**

February 13, 2016

**What are
the challenges
of the
question about
mathematics
in teaching?**

- While observing, we often feel that what is being taught in different classrooms, although supposed to be “**the same mathematics**”, is not the same, after all

What are the challenges of the question about mathematics

We also find one preferable to the other

- While observing, we often feel that what is being taught in different classrooms, although supposed to be “**the same mathematics**”, is not the same, after all

What are
the challenges
of the
question about

We also find
one
preferable
to the other

- While observing, we often feel that what is being taught in different classrooms, although supposed to be “**the same mathematics**”, is not the same, after all

Think about what you felt while reading the transcripts from Johannesburg and from Montreal

**What are
the challenges
of the
question about
mathematics
in teaching?**

- While observing, we often feel that what is being taught in different classrooms, although supposed to be “**the same mathematics**”, is not the same, after all

What are the challenges of the question about mathematics in teaching?

- While observing, we often feel that what is being taught in different classrooms, although supposed to be “**the same mathematics**”, is not the same, at least not for all

but we cannot pinpoint the difference

The main suspect:
our vocabulary
as observers

the
que
mathematics
in teaching?

...ing, we
at what is
in
classrooms,
supposed to
be “the same
mathematics”, is not
the same, a
er all

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difference

What are the challenges of the question about mathematics in teaching?

- While observing, we often feel that what is being taught in different classrooms, although supposed to be “**the same mathematics**”, is not the same, after all

What are the challenges of the question about

- While observing, we often feel that what is being taught in different classrooms,

Claim:

The words we use to describe teaching (and learning) often get in our eyes and make us see **less and **worse** rather than more and better.**

Problem 1

Observer's words are vague

February 13, 2016

Problem 1

Observer's words are vague

When you say

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Observer's words are vague

When you say

“I want to see the mathematics that is being taught in the classroom”,

Problem 1

Observer's words are vague

When you say

“I want to see the mathematics that is being taught in the classroom”,

what do you mean?

Problem 1

Observer's words are vague

When you say

“I want to see the mathematics that is being taught in the classroom”

what do you mean?

Problem 1

Observer's words are vague

When you say

"I want to see the mathematics that is being taught in the classroom",

what do you mean?

A light orange speech bubble with a tail pointing towards the word "classroom" in the text above. The text inside the bubble is written in a bold, black, sans-serif font and is rotated slightly counter-clockwise.

What is it? How do I find it? Where is it?

Problem 1

Observer's words are vague

When you say

"I want to see the mathematics that is being taught in the classroom",

what do you mean?

What is it? How do I find it? Where is it?

"mathematical"

Problem 1

Observer's words are vague

When you say

"I want to see the mathematics that is being taught in the classroom",

what do you mean?

What is it? How do I find it? Where is it?

"mathematical

"concept(ion) of function"

February 16

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**Indeed, we speak about what is
happening the classroom in terms
of **form** and **content****

**Indeed, we speak about what is
happening the classroom in terms
of **form** and **content****

**a story about how
the teacher
manages
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Indeed, we speak about what is happening the classroom in terms of **form** and **content**

a story about how the teacher manages **classroom interactions**

a story about **mathematics** that is being taught

Indeed, we speak about what is happening the classroom in terms of **form** and **content**

form
(how we teach)

a story about
mathematics
that is being taught

Indeed, we speak about what is happening the classroom in terms of **form** and **content**

form
(how we teach)

content
 (“the
mathematics”)

Indeed, we speak about **what is** happening in terms

But how can you see the content separately from its form?

form
(how we teach)

content
 (“the mathematics”)

Problem 1

Observer's words are vague

February 13, 2016

Problem 1

Observer's words are vague

When the words the observer uses to describe what she is looking for are not defined in clear terms,

Problem 1

Observer's words are vague

When the words the observer uses to describe what she is looking for are not defined in clear terms, how can she know what to look at?

Problem 1

Observer's words are vague

**When the words the observer uses to describe what she is looking for are not defined in clear terms,
how can she know
what to look at?**

**How can the recipient know from her report
what was actually done in the classroom?**

Problem 2

Observer's words are too broad

February 13, 2016

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Problem 2

Observer's words are too broad

Look at each of the two pairs:

Problem 2

Observer's words are too broad

Look at each of the two pairs:

Half times one fifth
equals one tenth

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Half of one fifth
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Problem 2

Observer's words are too broad

Look at each of the two pairs:

Half times one fifth
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If I **extract** a square root from x and **raise** the result to the third power, I **get** the same result as when I **raise** x to the 3rd power and **extract** square root from it

Problem 2

Observer's words are too broad

Look at each of the two pairs:

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equals one tenth

Half of one fifth
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If I **extract** a square root from x and
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3rd power and **extract** square root from
it

The 3rd power of
square root
equals **square**
root of the 3rd
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Are these pieces of “the same mathematics”?

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Observer's words are too broad

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Are these pieces of “the same mathematics”?

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Depends on whom you ask!

February 13, 2016

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Depends on whom you ask!

From a lesson:

Depends on whom you ask!

From a lesson:

**T: What is half times
one**

Depends on whom you ask!

From a lesson:

**T: What is half times
one**

fifth?

Depends on whom you ask!

From a lesson:

T: What is half times
one

fifth?

S: <remains silent>

Depends on whom you ask!

From a lesson:

T: What is half times
one

fifth?

S: <remains silent>

T: Half times one
fifth?

Depends on whom you ask!

From a lesson:

T: What is half times
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fifth?

S: <remains silent>

T: Half times one
fifth?

S: aaa...

Depends on whom you ask!

From a lesson:

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T: What is half times one fifth?

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**An expert has
difficulty seeing
as different
what a novice
cannot see as
the same**

Depends on whom you ask!

From a lesson:

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**An expert has
difficulty seeing
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Problem 2

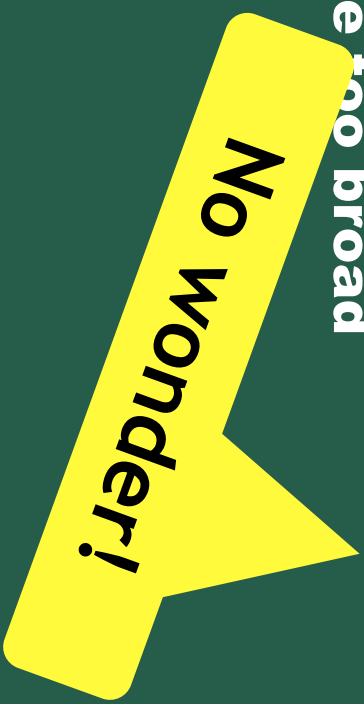
Observer's words are too broad

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Problem 2

Observer's words are too broad



Problem 2

Observer's words are too broad

No wonder!



Henri Poincaré

Problem 2

Observer's words are too broad

No wonder!

**Mathematics
is the science
of giving
the same name
to different
things**



Henri Poincaré

Observer's dilemma 1

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Observer's dilemma 1

How does
observer
“bracket” her
own
understanding of
mathematical
words

Observer's dilemma 1

**How does
observer
“bracket” her
own
understanding of
mathematical
words**

**so as to be able
to notice the
(different) use of
the same words
by the observed
(teacher,
student)?**

Observer's dilemma 1

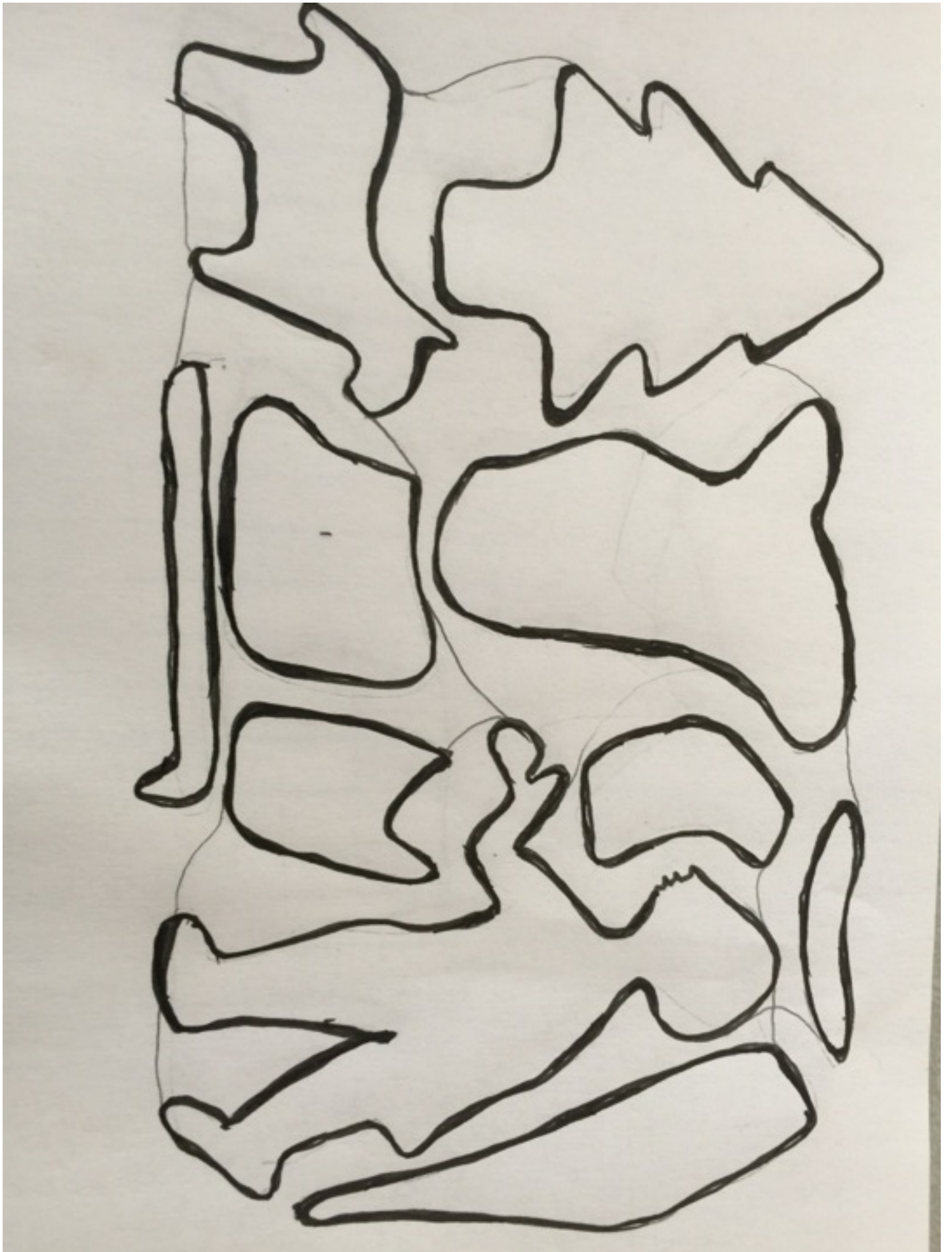
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Observer's dilemma 1

**How does
observer
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own
understanding of
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words**

**so as to be able
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(different) use of
the same words
by the observed
(teacher,
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Problem 3

Observer's words blind them to the unnamed

February 13, 2016

Problem 3

**Observer's words blind them
to the unnamed**

**Our words hide more than they
reveal.**

Problem 3

**Observer's words blind them
to the unnamed**

**Our words hide more than they
reveal.**

**When we look for “a concept”, we
can only notice its presence or
absence - but not what's**

happening around!

Problem 3

**Observer's words blind them to the
unnamed**

**Our words hide more than they
reveal.**

Observer's dilemma 2

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Observer's dilemma 2

To see
things, you
have to
have words
for them

Observer's dilemma 2

**To see things, you
have to have words
for them**

**But how can
you have words for
things that
you don't
see?**

Observer's dilemma 2

To see things, you
have to have words
for them

But how can
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see?

Observer's dilemma 2

**To see things, you
have to have words
for them**

**But how can
you have words for
things that
you don't
see?**

**In short, the words
we use to describe teaching are**

In short, the words

we use to describe teaching are

- **non-operational – we do not communicate with sufficient precision**

In short, the words

we use to describe teaching are

- **non-operational** – we do not communicate with sufficient precision
- **too crude** – make us oblivious to potentially consequential aspects of teaching

In short, the words

we use to describe teaching are

- **non-operational** – we do not communicate with sufficient precision
- **too crude** – make us oblivious to potentially consequential aspects of teaching
- **blind us to as-yet-unnamed and turn our reports into accounts of deficits**

How to solve it?

February 13, 2016

How to solve it?

**Change our way of talking about mathematics,
its learning and teaching**

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How to solve it?

Change our way of talking about mathematics,
its learning and teaching

so that our words become

How to solve it?

Change our way of talking about mathematics,
its learning and teaching

so that our words become

- **fully operational** (refer to things we can see, not to undefined abstract entities)

How to solve it?

Change our way of talking about mathematics,
its learning and teaching

so that our words become

- **fully operational** (refer to things we can see, not to undefined abstract entities)
- **highly sensitive** – allow to arrive at high-resolution picture of what is happening in the classroom

Plan of this talk

1. **Our task as observers of teaching**
What is there to be observed?
2. **What are the challenges?**
Our words as (possibly imperfect) tools
3. **Getting what we need**
Changing the way we talk
4. **Testing the new way of talking**
Can we see more?
5. **Conclusions**
What have we done and was it worth doing?

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Question:

Question:

What is mathematics?

Question:

What is **mathematics**?

- Just like **biology** is the study of **living things** (plants, animals)

Question:

What is mathematics?

- Just like **biology** is the study of **living things** (plants, animals)
- and as **physics** is a study of **natural things** (moving bodies, light, etc.)

Question:

What is mathematics?

- Just like **biology** is the study of **living things** (plants, animals)
- and as **physics** is a study of **natural things** (moving bodies, light, etc.)
- so is **mathematics** a study of **mathematical objects**

Question:

What is **mathematics**?

- Just like **biology** is the study of **living things** (plants, animals)
- and as **physics** is a study of **natural things** (moving bodies, light, etc.)
- so is **mathematics** a study of **mathematical objects**

But what are **mathematical objects**?

Question:

But what are mathematical objects?

Question:

But what are mathematical objects?

$$(x + 1)(x - 1) = x^2 - 1$$

What does it say?

Question:

But what are mathematical objects?

$$(x + 1)(x - 1) = x^2 - 1$$

What does it say?

When you open the brackets, you get the difference of the squares of x and 1

Question:

But what are mathematical objects?

$$(x + 1)(x - 1) = x^2 - 1$$

What does it say?

When you open the brackets, you get the difference of the squares of x and 1

When you multiply a predecessor of a number by its successor, you get the predecessor of the square of this number.

Question:

But what are mathematical objects?

$$(x + 1)(x - 1) = x^2 - 1$$

What does it say?

When you open the brackets, you get the difference of the squares of x and 1

When you multiply a predecessor of a number by its successor, you get the predecessor of the square of this number.

The product of the sum and difference of two functions is equal to the difference between their squares

Question:

But what are mathematical objects?

$$(x + 1)(x - 1) = x^2 - 1$$

What does it say?

When you open the **brackets**, you get the difference of the squares of x and 1

When you multiply a predecessor of a number by its successor, you get the predecessor of the square of this number.

The product of the sum and difference of two functions is equal to the difference between their squares

Question:

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$$(x + 1)(x - 1) = x^2 - 1$$

What does it say?

symbol

When you open the brackets, you get the difference of the squares of x and 1

When you multiply a predecessor of a number by its successor, you get the predecessor of the square of this number.

The product of the sum and difference of two functions is equal to the difference between their squares

Question:

But what are mathematical objects?

symbol

mathematical object

$$(x + 1)(x - 1) = x^2 - 1$$

When you open the brackets, you get the difference of the squares of x and 1. When you multiply a number by its predecessor, you get the difference of the squares of this number.

The product of the sum and difference of two functions is equal to the difference between their squares.

Question:

But what are mathematical objects?

$$(x + 1)(x - 1) = x^2 - 1$$

What does it say?

When you open the brackets, you get the difference of the squares of x and 1

When you multiply a predecessor of a number by its successor, you get the predecessor of the square of this number.

The product of the sum and difference of two functions is equal to the difference between their squares

Question:

But what are mathematical objects?

$$(x + 1)(x - 1) = x^2 - 1$$

What does it say?

When you open the brackets, you get the difference of the squares of x and 1

When you multiply a predecessor of a number by its successor, you get the predecessor of the square of this number

What's the difference between mathematical symbol and

mathematical objects?

The relation between symbols and mathematical objects

We say:

The relation between symbols and mathematical objects

We say:

the symbols

$(x+1)(x-1)$ and x^2-1

The relation between symbols and mathematical objects

We say:

the symbols

$(x+1)(x-1)$ and x^2-1

“represent the
same function”

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But if mathematical objects are not symbols, what are they?

The relation between symbols and mathematical objects

The relation between symbols and mathematical objects

**Perhaps this is because of this question
that Bertrand Russell famously stated:**

The relation between symbols and mathematical objects

Perhaps this is because of this question
that Bertrand Russell famously stated:

Mathematics can be described
“as a subject in which we never know
what we are talking about, nor
whether what we are saying is true.”

What makes us say:

“These pictures present the same person”?

