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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Neelesh Tiruviluamala **Email/Phone:** tiruvilu@usc.edu

Speaker's Name: Chris Rasmussen

Talk Title: Attending to student thinking and their interactions when working in small groups

Date: 2 / 11 / 16 **Time:** 11 : 15 [✓ am] / pm **(circle one)**

List 6-12 key words for the talk: Inquiry Oriented, peer to peer collaboration, collective mathematical progress, mathematical conceptions and activity

Please summarize the lecture in 5 or fewer sentences:

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

■ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

■ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **<u>Computer Presentations</u>**: Obtain a copy of their presentation
- **<u>Overhead</u>**: Obtain a copy or use the originals and scan them
- **<u>Blackboard</u>**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- **<u>Handouts</u>**: Obtain copies of and scan all handouts

■ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack <u>with this completed sheet on top</u> and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

■ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)

■ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Attending to student thinking and their interactions when working in small groups

Chris Rasmussen
San Diego State University

An inquiry-oriented differential equations class

Inquiry-oriented (IO)

- Deep engagement in the mathematics
- Peer to peer collaboration
- Teacher interest and curiosity into student work

In IO classrooms teachers typically circulate around the class and listen to what their students are doing. What might a teacher or researcher pay attention to?

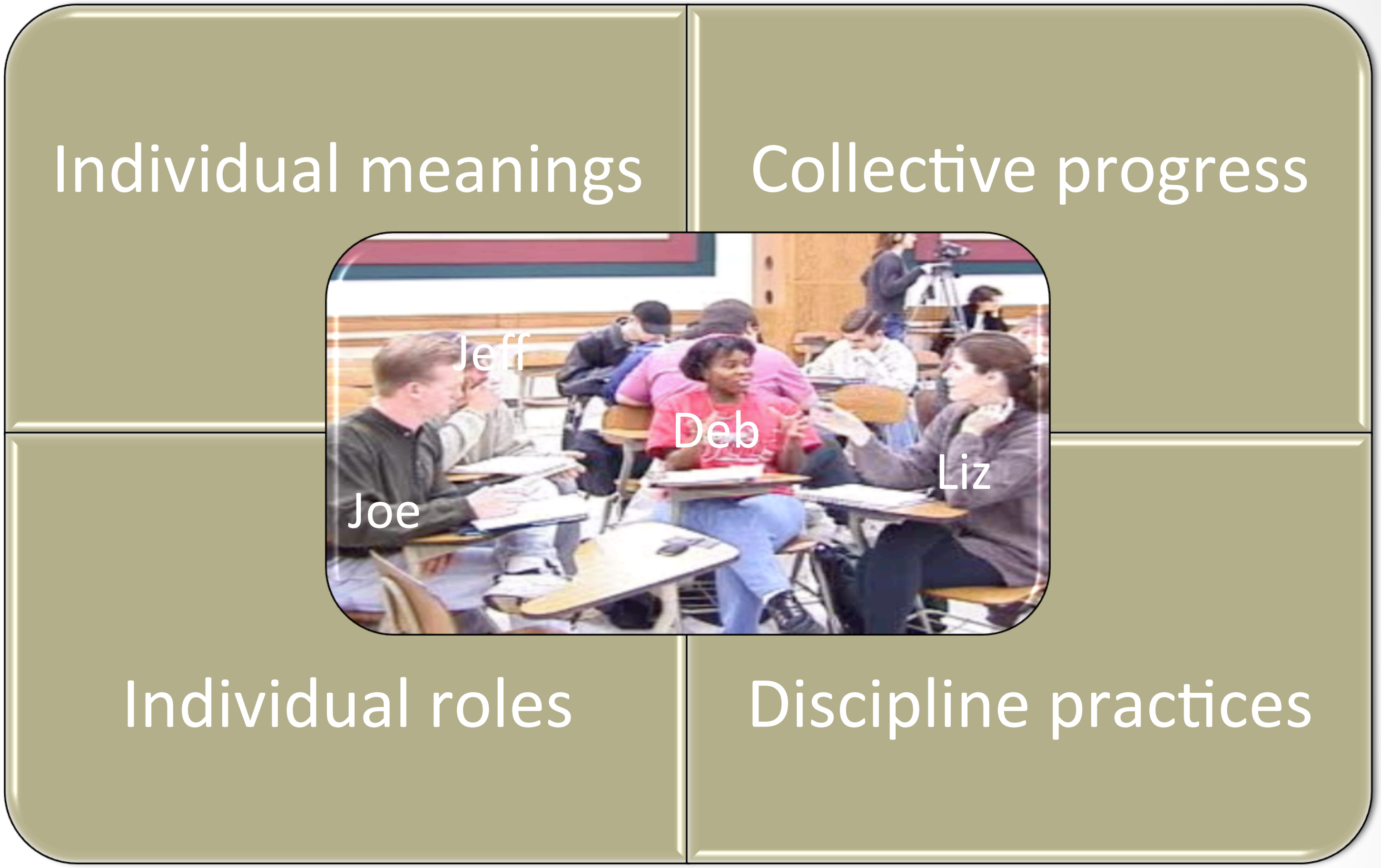
What are you attending to in this 2 minute video clip?