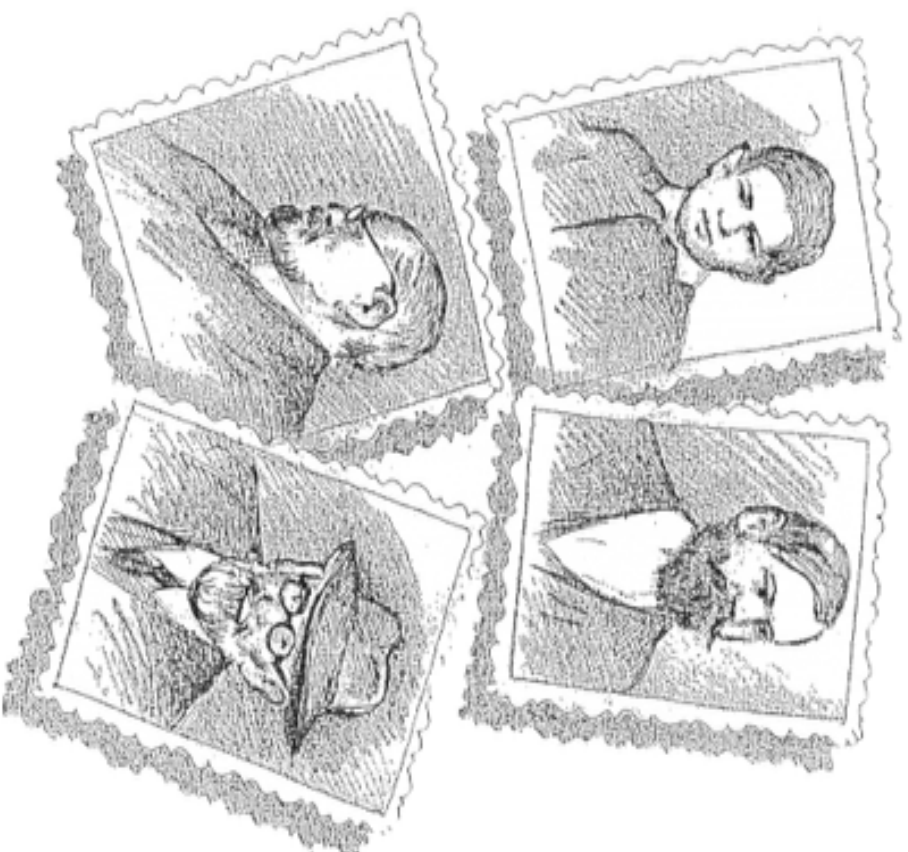
What makes us say:

“These pictures present the same person”?



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What makes us say:

“These pictures present the same person”?



**What is “the same”
about these six things?**

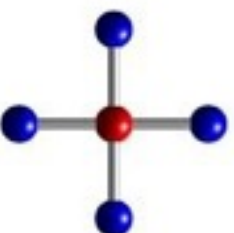
**What is “the same”
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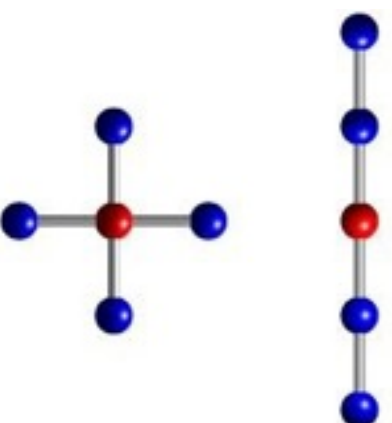
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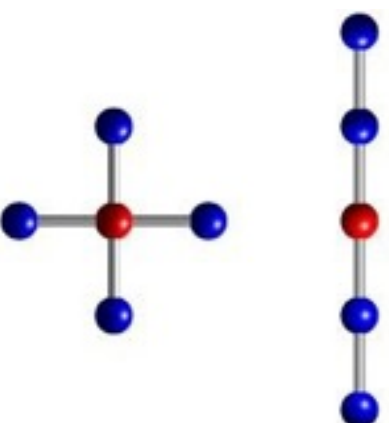
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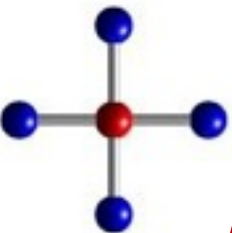
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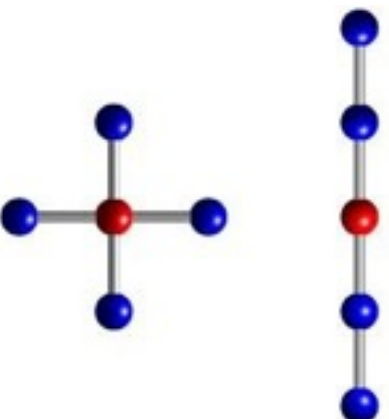


**What is “the same”
about these six things?**



**The same object is shown
in all these pictures?**

**What is “the same”
about these six things?**



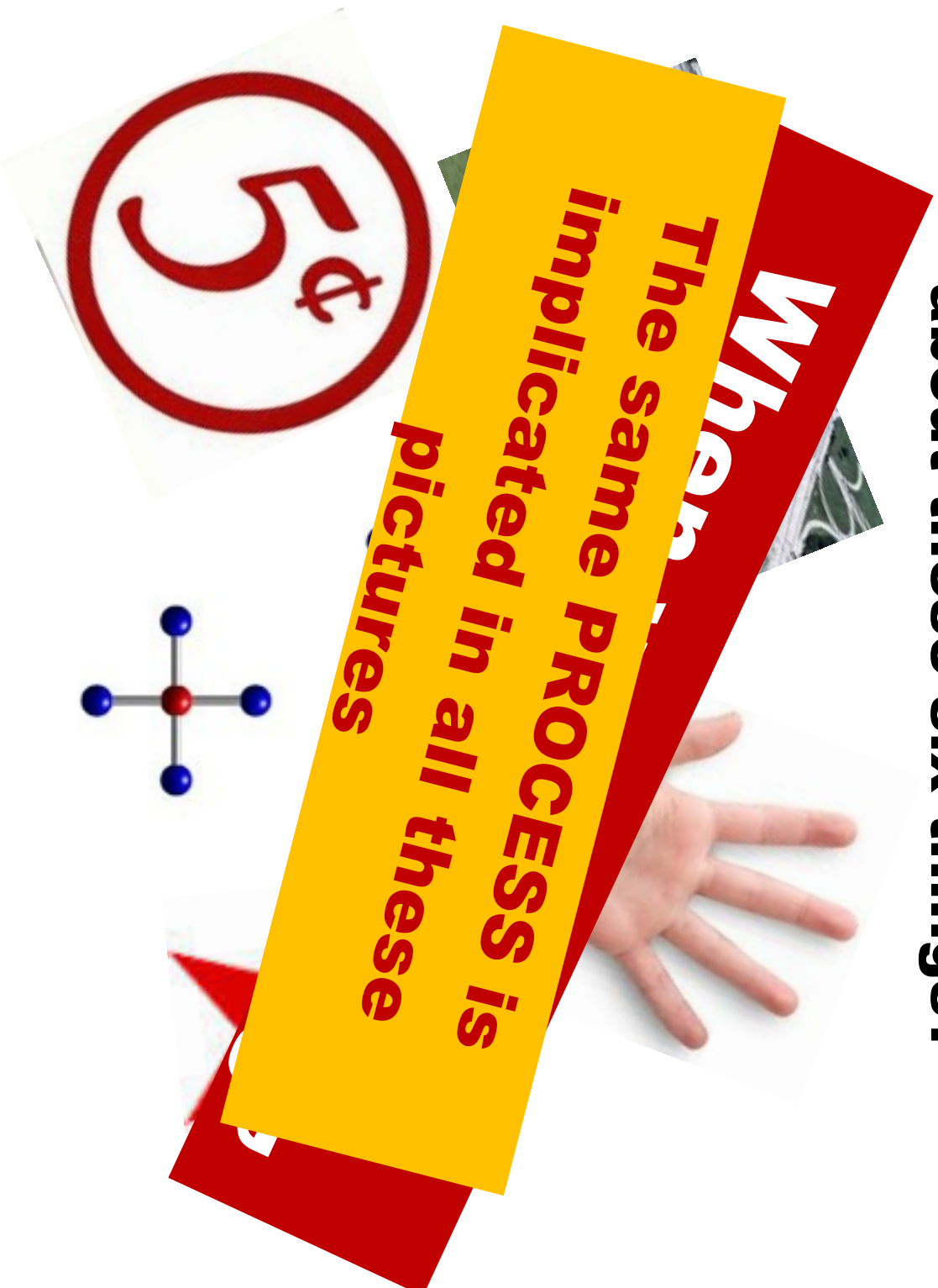
**What is “the same”
about these six things?**

**When you count,
you end with “fives”**



**What is “the same”
about these six things?**

**The same PROCESS is
implicated in all these
pictures**



conclusion

2/13/2016

conclusion

**Abstract objects are
we create them
by “recycling” forms
of speech that we
use in conversations
on material objects-**

conclusion

**Abstract objects are
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Abstract objects are
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metaphors:
by “recycling” forms
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We create
mathematical
objects by
talking about
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Abstract objects are
we create them
by “recycling” forms
of speech that we
use in conversations
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metaphors:

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talking about
them

But why should we
do such a thing?

**Why is this important whether we
objectify? What is preferable?**

Why is this important whether we objectify? What is preferable?

**What is this talking
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**This form
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the ways
we do
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Why is this important whether we objectify? What is preferable?

What is this talking in terms of mathematical objects good for?

This form of talk changes the ways we do things

For one thing, we can now say more with less

**Mathematical objects
let you say more with less**

Mathematical objects

let you say more with less

If I **extract** a square root from x and **raise** the result to the third power, I **get** the same result as when I **raise** x to the 3rd power and **extract** square root from it

Mathematical objects

let you say more with less

If I **extract** a square root from x and **raise** the result to the third power, I **get** the same result as when I **raise** x to the 3rd power and **extract** square root from it

The 3rd power of **square root** equals **square root of the 3rd power**

**Why is this important whether we
objectify? What is preferable?**

Why is this important whether we objectify? What is preferable?

If mathematics is the study of mathematical objects, you must “see” these objects to be able to do mathematics (explore and find new facts about them)



**Mathematics teacher as a juggler of
mathematical objects**

2/13/2016



**Mathematics teacher
as seen through his students' eyes**



**Mathematics teacher
as seen through his students' eyes**

2/13/2016

**Without mathematical objects, you can only do
ritualized mathematizing**

**Without mathematical objects, you can only do
ritualized mathematizing**

- **undertaken for social reasons**

**Without mathematical objects, you can only do
ritualized mathematizing**

- undertaken for social reasons
- as a discourse-for-others

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- undertaken for social reasons
- as a discourse-for-others
- performed mainly through imitation

Without mathematical objects, you can only do ritualized mathematizing

- undertaken for social reasons
- as a discourse-for-others
- performed mainly through imitation
- usually scaffolded by others

Without mathematical objects, you can only do
ritualized mathematizing

- undertaken for **social reasons**
- as a **discourse-for-others**
- performed mainly through **imitation**

**But the kind of mathematics we
wish the students to perform is**

Ritualized mathematizing

- undertaken for social reasons
- as a discourse-for-others
- performed mainly through imitation
- usually scaffolded by others

explorative mathematizing

- undertaken for social reasons
- as a discourse-for-others
- performed mainly through imitation
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explorative mathematizing

- undertaken to know more
- as a discourse-for-others
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explorative mathematizing

- undertaken to know more
- as a discourse-for-oneself
- performed mainly through imitation
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explorative mathematizing

- undertaken to know more
- as a discourse-for-oneself
- performed mainly through asking one's own questions
- usually scaffolded by others

explorative mathematizing

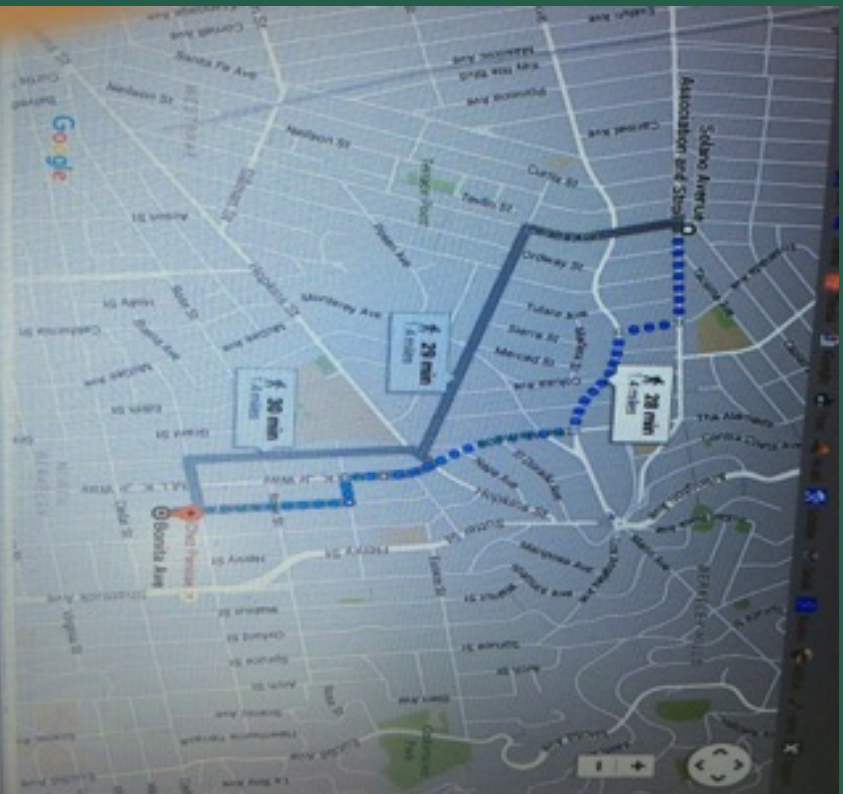
- undertaken to know more
- as a discourse-for-oneself
- performed mainly through asking one's own questions
- | **unscaffolded** by others

Types of mathematics

2/13/2016

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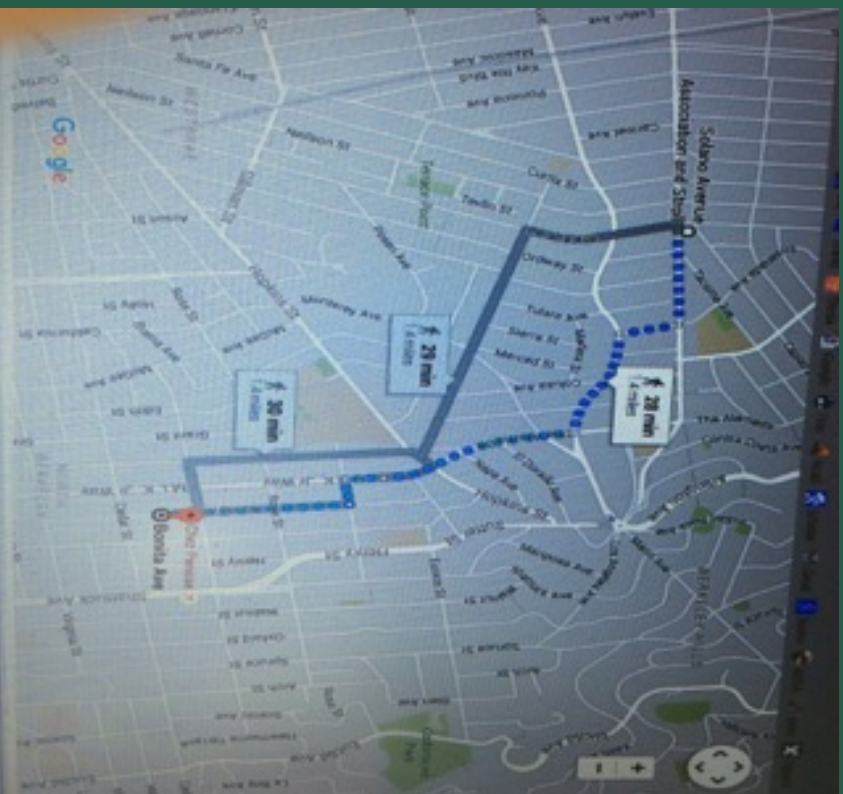
Types of mathematics



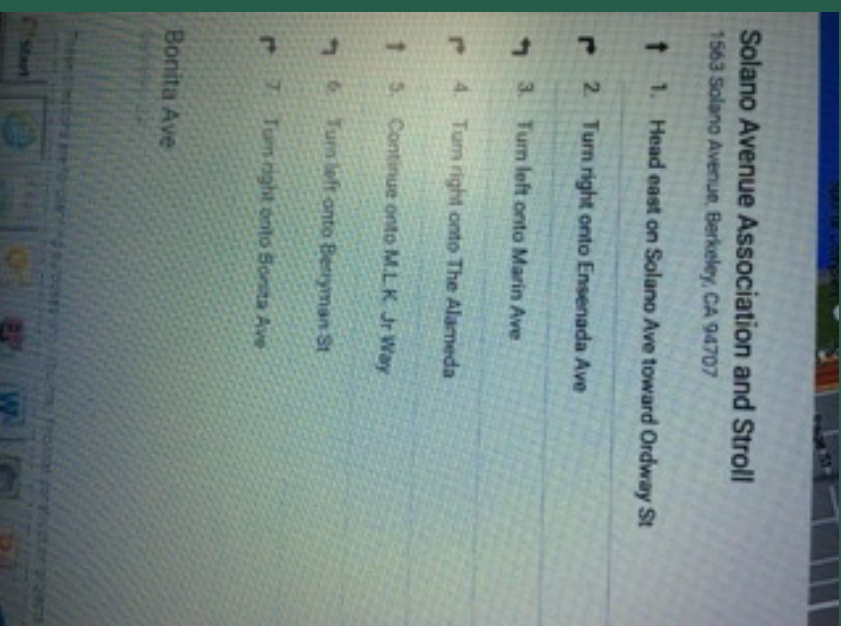
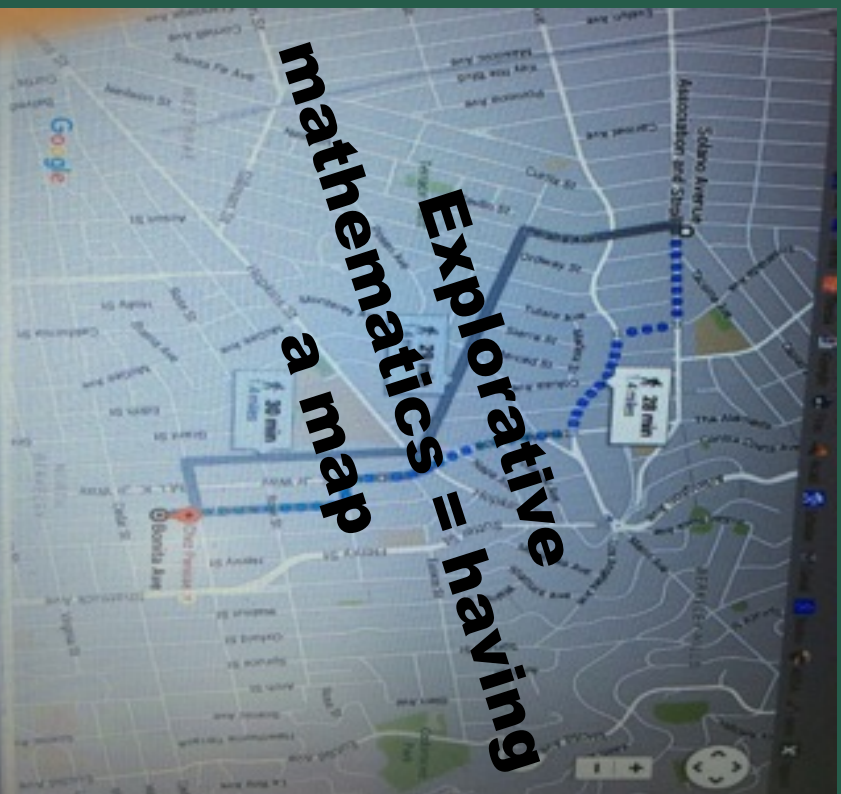
2/13/2016

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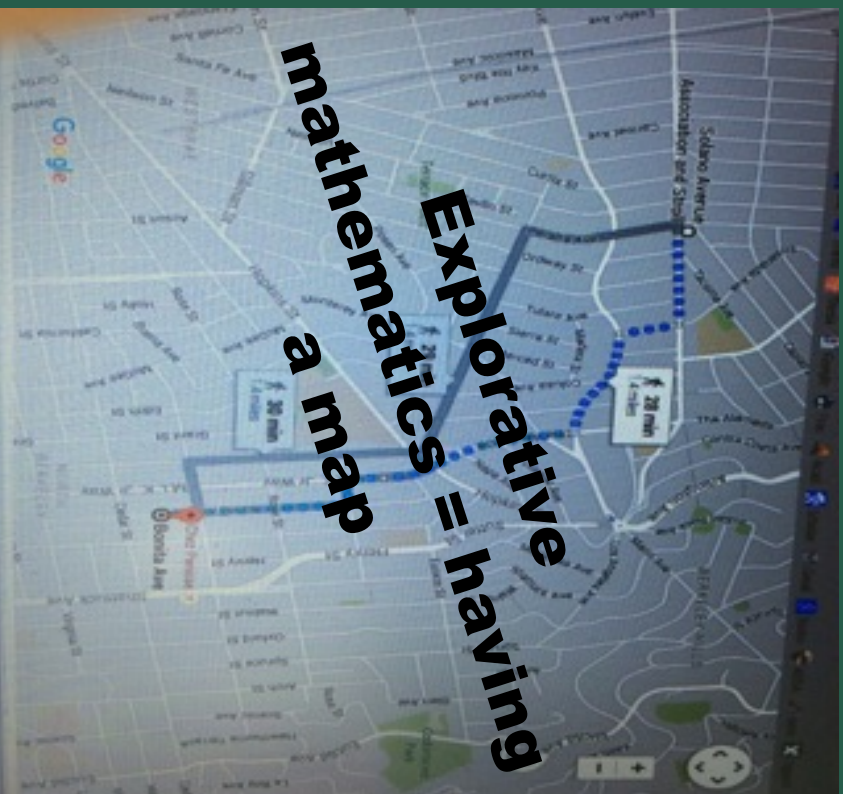
Types of mathematics



Types of mathematics



Types of mathematics



Plan of this talk

1. **Our task as observers of teaching**
What is there to be observed?
2. **What are the challenges?**
Our words as (possibly imperfect) tools
3. **Getting what we need**
Changing the way we talk
4. **Testing the new way of talking**
Can we see more?
5. **Conclusions**
What have we done and was it worth doing?

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Questions to guide observations of teaching

2/13/2016

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Questions to guide observations of teaching

What was the teacher's

Questions to guide observations of teaching

What was the teacher's

mathematizing:

Was the
teacher's own
mathematics
exploratory?

Questions to guide observations of teaching

mathematics:
Was the
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Questions to guide observations of teaching

In what kind of mathematics did the teacher try
to

mathematics:
Was the
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Questions to guide observations of teaching

In what kind of mathematics did the teacher try to

mathematics:
Was the teacher's own mathematics exploratory?

involve students:
Did the teacher's encourage student to engage in exploratory or ritualized mathematics?

Questions to guide observations of teaching

mathematics: Was the teacher's own mathematics exploratory?

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Questions to guide observations of teaching

**mathematics:
Was the
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How to tell

explorative discourse from **ritualized**?

ritualized **explorative**

How to tell

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ritualized

explorative

What is
mathematics all
about?

one's actions
with signifiers

properties of
mathematical
objects

How to tell

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What is mathematics all about?

one's actions with signifiers

properties of mathematical objects

Where do mathematical claims come from?

another person & her approval; memory

logical derivation; exploring objects, one's own argument

How to tell
explorative discourse from ritualized?

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What are the
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to become able to
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to turn
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What was the discourse all about?

The Johannesburg teacher said

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“We want to solve
[$x^2 = 4$] for x ”

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“We want to solve
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“We are saying any of
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 $[(x-2)(x+2)]$ is equal to
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The teacher said

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“We want to solve
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The teacher could have said

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"When is the product
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“We subtract 2 from [the numbers on] both sides of the equation”

What was the discourse all about?

The teacher said

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The teacher could have said

“What are the numbers x the square of which is greater than 4 ?”

“When is the product of two numbers equal 0 ?”

“We subtract 2 from [the numbers on] both sides of the equation”

“For what numbers x is the value of function x^2 greater than 4 ?”

“Two functions remain equal if we subtract the same number from both of them”

What was the discourse all about?

The teacher said

“We want to solve $[x^2 > 4]$ for x ”

“We are saying any of these brackets is equal to zero”

“And then we transpose them $[2, -2]$ ”

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The teacher

Discourse on our actions with symbols

“We are saying any of these brackets is equal to zero”

“And then we transpose them [2, -2]”

The teacher could have said

“What are the numbers x the square of which is greater than 4?”

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What was the discourse all about?

The teacher

Discourse on our actions with symbols

“We are saying any of these brackets is equal to zero”

“And then we transpose them [2, -2]”

The teacher could have said

Discourse on the properties of numbers

“What are the numbers x the square of which is greater than 4?”

“For what numbers x is the value of function x^2 greater than 4?”

“We subtract 2 from [the numbers on] both sides of the equation”

“Two functions remain equal if we subtract the same number from both of them”

What was the discourse all about?

The teacher

Discourse on our actions with symbols

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Ritualized discourse

The teacher said

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"We subtract 2 from [the numbers on] both sides of the equation"

"Two functions remain equal if we subtract the same number from both of them"

"And then we transpose them [2, -2]"

What was the discourse all about?

The teacher said

**Ritualized
discourse**

"We are saying
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to zero"

"And then we
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-2]"

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"What are the
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**Explorative
discourse**

"We subtract 2 from
[the numbers on]
both sides of the
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two functions
remain equal if we
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“We want to solve $[x^2 > 4]$ for x ”

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The teacher said

could have said

Note: The abbreviated expressions are not “forbidden”!

The problem arises only if they are given exclusivity or dominance.

“We w-

[x²

numbers x

-2]

system”

from both of

same

re

Conclusion (tentative)

2/13/2016

53

Conclusion (tentative)

The mathematical
discourse of J
Teacher tended to be
ritualized

What was the discourse all about?

The Montreal teacher said

What was the discourse all about?

The Montreal teacher said

"For what number of days would renting a pump from this be a better deal than renting from that?", I'm asking you..."

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"when is $C(x)$, which is Cheap Tools, less than $T(x)$, which is Tools 4 U"

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"Or I'm also asking you "when is $50 + 3x$, this function, less than $30 + 10x$?" . It's the same thing. When x is less than 9 this happens."

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All these
are
utterances
on
mathematical
objects
and their
own
"actions"

Conclusion (tentative)

2/13/2016

55


Conclusion (tentative)

The mathematical
discourse of M
Teacher tended to be
explorative

comparison

February 16

comparison



**J Teacher
spoke about
people's
actions with
symbols**

February 16

comparison

J Teacher
spoke about
people's
actions with
symbols

M Teacher
spoke about
mathematical
objects and
their own
“actions”

comparison



comparison

J Teacher spoke about math with symbols

ritualized

M Teacher spoke about math objects

explorative

comparison

J Teacher spoke about math with symbols

ritualized

M Teacher spoke about math objects

explorative

For reasons already explained,
I prefer this way of teaching

J Teacher
spoke
with
symbols

M Teacher
spoke about
mathematical
objects
the

ritualized

explorative

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Questions to guide observations of teaching

**mathematics:
Was the
teacher's own
mathematics
exploratory?**

**involve students:
Did the teacher's
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exploratory or
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Questions to guide observations of teaching

mathematics:
Was the teacher's own mathematics exploratory?

involve
Did the teacher encourage you to do the same for the M
to engage
exploratory
ritualized
mathematics?

I will perform the analysis for the J Teacher only. I encourage you to do the same for the M Teacher.

The teacher's overtures

The teacher's overtures



The teacher's overtures



mathematizing

The teacher's overtures

Referring to
mathematics

mathematizing

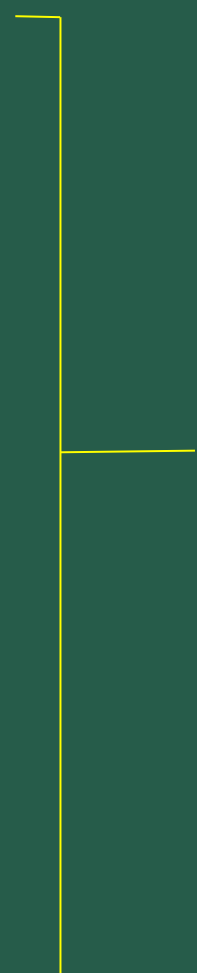


The teacher's overtures

Referring to
mathematics

mathematizing

subjectifying



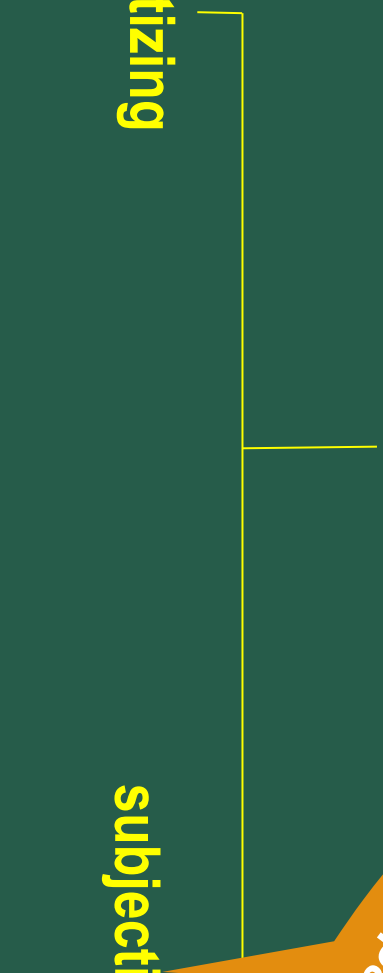
The teacher's overtures

Referring to mathematics

mathematizing

Referring to participants

subjectifying



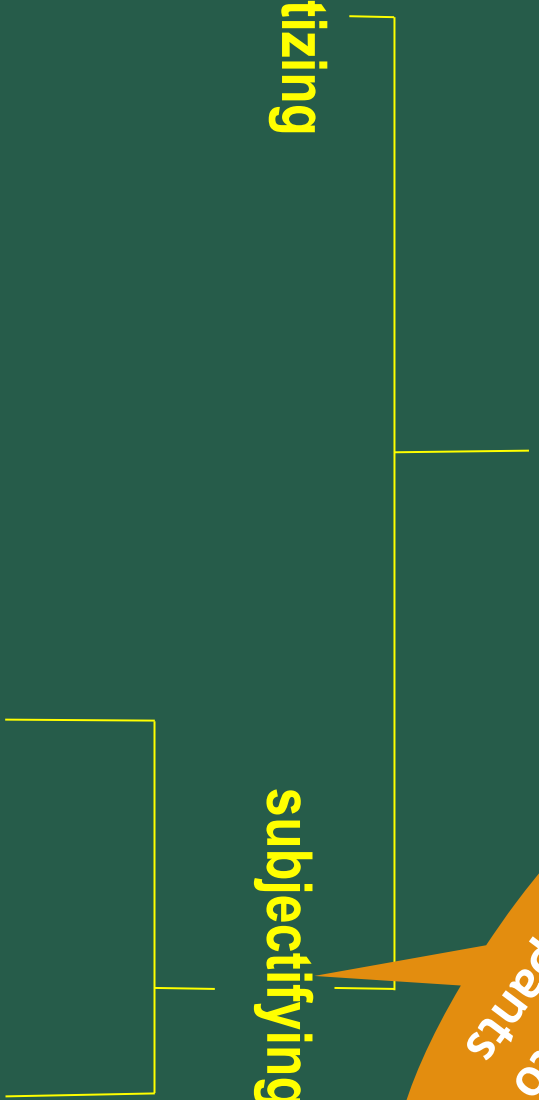
The teacher's overtures

Referring to mathematics

mathematizing

Referring to participants

subjectifying



The teacher's overtures

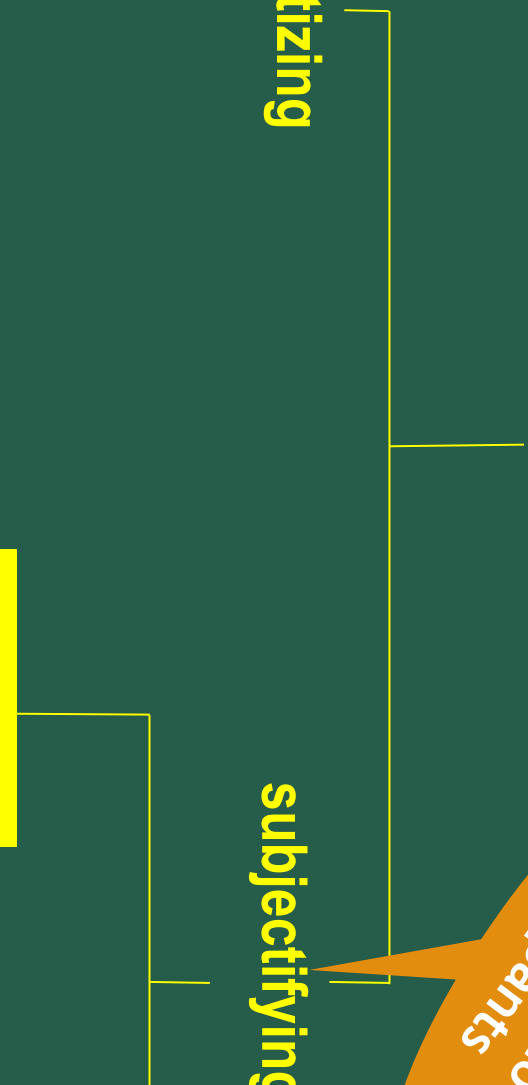
Referring to mathematics

mathematizing

Referring to participants

subjectifying

inter-
personal



The teacher's overtures

Referring to mathematics

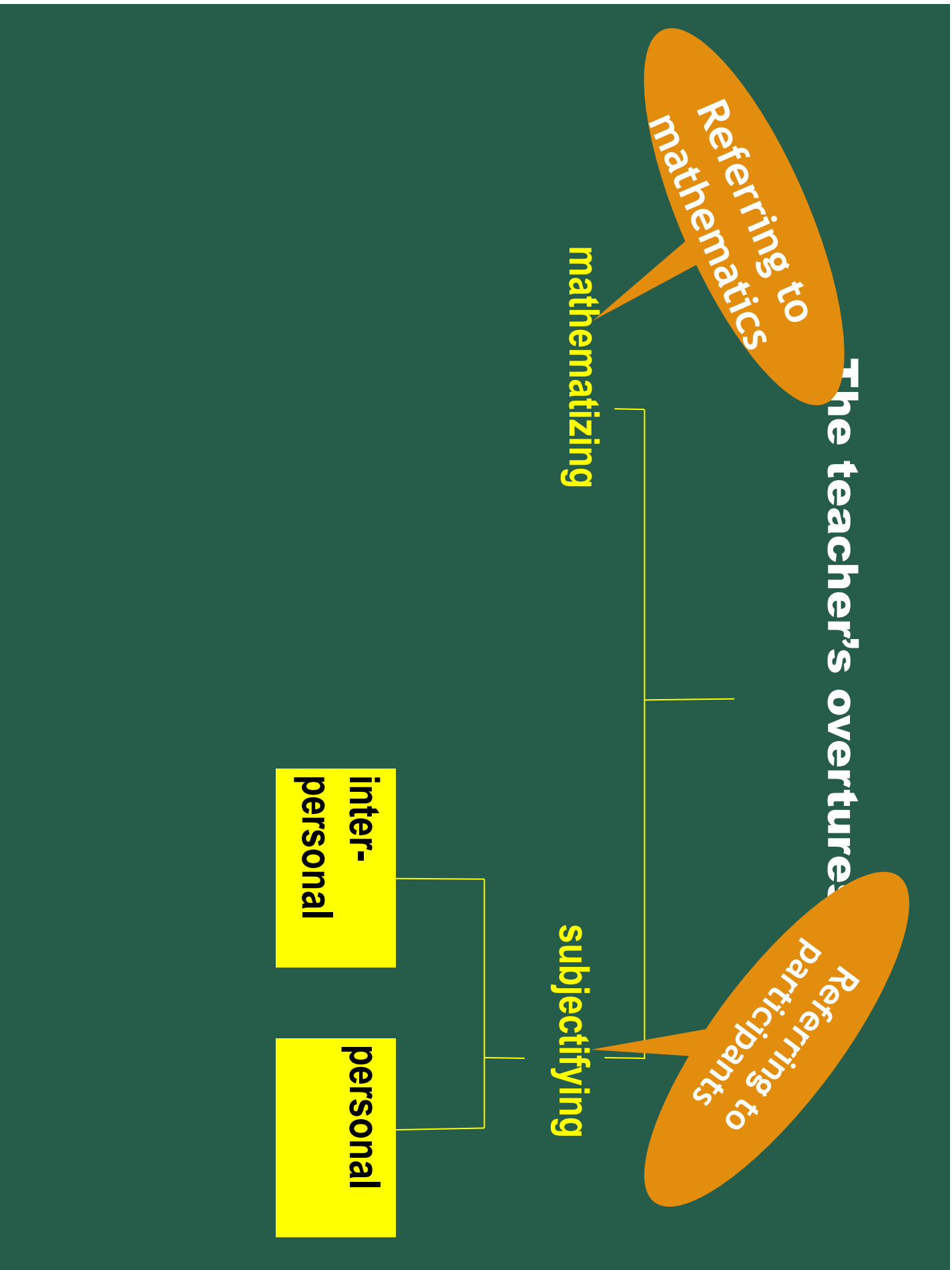
mathematizing

Referring to participants

subjectifying

inter-personal

personal



The teacher's overtures

Referring to mathematics

mathematizing

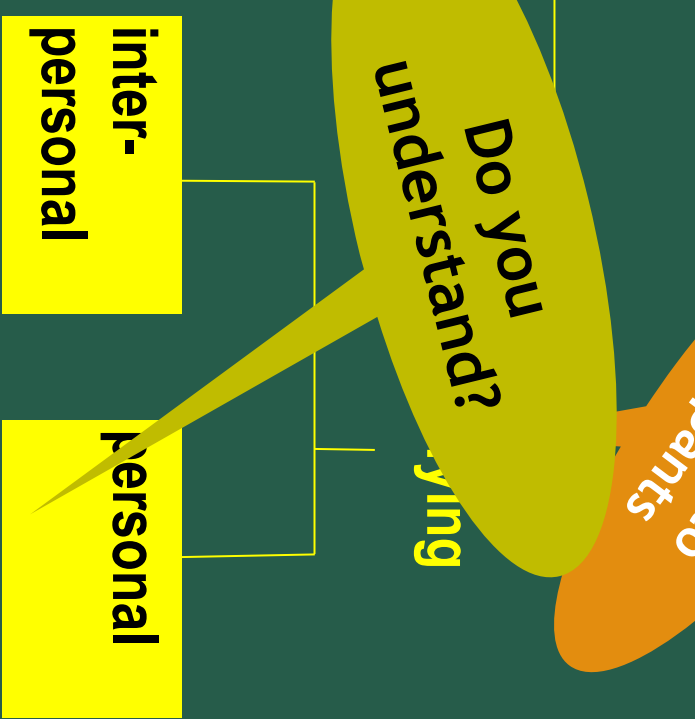
Referring to participants

Do you understand?