Attending to Student Thinking

A goal of the CIME 2016 workshop is "to develop language and methods for describing, analyzing and evaluating what can be seen in the classroom."

Miriam Sherin asked:

- What lens are we using?
- What tools are we using?



Individual meanings

Collective progress

Individual roles

Discipline practices

A compelling small group episode in which students made considerable progress in reinventing Euler's method

Emergent perspective and the (classroom) interpretive framework (Cobb & Yackel, 1996)

Social Perspective	Individual Perspective
Social norms	Beliefs about one's own role, others' role, and the general nature of mathematical activity
Sociomathematical norms	Mathematical beliefs and values
Collective mathematical progress	Mathematical conceptions and activity

Social Perspective	Individual Perspective
Collective mathematical progress	Mathematical conceptions and activity

The need to expand the bottom row of the interpretive framework

- “Mathematical conceptions and activity” has primarily been operationalized in terms of individual participation in classroom mathematical practices
- Desire to be more inclusive of cognitive framing and draw on expansive literature that examines the meanings that individuals bring to bear and develop
- Work in undergraduate mathematics foregrounds disciplinary nature of students’ mathematical activity

Expanded Interpretive Framework

<table>
<tr><th colspan="2">Social Perspective</th><th colspan="2">Individual Perspective</th></tr>
<tr><td colspan="2">Social norms</td><td colspan="2">Beliefs about one’s own role, others’ role, and the general nature of mathematical activity</td></tr>
<tr><td colspan="2">Sociomathematical norms</td><td colspan="2">Mathematical beliefs and values</td></tr>
<tr><td>Disciplinary practices</td><td>Collective mathematical progress</td><td>Participation in mathematical activity</td><td>Mathematical meanings</td></tr>
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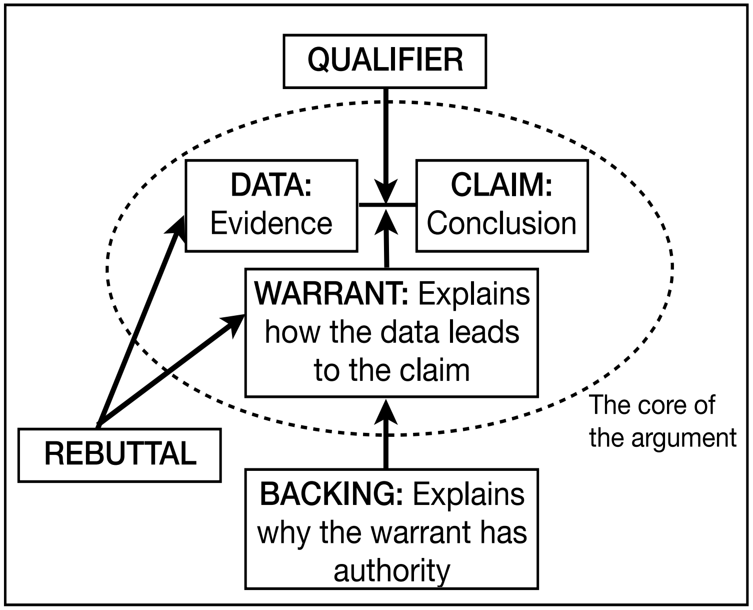
4 constructs and research questions

Disciplinary Practices	Collective mathematical progress	Participation in mathematical activity	Mathematical meanings
What is the mathematical progress of the classroom community in terms of the disciplinary practices of mathematics?	What are the normative ways of reasoning that emerge in a particular small groups or classrooms?	How do individual students contribute to collective mathematical progress?	What meanings do individual students bring to bear and develop in their mathematical work?

and coordination of analyses

Collective mathematical progress: Ways of reasoning that function as if shared



QUALIFIER

DATA: Evidence

CLAIM: Conclusion

WARRANT: Explains how the data leads to the claim

REBUTTAL

BACKING: Explains why the warrant has authority

The core of the argument

Toulmin (1958)

Three criteria

Criterion 1: When the backing and/or warrants for particular claim are initially present but then drop off

Criterion 2: When certain parts of an argument (the warrant, claim, data, or backing) shift position within subsequent arguments

Criterion 3: When a particular idea is repeatedly used as either data or warrant for different claims across multiple days

Rasmussen & Stephan (2008)

Mathematical meanings

As students solve problems, explain their thinking, represent their ideas, and make sense of others' ideas, they necessarily bring forth various meanings of the ideas being discussed and potentially enlarge or modify these meanings.

- When feasible, make use of prior work that characterizes student meanings of particular ideas: concept image of limit (Williams), covariational reasoning (Carlson), rate of change (Thompson; Zandieh), etc.
- In less traversed domains, one will need to develop new characterizations of the meanings that individuals bring to bear and develop.

Participation in mathematical activity

Individual progress as participation in mathematics is operationalized in terms of production and recipient design (Krummheuer; 2007, 2011).