explaining

inter-personal

personal



The teacher's overtures

Referring to mathematics

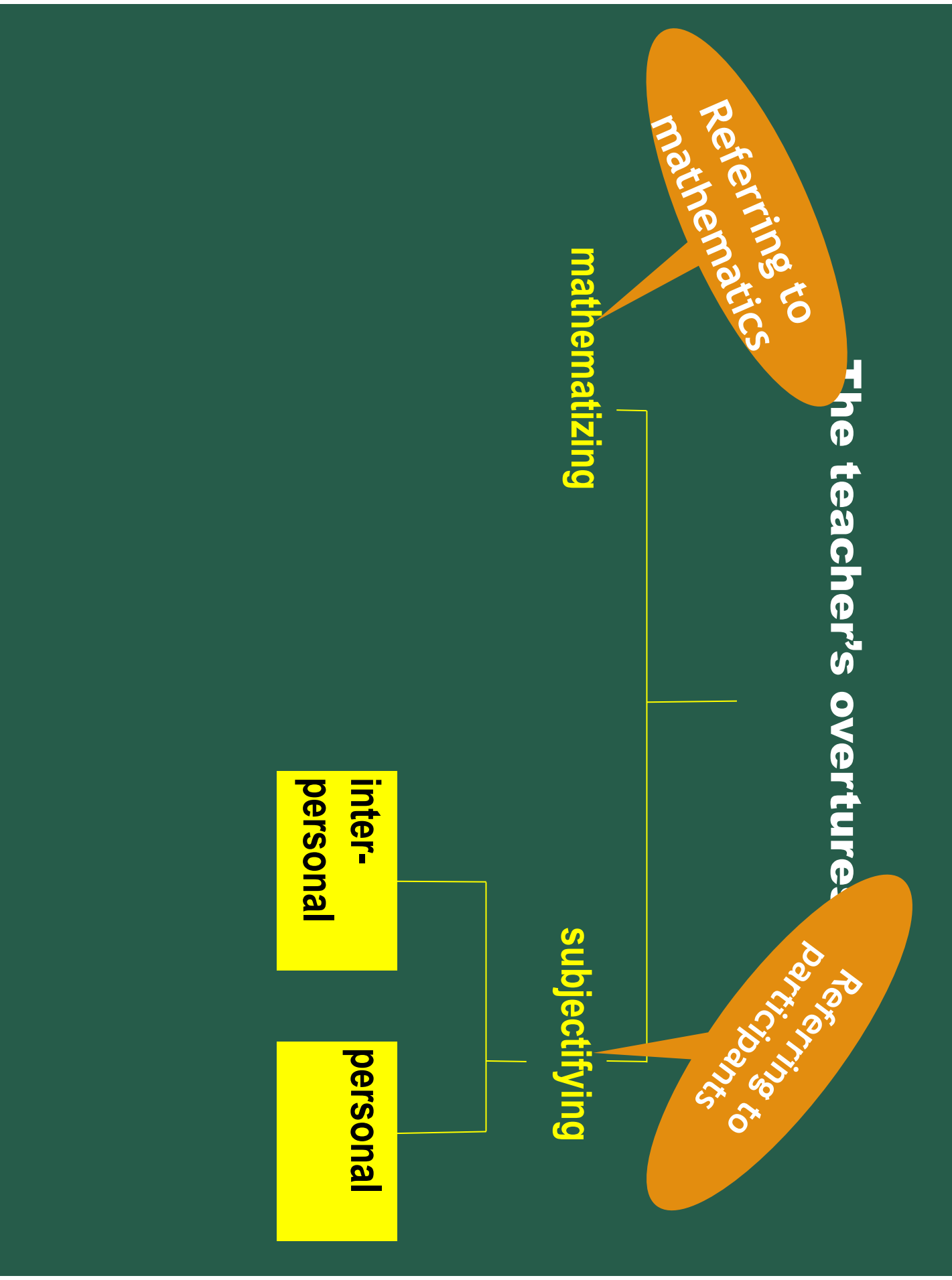
mathematizing

Referring to participants

subjectifying

inter-
personal

personal



The teacher's overtures

Referring to mathematics

mathematizing

Referring to participants

subjectifying

Are you happy?

inter-personal

personal

The teacher's overtures

Referring to mathematics

mathematizing

Referring to participants

subjectifying

inter-
personal

personal

The teacher's overtures

Referring to mathematics

mathematizing

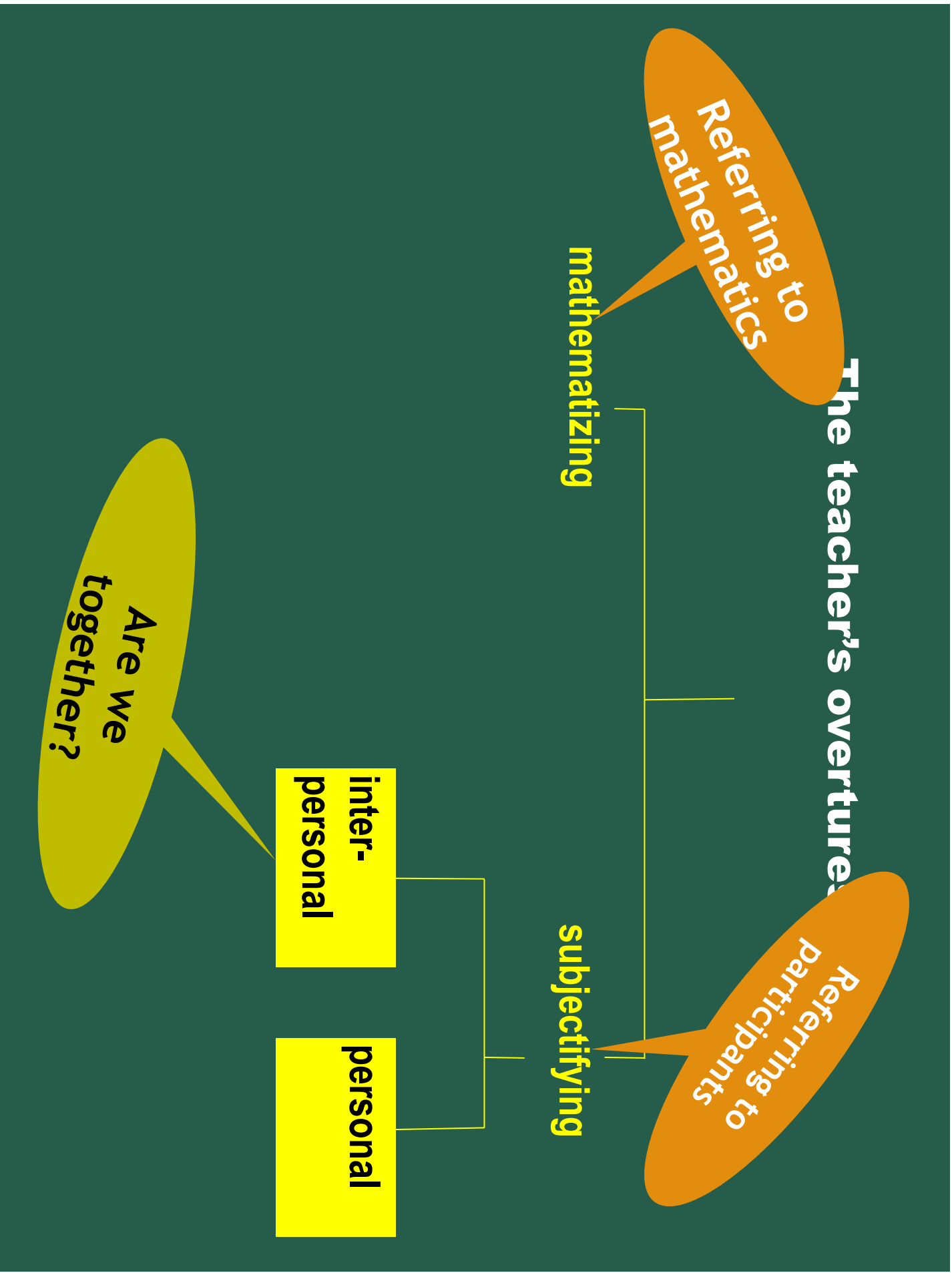
Referring to participants

subjectifying

inter-
personal

personal

Are we
together?



The teacher's overtures

Referring to mathematics

mathematizing

Referring to participants

subjectifying

inter-
personal

personal

The teacher's overtures

Referring to mathematics

mathematizing

Referring to participants

subjectifying

inter-
personal

personal

The teacher's overtures

Referring to mathematics

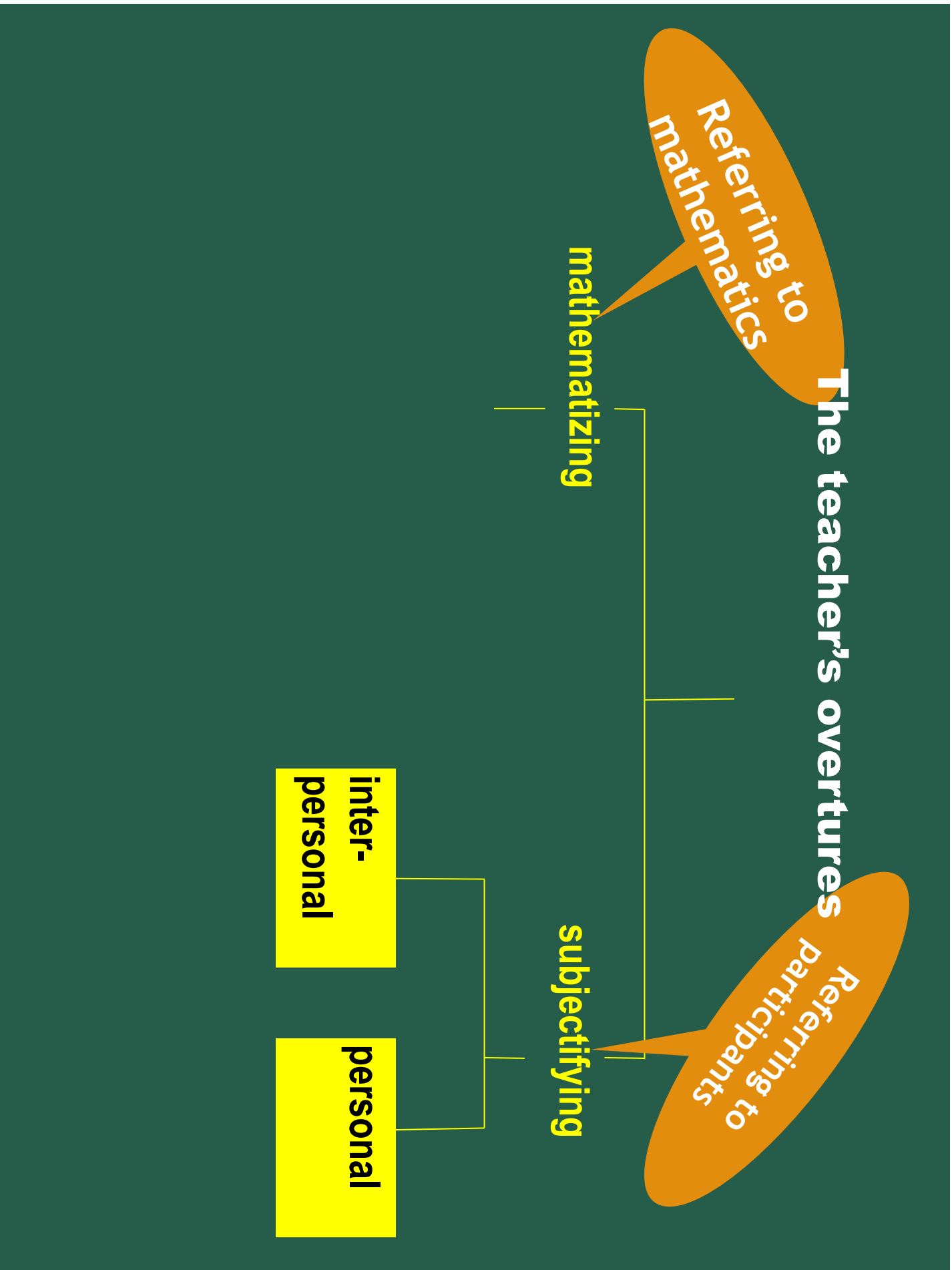
mathematizing

Referring to participants

subjectifying

inter-
personal

personal



The teacher's overtures

Referring to mathematics

mathematizing

Referring to participants

subjectifying

inter-
personal

personal

The teacher's overtures

Referring to mathematics

mathematizing

closed question

Referring to participants

subjectifying

inter-personal

personal

The teacher's overtures

Referring to mathematics

mathematizing

closed question

open question

subjectifying

inter-personal

personal

Referring to participants

The teacher's overtures

Referring to mathematics

mathematizing

closed question

open question

subjectifying

Referring to participants

inter-personal

personal

The teacher's overtures

Referring to mathematics

mathematizing

closed question

open question

subjectifying

inter-personal

personal

Referring to participants

The teacher's overtures

Referring to mathematics

mathematizing

closed question

open question

subjectifying

inter-personal

personal

Referring to participants

The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

open question

inter-personal

personal

Confirm narrative

The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

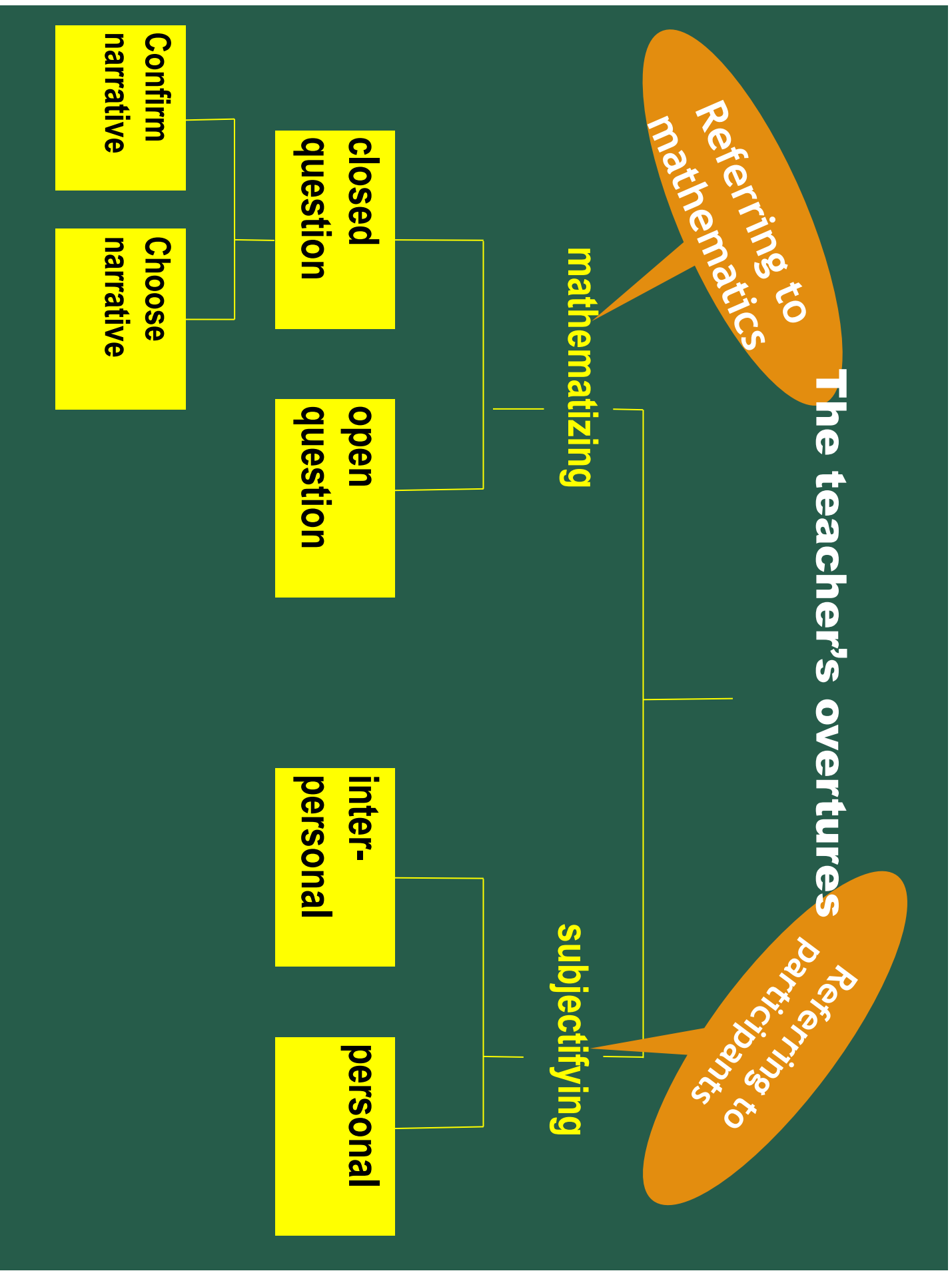
open question

inter-personal

personal

Confirm narrative

Choose narrative



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

Right?

closed question

question

inter-personal

personal

Confirm narrative

Choose narrative

The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

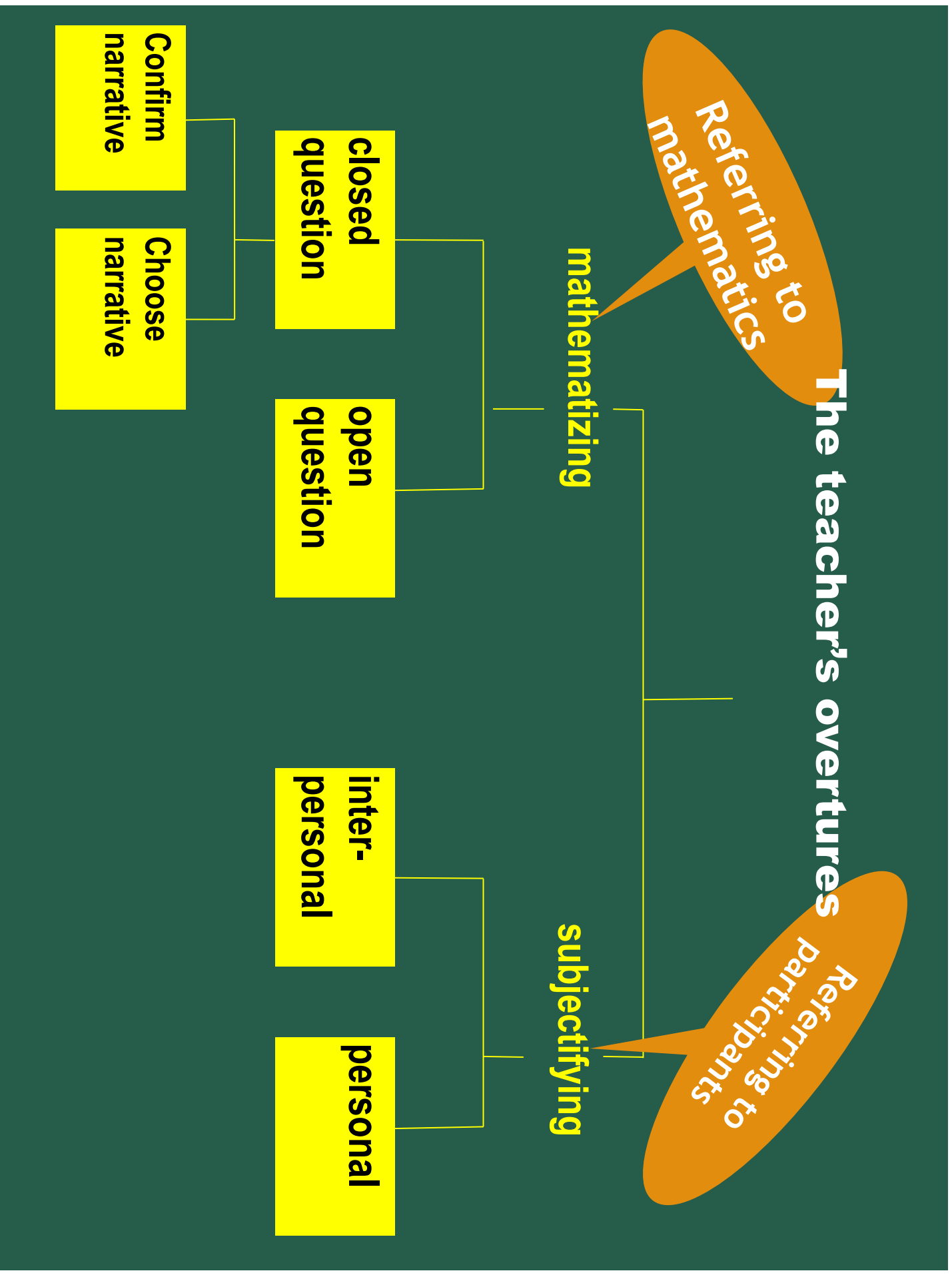
open question

inter-personal

personal

Confirm narrative

Choose narrative



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

Do you agree?

closed question

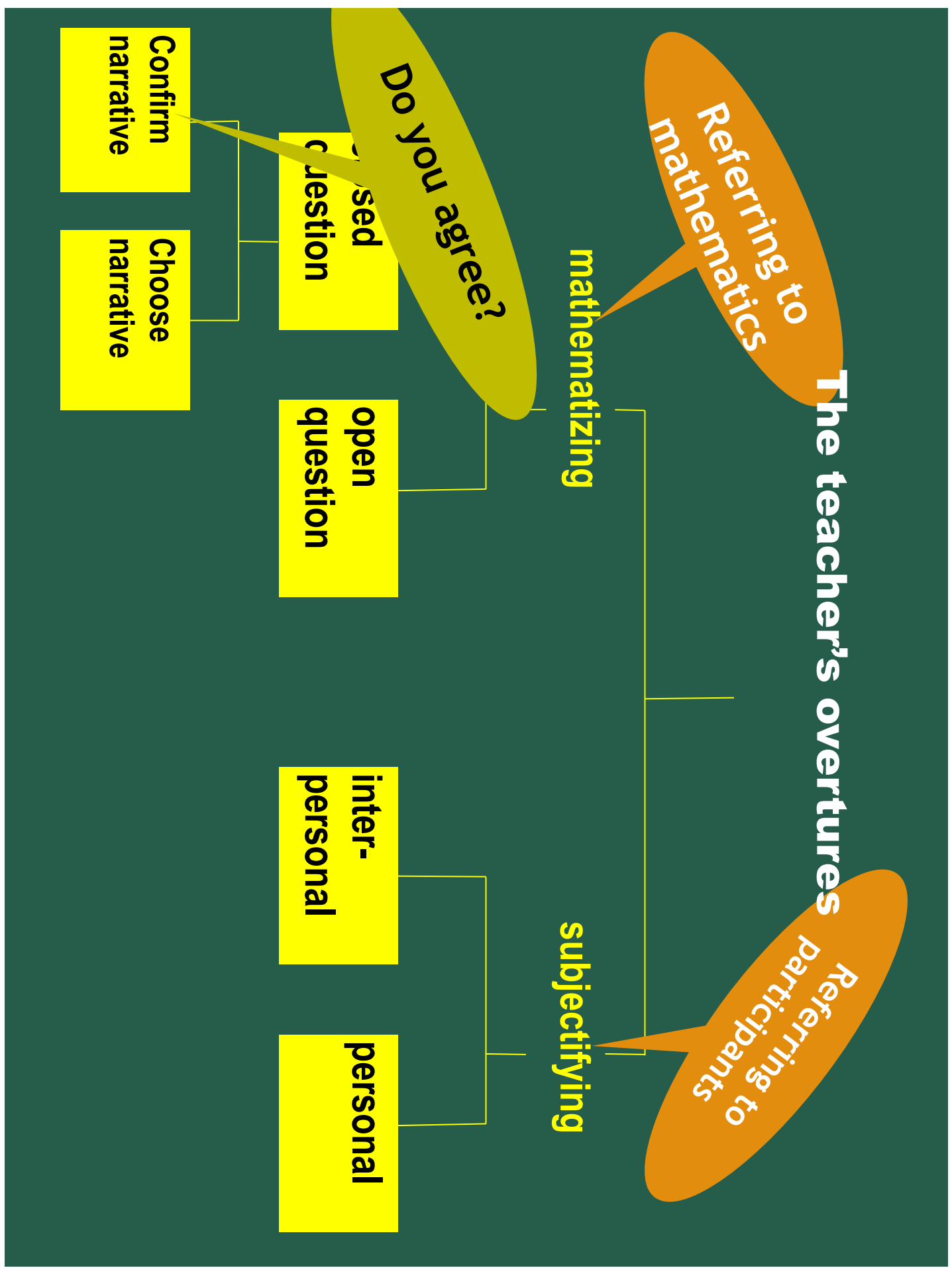
open question

inter-personal

personal

Confirm narrative

Choose narrative



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

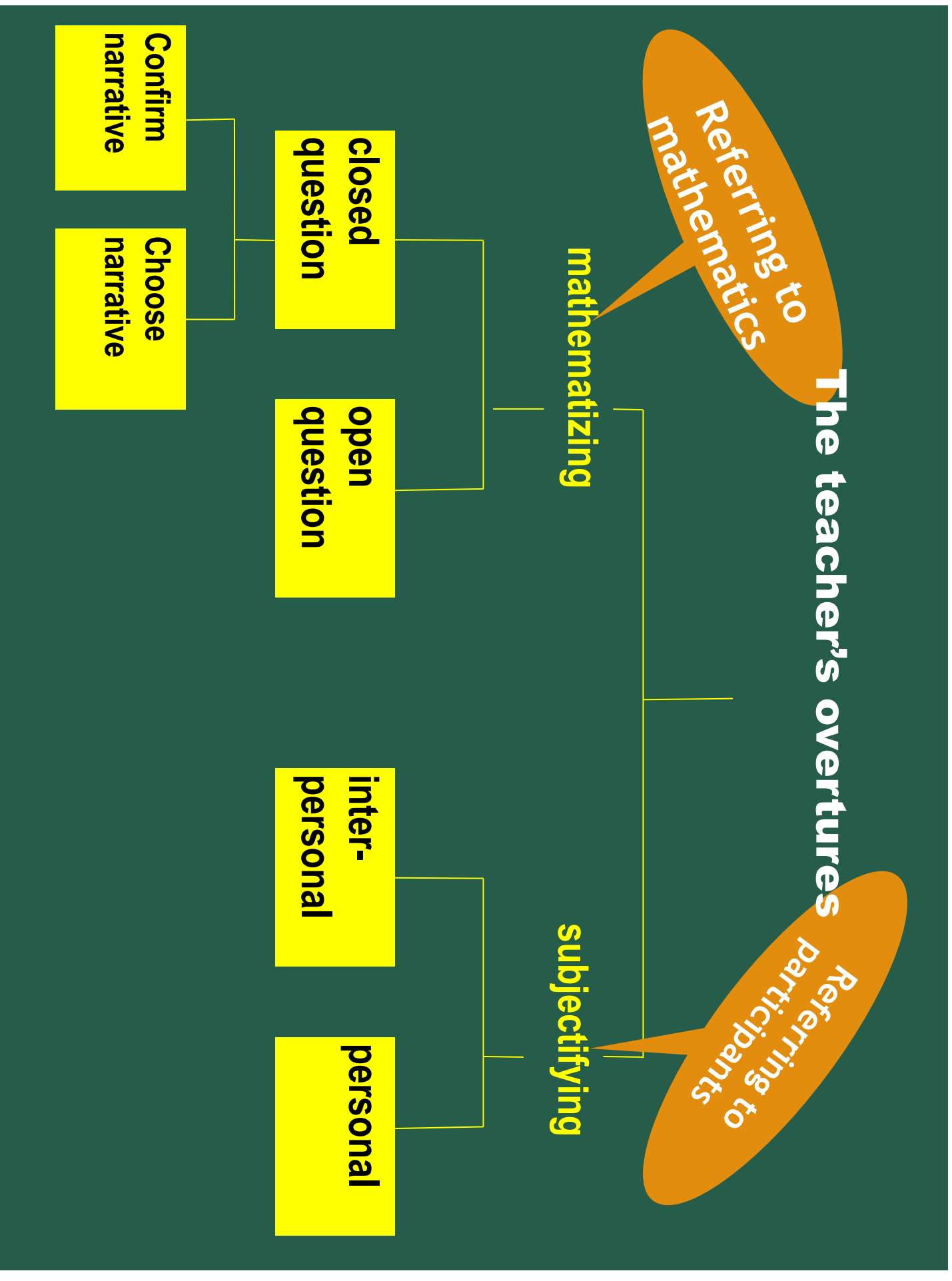
open question

inter-personal

personal

Confirm narrative

Choose narrative



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

Is 4 greater than 4?

closed question

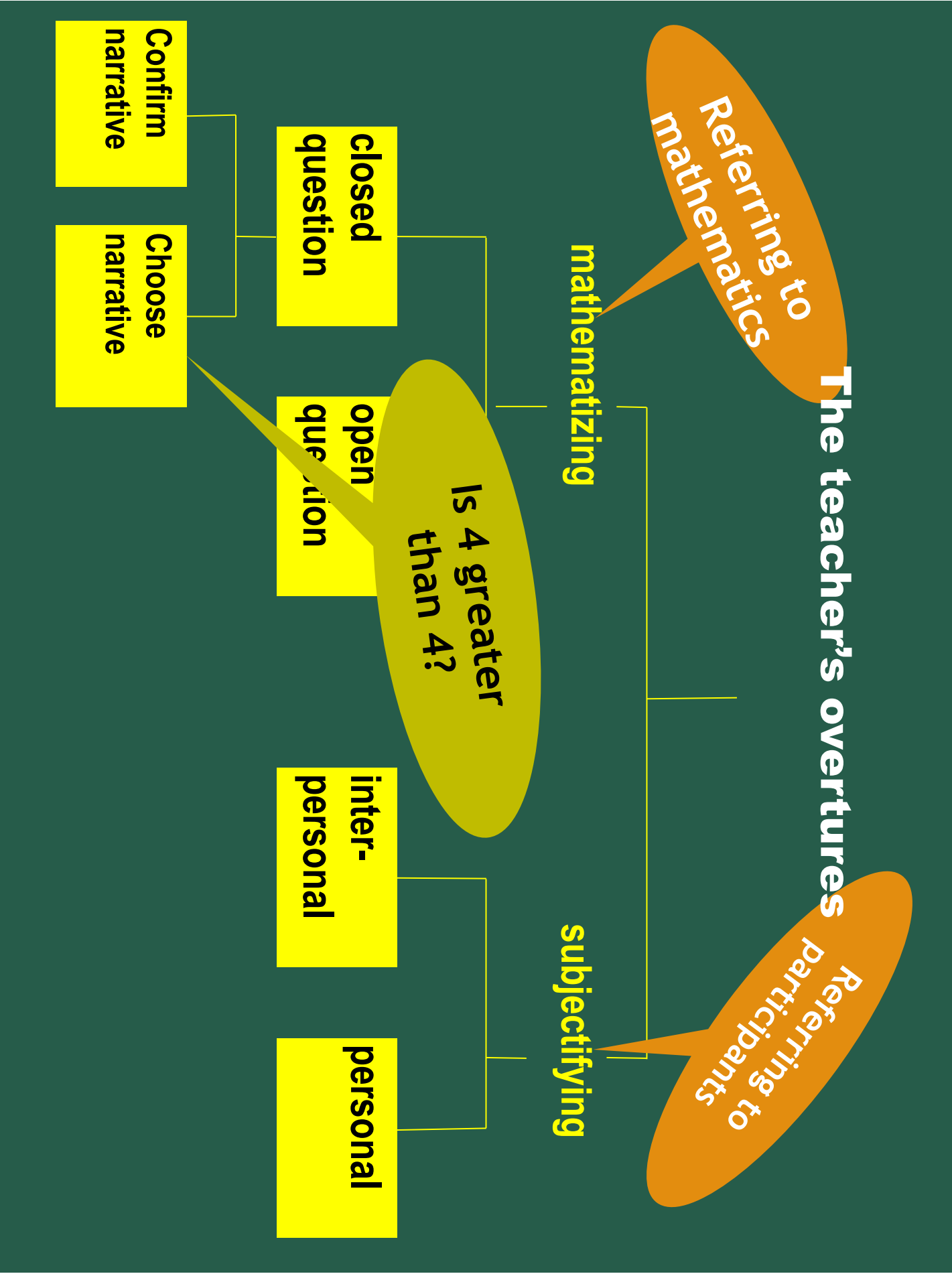
open question

inter-personal

personal

Confirm narrative

Choose narrative



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

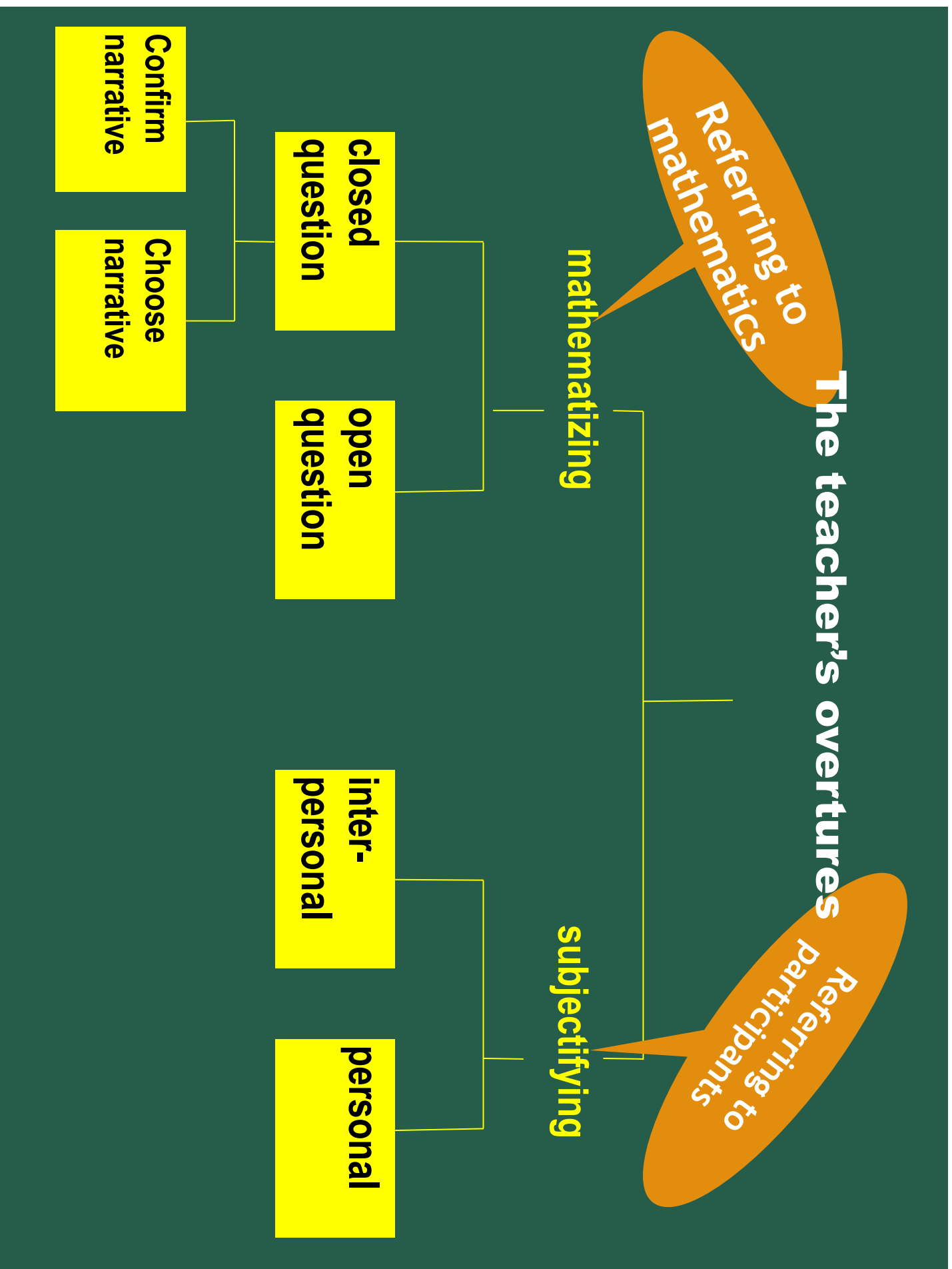
open question

inter-personal

personal

Confirm narrative

Choose narrative



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

open question

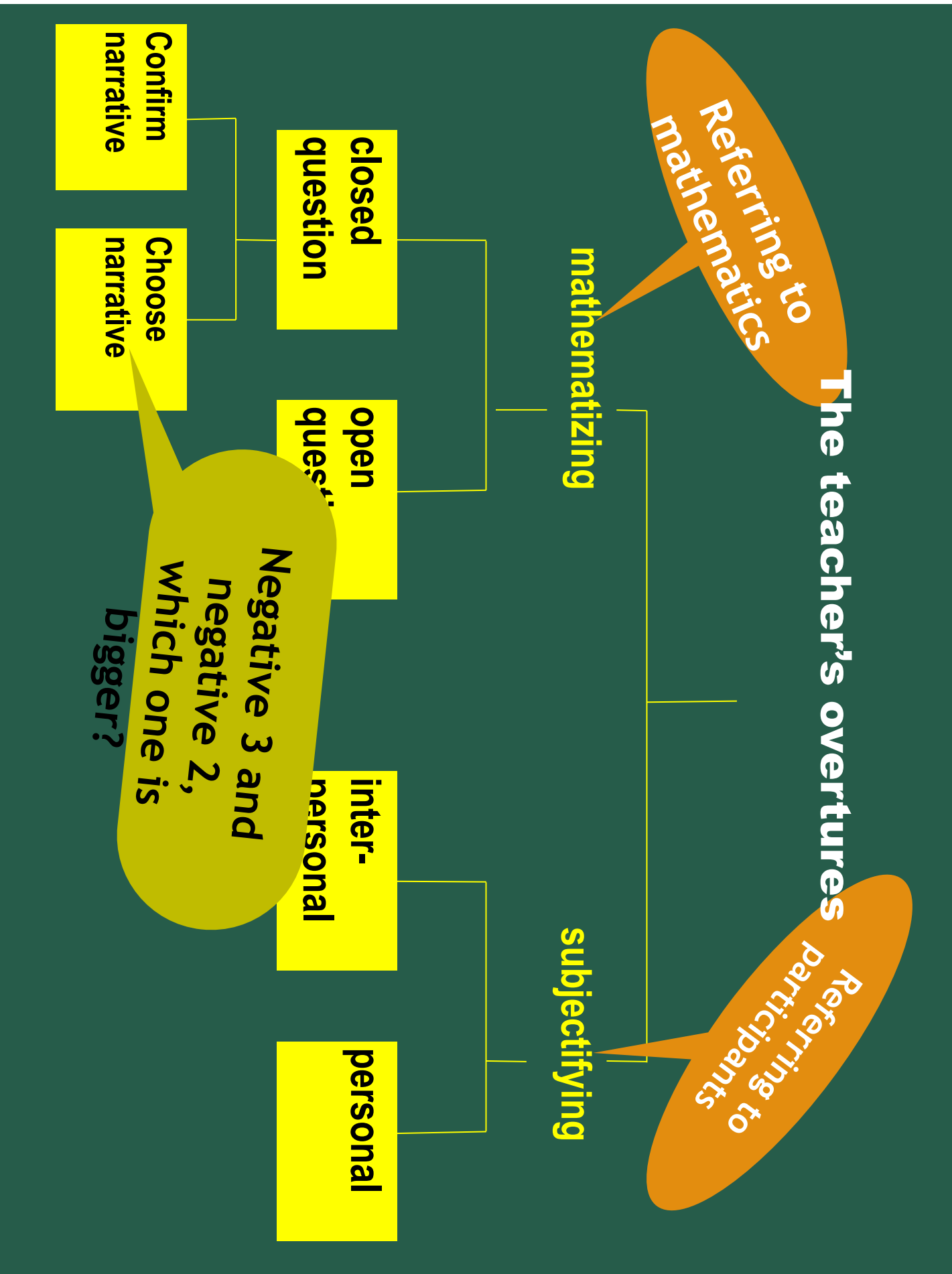
inter-personal

personal

Confirm narrative

Choose narrative

Negative 3 and negative 2, which one is bigger?



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

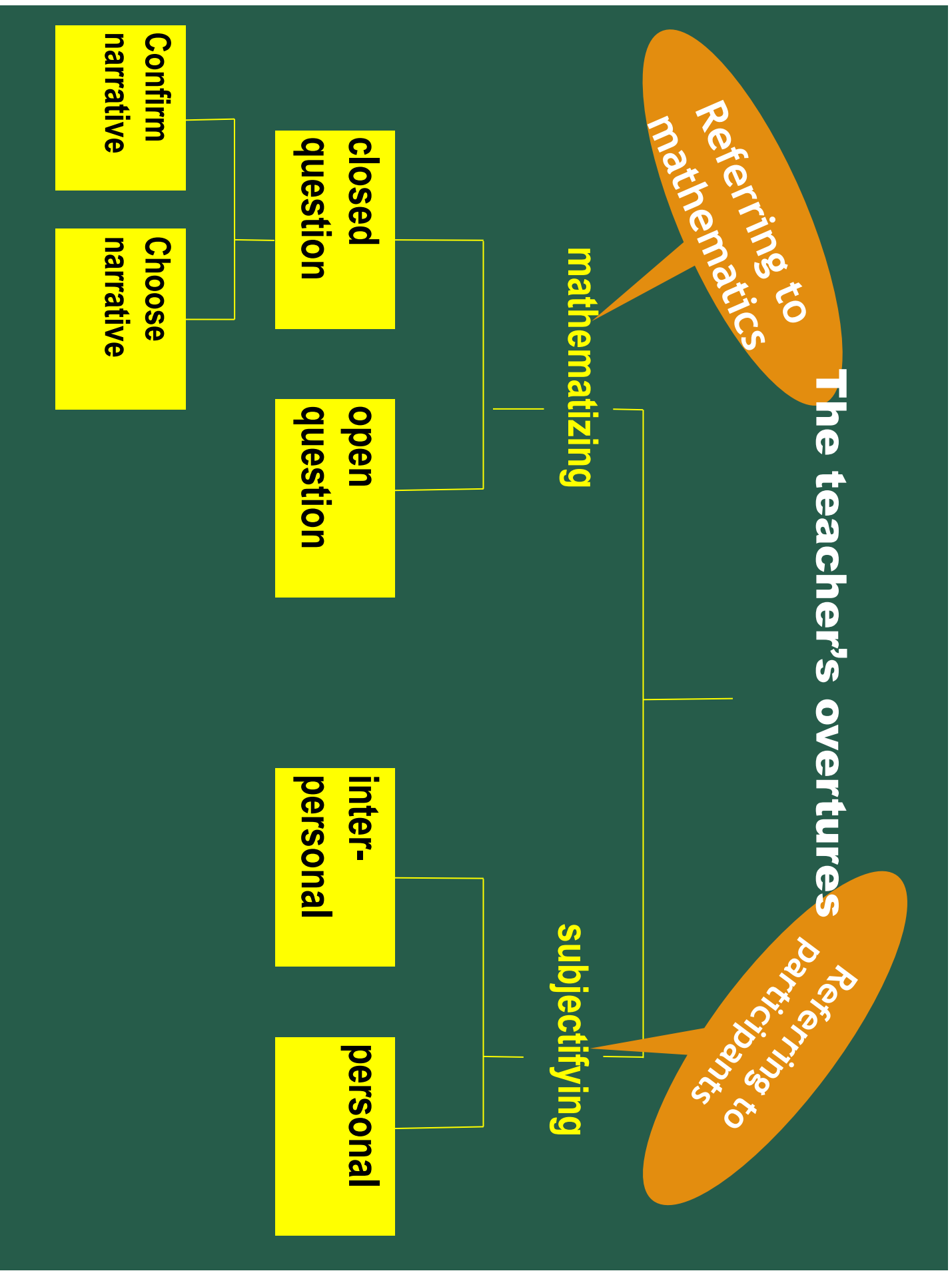
open question

inter-personal

personal

Confirm narrative

Choose narrative



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

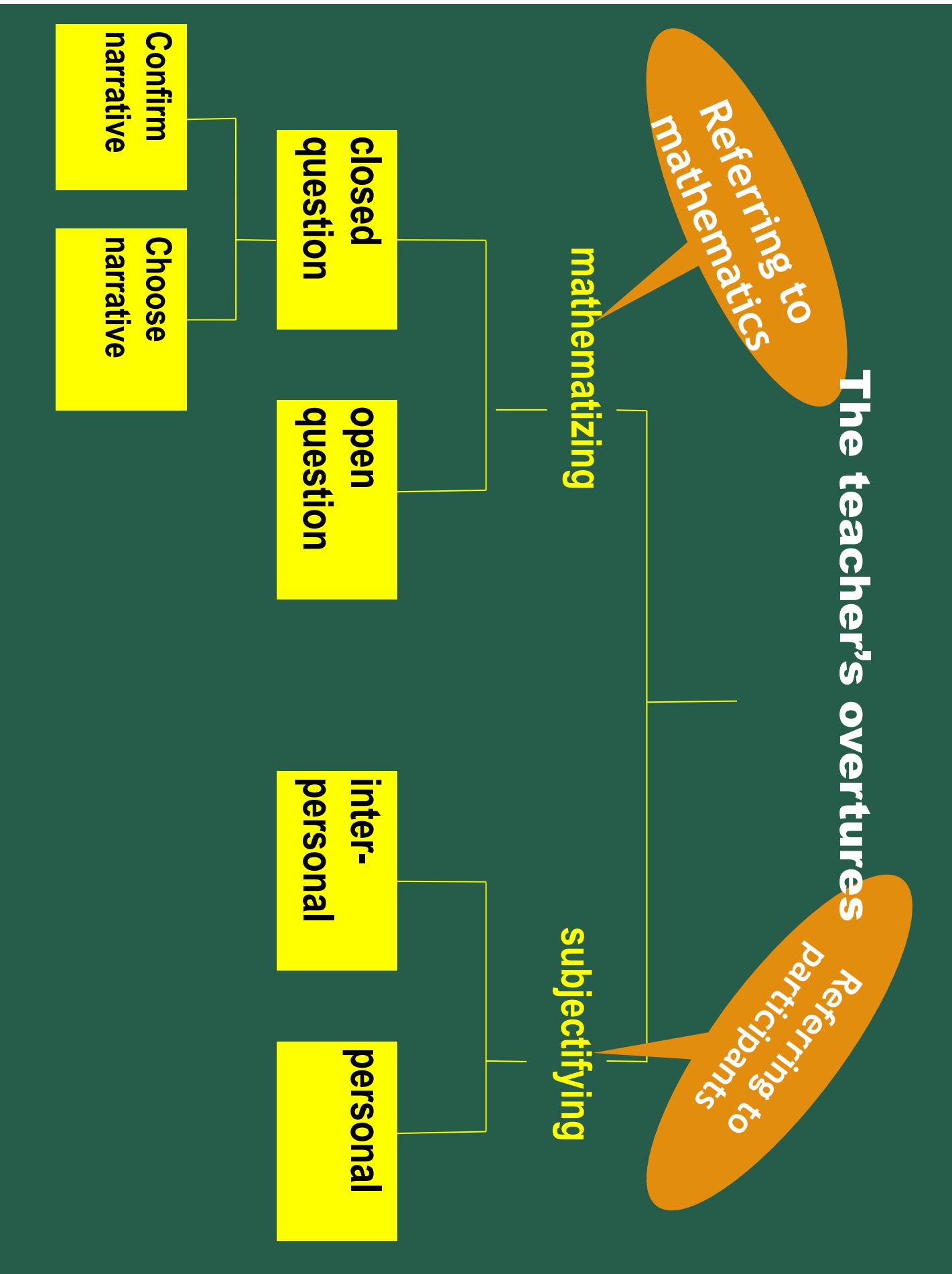
open question

inter-personal

personal

Confirm narrative

Choose narrative



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

open question

inter-personal

personal

Confirm narrative

Choose narrative

The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

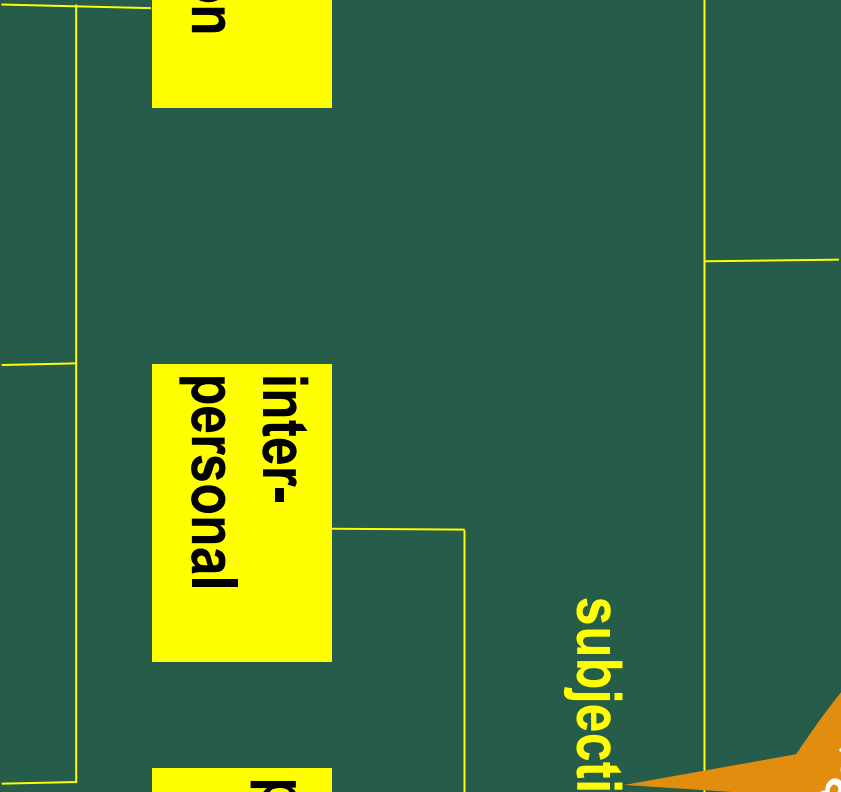
open question

inter-personal

personal

Confirm narrative

Choose narrative



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

open question

inter-personal

personal

Confirm narrative

Choose narrative

Offer narrative (answer)

The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

open question

inter-personal

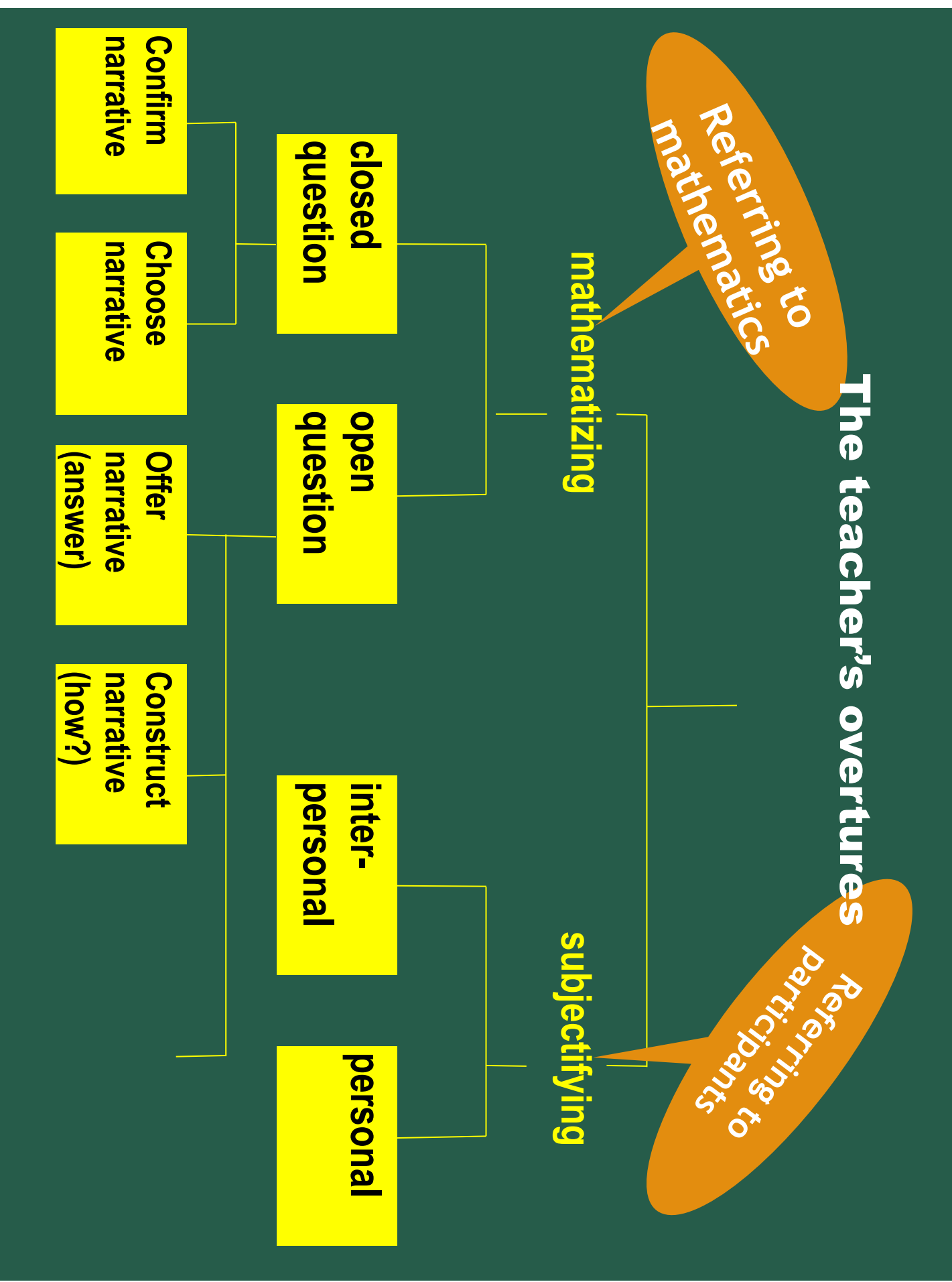
personal

Confirm narrative

Choose narrative

Offer narrative (answer)

Construct narrative (how?)



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

open question

inter-personal

personal

Confirm narrative

Choose narrative

Offer narrative (answer)

Construct narrative (how?)

Substantiate narrative (why?)

The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

open question

personal

personal

Confirm narrative

Choose narrative

Offer narrative (answer)

Construct narrative (how?)

Substantiate narrative (why?)

What are the answers?

The teacher's overtures

Referring to mathematics

mathematizing

Referring to participants

subjectifying

closed question

Confirm narrative

Choose narrative

open question

Offer narrative (answer)

Construct narrative (how?)

Substantiate narrative (why?)

inter-personal

personal

The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

What is our x equal to?

closed question

open question

inter-personal

personal

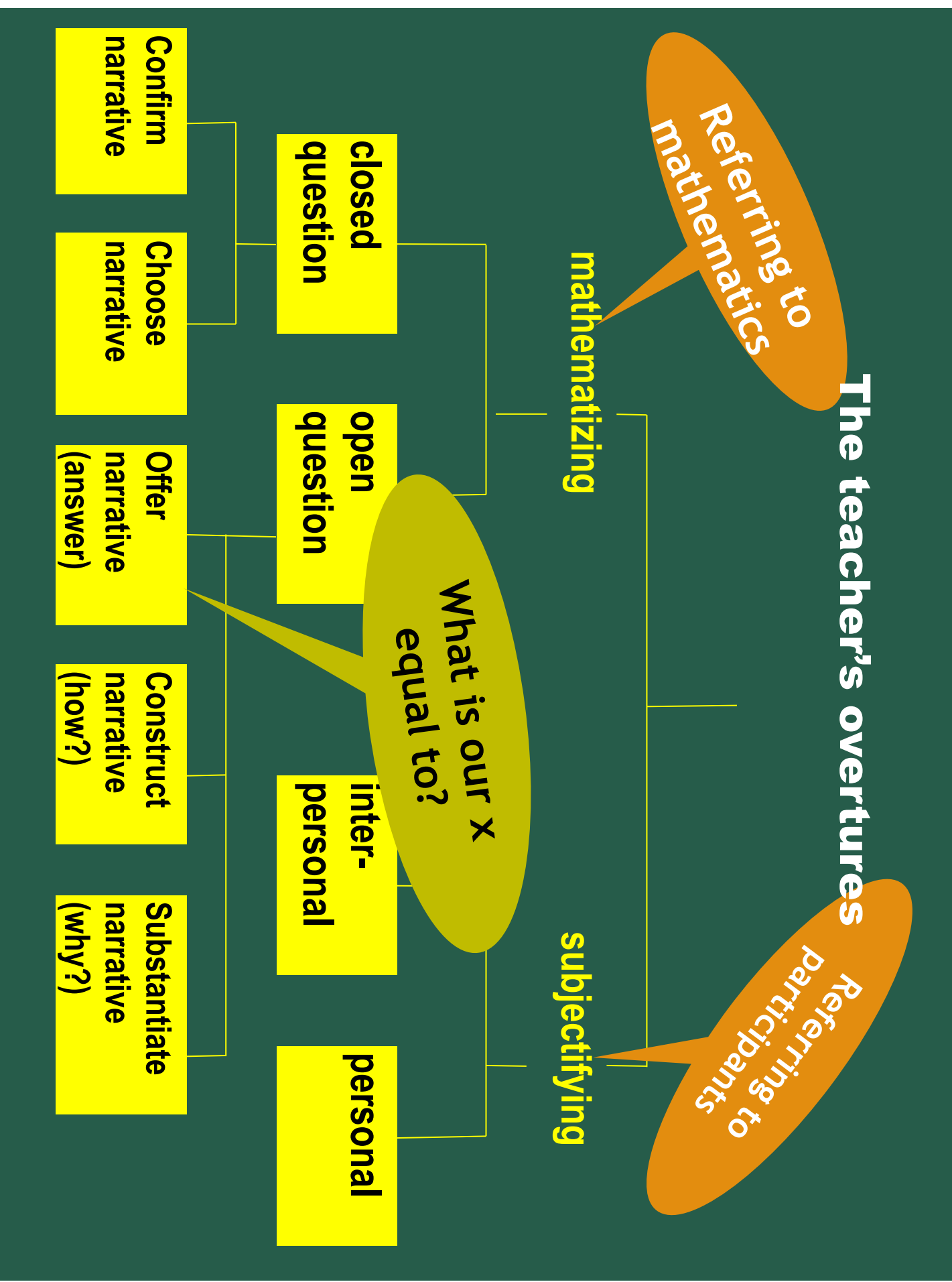
Confirm narrative

Choose narrative

Offer narrative (answer)

Construct narrative (how?)

Substantiate narrative (why?)



The teacher's overtures

Referring to mathematics

mathematizing

Referring to participants

subjectifying

closed question

Confirm narrative

Choose narrative

open question

Offer narrative (answer)

Construct narrative (how?)

inter-personal

Substantiate narrative (why?)

personal

The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

How did you decide that our x should be less than negative 2?

ing

closed question

open question

inter-personal

personal

Confirm narrative

Choose narrative

Offer narrative (answer)

Construct narrative (how?)

Substantiate narrative (why?)

The teacher's overtures

Referring to mathematics

mathematizing

Referring to participants

subjectifying

closed question

Confirm narrative

Choose narrative

open question

Offer narrative (answer)

Construct narrative (how?)

inter-personal

Substantiate narrative (why?)

personal

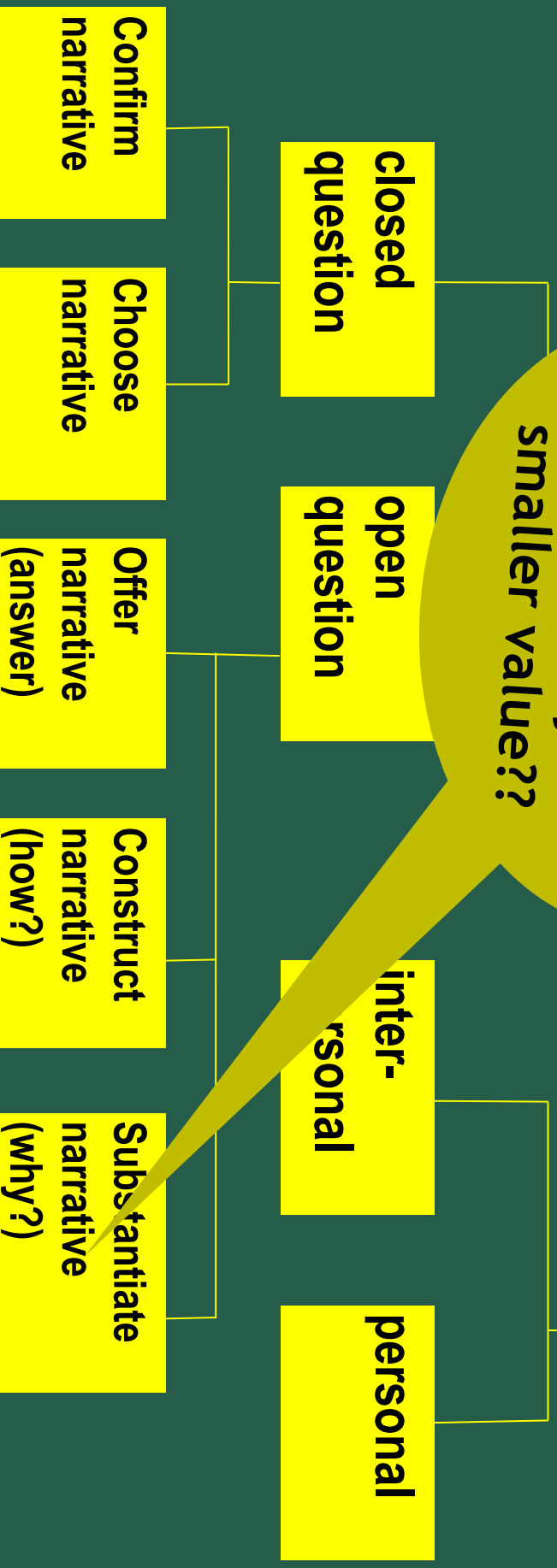
The teacher's overtures

Referring to mathematics

Why are we starting with a bigger value followed by a smaller value??

Referring to participants

subjectifying



The teacher's overtures

Referring to mathematics

Referring to participants

mathematizing

subjectifying

closed question

open question

inter-personal

personal

Confirm narrative

Choose narrative

Offer narrative (answer)

Construct narrative (how?)

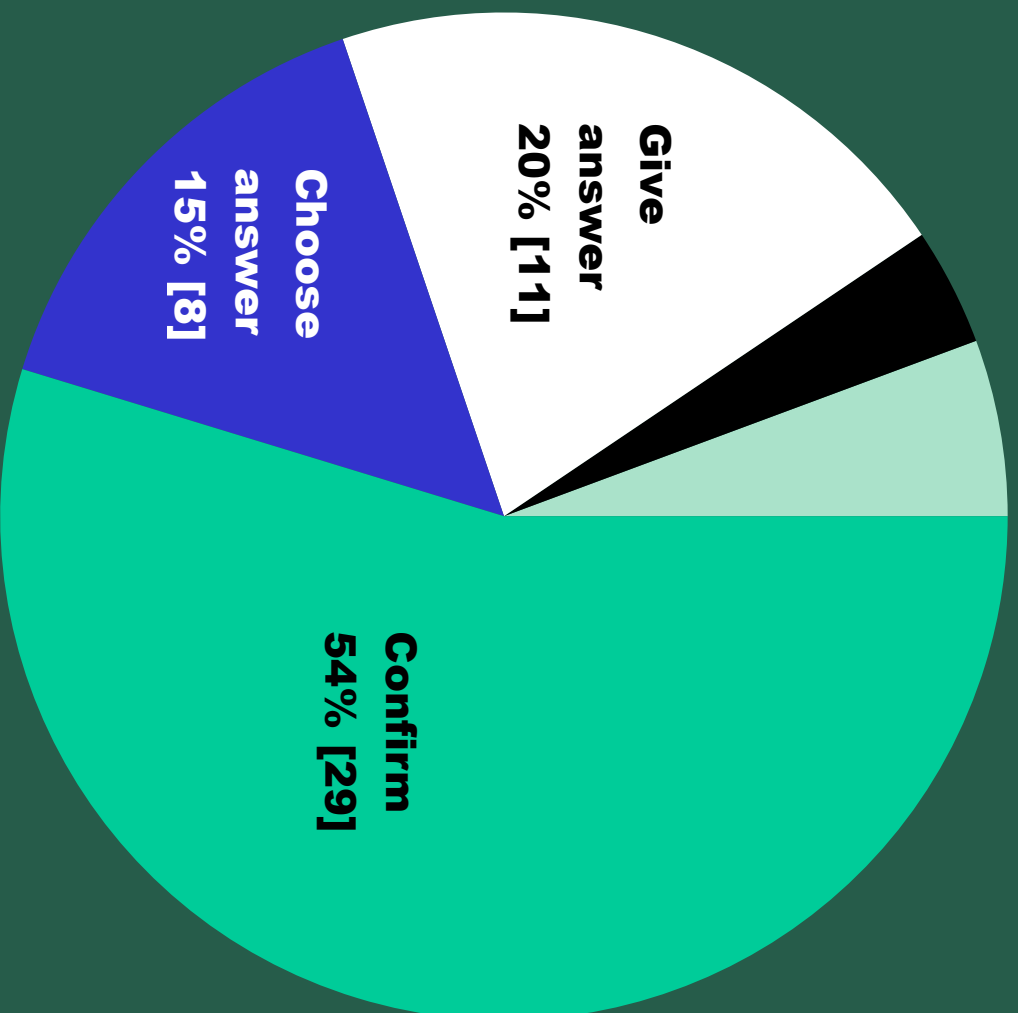
Substantiate narrative (why?)

**Give
answer
20% [11]**

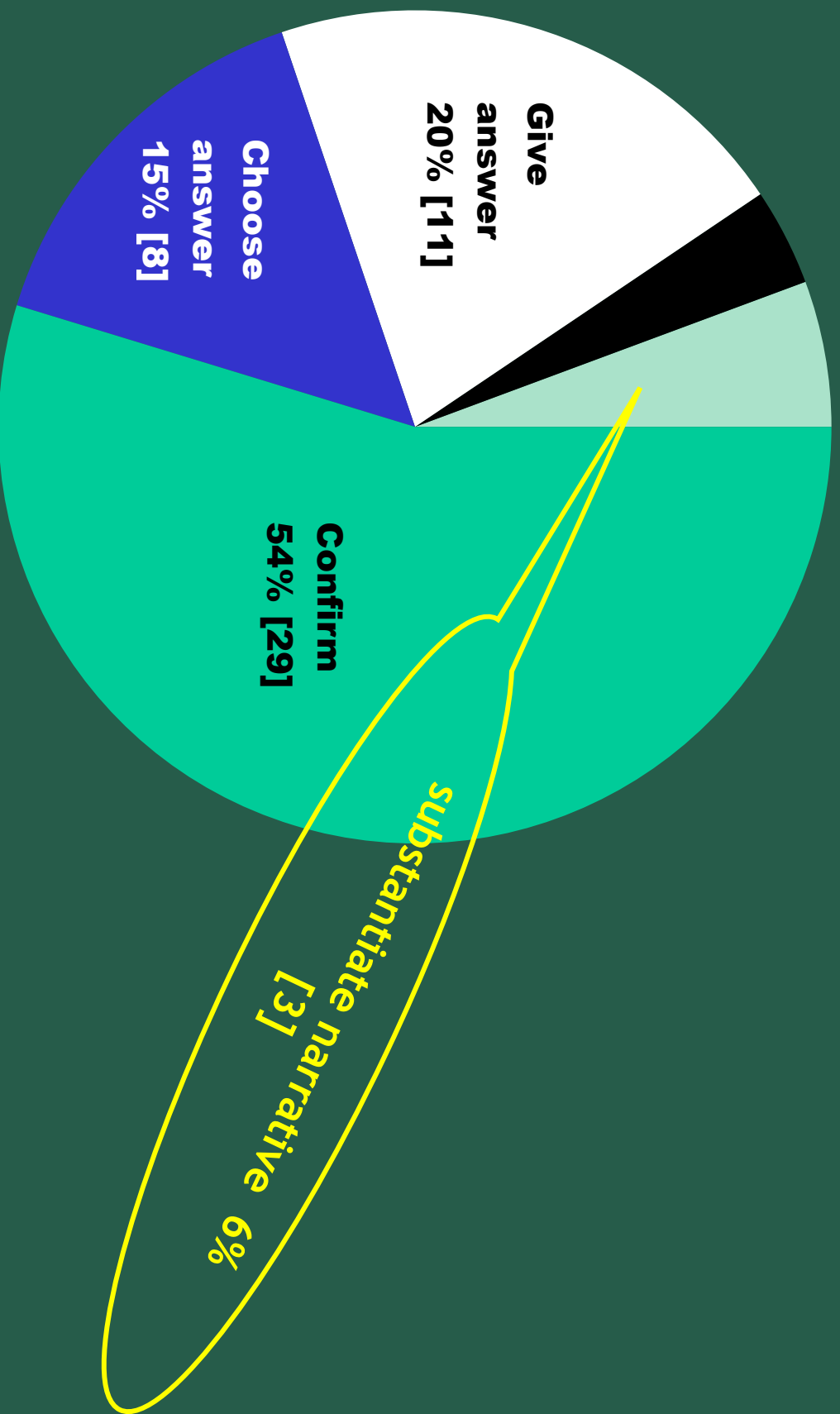
**Confirm
54% [29]**

**Choose
answer
15% [8]**

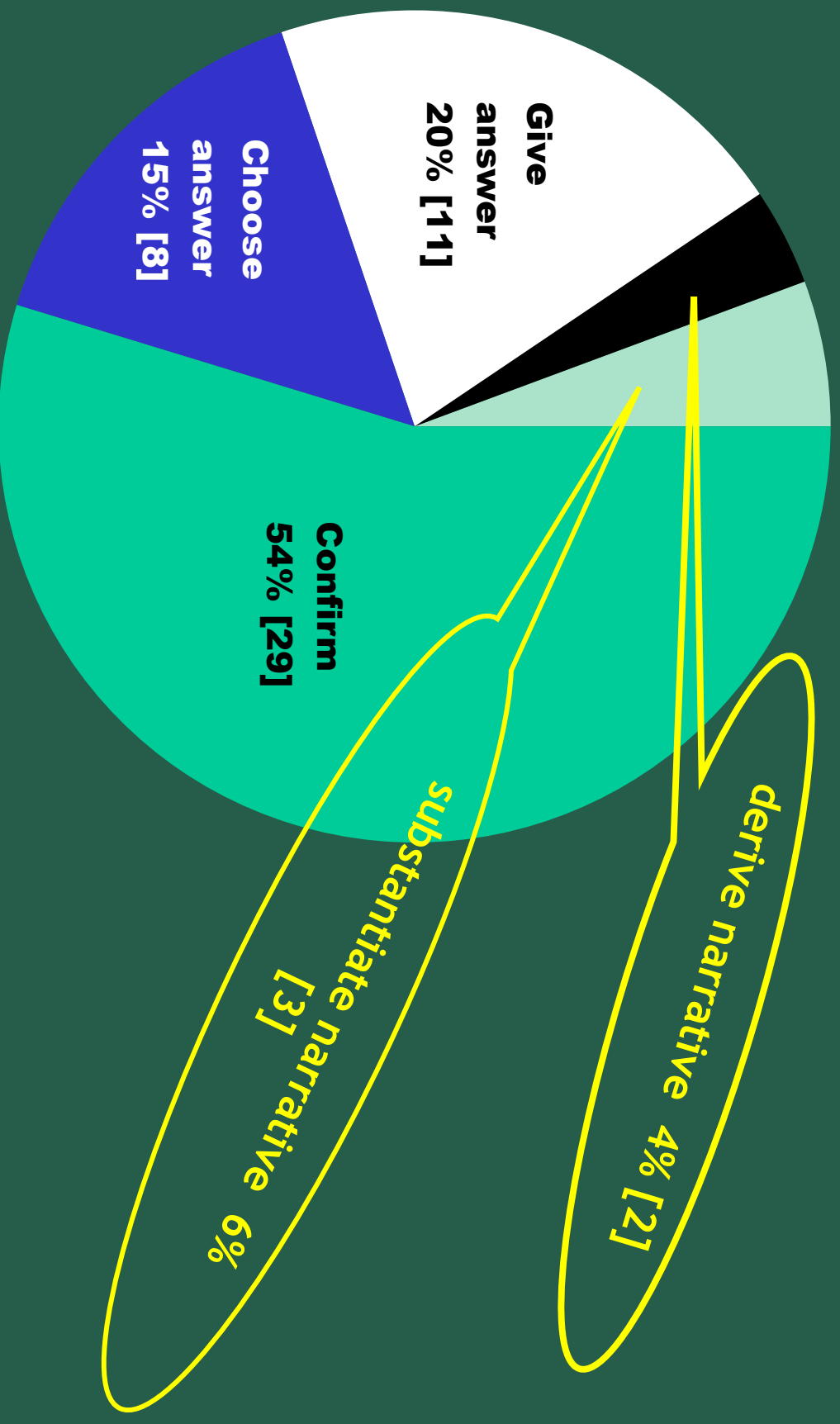
Teacher's overtures to mathematizing [53]



Teacher's overtures to mathematizing [53]



Teacher's overtures to mathematizing [53]



In addition

2/13/2016

64

In addition

- **Most of the Teacher's questions contained their own answers**

In addition

- **Most of the Teacher's questions contained their own answers**
- **The Teacher never really waited for the learners' answers**

In addition

- **Most of the Teacher's questions contained their own answers**
- **The Teacher never really waited for the learners' answers**
- **He never addressed a question at an individual**

2/13/2016

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In addition

- Most of the Teacher's questions contained their own answers

- **Teacher's questions were rhetorical**

- He never addressed a question at an individual

2/13/2016

His main message to
the learners seemed to
be

“do as we all are
doing” (see also his frequent use
of the phrases “we are saying ...”,
“are we together?” that in this
context appear as aiming at a chorus)

2/13/2016

question at an individual

64

Conclusion (tentative)

2/13/2016

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Conclusion (tentative)

The teacher's mathematizing
overtures did not really ask
for either derivation or
substantiation,
they thus did not promote
explorative discourse

Conclusion (tentative)

The teacher's mathematizing
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Plan of this talk

1. **Our task as observers of teaching**
What is there to be observed?
2. **What are the challenges?**
Our words as (possibly imperfect) tools
3. **Getting what we need**
Changing the way we talk
4. **Testing the new way of talking**
Can we see more?
5. **Conclusions**
What have we done and was it worth doing?

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Looking back

2/13/2016

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Looking back

What was the
problem?

2/13/2016

67

Looking back

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problem?

As an observer, how do I decide whether
the mathematics that is being taught
is the mathematics I would like to see?

Looking back

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Looking back

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What was the suggested solution?

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 - not sensitive enough - hides differences
 - too selective - leaves many things out of ~~change~~ our language so that
 - there is no content-form duality
 - all notions are operational
 - it forces us to attend to words rather than looking “through” the words
- What was the suggested solution?

Looking forward

2/13/2016

Looking forward

In sum,

2/13/2016

Looking forward

In sum, This talk was about the importance of attending closely to language at any level: that of the observer and that of the observed

Looking forward

In sum, This talk was about the importance of attending closely to language at any level: that of the observer and that of the observed

What may
be our
further
problems?

One problem:

2/13/2016

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One problem:

**As observers,
we have to unlearn
what we have
learned as
mathematicians**

One problem:

**As observers,
we have to unlearn
what we have
learned as
mathematicians**

Another problem:

2/13/2016

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Another problem:

This form of
observation seems
extremely
work-consuming.

Is it worth it?

Another problem:



**My personal answer:
My resounding YES**

**obsession seems
extremely**

work-consuming.

Is it worth it?

Another problem:

**My personal answer:
My resounding YES**

**My observation seems
extremely**

Why?

Is it worth it?

This is what we (I) gain:

2/13/2016

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This is what we (I) gain:

**Some things we all intuitively feel
are true I now know differently**

This is what we (I) gain:

**Some things we all intuitively feel
are true I now know differently**

**Knowing THAT something
is a case is not the same as
knowing HOW and WHY
this thing happens**

This is what we (I) gain:

**Some things we all intuitively feel
are true I now know differently**

This is what we (I) gain:

**Some things we all intuitively feel
are true I now know differently**

**I also now know some totally new
things**

This is what we (I) gain:

**Some things we all intuitively feel
are true I now know differently**

**I also now know some totally new
things
I understand more about
mathematics, what it is and how it
works**

Different visions of the relation between mathematics and discourse

February 13, 2016

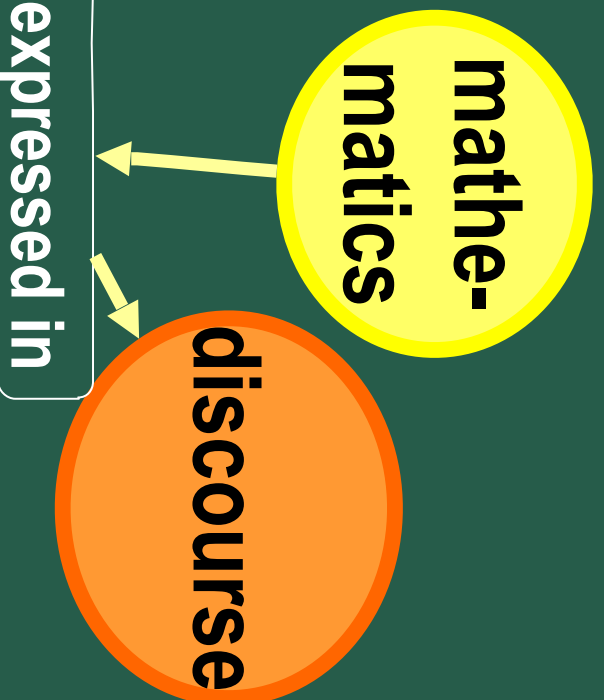
73

**Different visions of the relation between
mathematics and discourse**

dualist

Different visions of the relation between mathematics and discourse

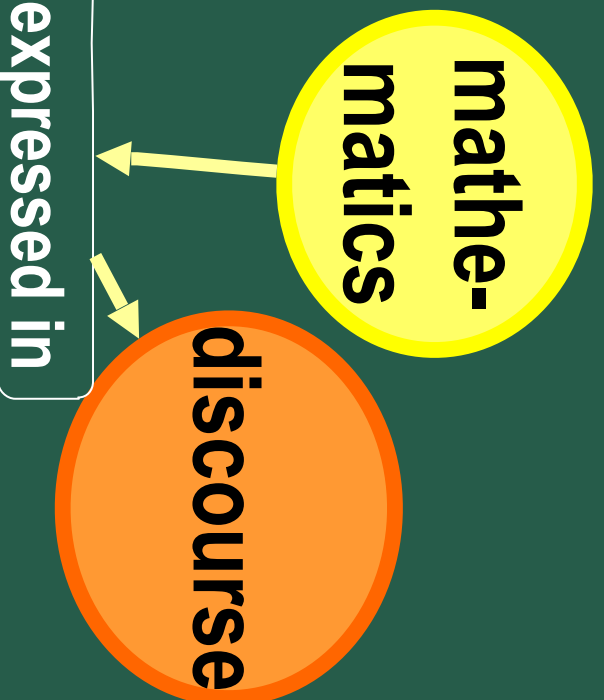
dualist



February 13, 2016

Different visions of the relation between mathematics and discourse

dualist

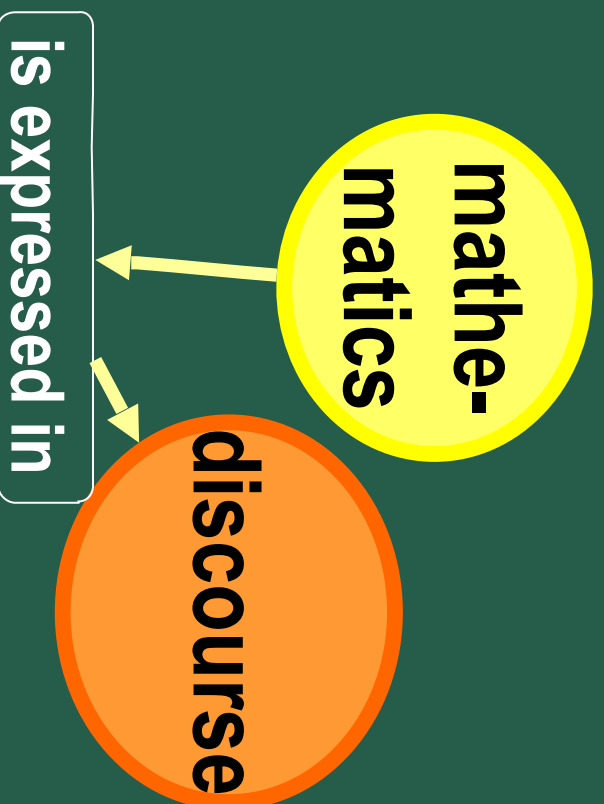


February 13, 2016

Different visions of the relation between mathematics and discourse

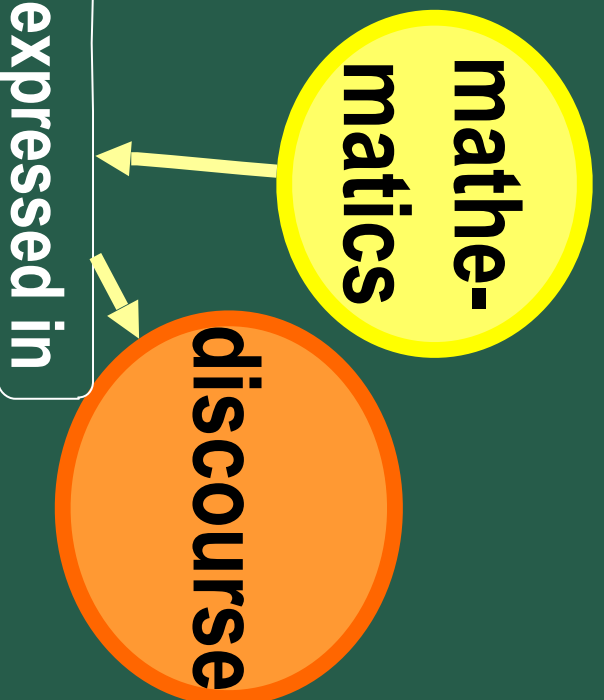
dualist

non-dualist



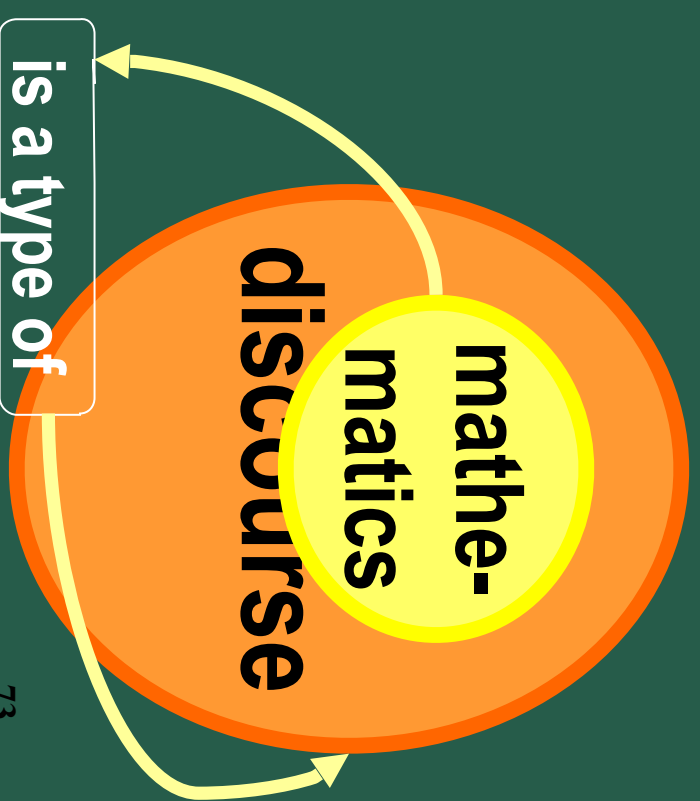
Different visions of the relation between mathematics and discourse

dualist



February 13, 2016

non-dualist



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This is what we (I) gain:

**Some things we all intuitively feel
are true I now know differently**

**I also now know some totally new
things
I understand more about
mathematics, what it is and how it
works**

This is what we (I) gain:

Some things we all intuitively feel
are true I now know differently

I also now know some totally new
things
I understand more about
mathematics, what it is and how it
works
I understand better what and
why
I value in mathematics

2/13/2016

This is what we (I) gain:

Some things we still find difficult to feel
are true

**And what about
YOU?**

I also
things
I understand
mathematics, what it is and how it
works
understand better what and
why
I value in mathematics

This is what we (I) gain:

**Some things we all intuitively feel
are true I now know differently**

**I understand more about
mathematics, what it is and how it
works**

This is what we (I) gain:

**Some things we all intuitively feel
are true I now know differently**

**I understand more about
mathematics, what it is and how it
works**

**I understand better what and why
I value in mathematics**

This is what we (I) gain:

Some things we still feel
are true

**And what about
YOU?**

I understand
mathematics
works

I understand better what and why
I value in mathematics

2/13/2016

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The teacher's overt to subjectifying

Referring to
participants

mathematizing

subjectifying

inter-
personal

personal

The teacher's overt to subjectifying

Referring to
participants

Do you
understand?

mathematizing

inter-
personal

personal

The teacher's overt to subjectifying

Referring to
participants

mathematizing

subjectifying

inter-
personal

personal

The teacher's overt to subjectifying

Referring to
participants

mathematizing

subjectifying

Are you happy?

inter-
personal

personal

The teacher's overt to subjectifying

Referring to
participants

mathematizing

subjectifying

inter-
personal

personal

The teacher's overt to subjectifying

mathematizing

subjectifying

Referring to
participants

inter-
personal

personal

Are we
together?

The teacher's overt to subjectifying

Referring to
participants

mathematizing

subjectifying

inter-
personal

personal

The teacher's overt to subjectifying

Referring to
participants

mathematizing

subjectifying

inter-
personal

personal

There were 14 “yes or no”
inquires about students’
understanding or
“happiness”; The teacher
did not probe the responses
and satisfied himself with
the learners’ choral answers.

Conclusion (tentstive)

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Conclusion (tentstive)

Throughout classroom discourse, the learners were identified as participants of a ritual rather than individual explorers

Conclusion (tentstive)

- Throughout classroom discourse, the learners were identified as **participants of a ritual** rather than **individual explorers**
- they were approached only as a group

Conclusion (tentstive)

Throughout classroom discourse, the learners were identified as **participants of a ritual rather than individual explorers**

- they were approached only as a group
- they were not invited to explorations

Conclusion (tentstive)

Throughout classroom discourse, the learners were identified as **participants of a ritual rather than individual explorers**

- they were approached only as a group
- they were not invited to explorations
- they were expected to be “happy” and “together” rather than “understanding”

Conclusion (tentstive)

Throughout classroom discourse, the learners were identified as **participants of a ritual rather than individual explorers**

- they were approached only as a group
- they were not invited to explorations
- they were expected to be “happy” and “together” rather than “understanding”

Conclusion (tentative)

Throughout the lesson, the learners
participated in choral responses rather than
individual explorations

- they were approached only as a group
- they were not invited to explorations
- they were expected to be “happy” and

“together” rather than

“understanding”

Conclusion (tentstive)

Throughout classroom discourse, the learners were identified as **participants of a ritual rather than individual explorers**

- they were approached only as a group
- they were not invited to explorations
- they were expected to be “happy” and “together” rather than “understanding”