Production design

- *Author* is given when a speaker is responsible for both the content and formulation of an utterance.
- *Relayer* is assigned when a speaker is not responsible for the originality of either the content or formulation of an utterance
- *Ghostee* takes part of the content of a previous utterance and attempts to express a new idea
- *Spokesman* is one who attempts to express the content of a previous utterance in his/her own words

Participation in mathematical activity

Recipient Design

- *Conversation partner* is the listener to whom the speaker seems to allocate the subsequent talking turn
- *Co-hearers* are listeners who are also directly addressed but do not seem to be treated as the next speaker
- *Over-hearers* are those who seem tolerated by the speaker but do not participate in the conversation
- *Eavesdroppers* are listeners who are deliberately excluded by the speaker from conversation

Disciplinary Practices

Moschkovich (2007) argues that disciplinary practices are “socially, culturally, and historically produced practices that have become normative”. From an a priori perspective, we have:

- Symbolizing
- Algorithmatizing
- Defining
- Modeling
- Theoremizing

Theoremizing: engaging in a mathematical setting, observing relationships, clarifying and refining stated relationships, arguing for (or against) claims, generalizing, and justifying generalizations.

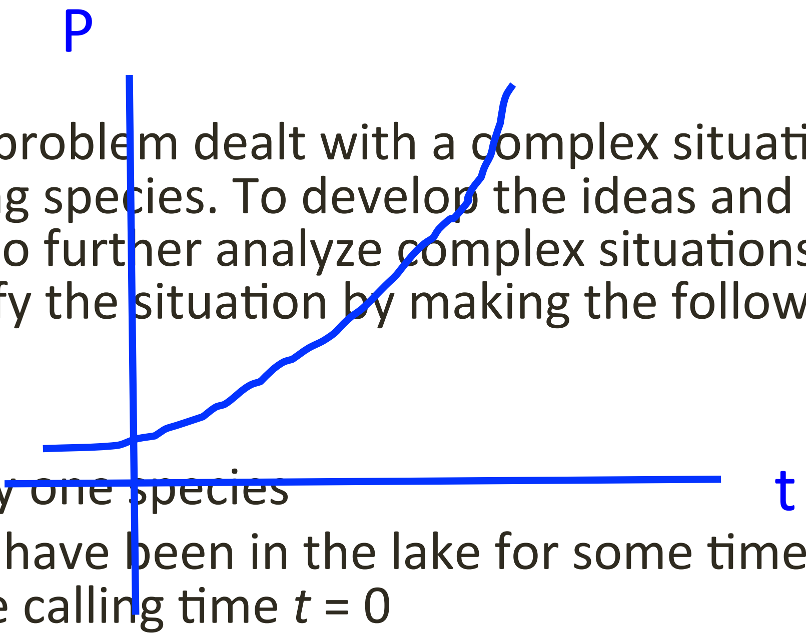
Using a grounded approach we allow the data to shape how we characterize the features of a disciplinary practice that emerge in a particular class.

Rasmussen, C., Wawro, M., & Zandieh, M. (2015). Examining individual and collective level mathematical progress. *Educational Studies in Mathematics, 88*(2), 259-281.

Design Based Research Setting

- Mid-sized university with “generous” admission policy
- Introductory course in differential equations
- 29 students in the class
- Mostly engineering majors and a few mathematics majors
- Curriculum inspired by Realistic Mathematics Education
- Inquiry-oriented instructional approach (Rasmussen & Kwon, 2007)
  - Deep engagement in the mathematics
  - Peer to peer interaction
  - Instructor inquiry into student thinking
- Video recordings of each class from two cameras, copies of student work, individual student interviews, debriefing meetings
- Day 2 of the course
- Small group episode lasting ≈ 10 minutes

Initial Task



The previous problem dealt with a complex situation with two interacting species. To develop the ideas and tools that we will need to further analyze complex situations like these, we will simplify the situation by making the following assumptions:

- There is only one species
- The species have been in the lake for some time before what we are calling time *t* = 0
- The resources (food, land, water, etc.) are unlimited
- The species reproduces continuously

$$dP/dt = 3P$$

Given these assumptions for a certain lake with fish, sketch three different population versus time graphs (one starting at *P* = 10, one starting at *P* = 20, and the third starting at *P* = 30).

The Task – Use dP/dt to Approximate Future Population Values

Consider the following rate of change equation, where $P(t)$ is the number of rabbits at time t (in years): $dP/dt = 3P(t)$ or in shorthand notation $dP/dt = 3P$. Suppose that at time $t = 0$ we have 10 rabbits (think of this as scaled, so we might actually have 1000 or 10,000 rabbits). Figure out a way to use this rate of change equation to approximate the future number of rabbits at $t = 0.5$ and $t = 1$.

4 Liz: Oh ok, so I get the rate of change at time, initially, the instantaneous rate of change would be 30. Did I multiply it right?

....

17 Liz: So if we have the 30, the question is how can we use that to help us figure out the population after a half unit elapsed?

A priori analysis: Intended knowledge elements (Tabach et al, 2015)

- C_{sy} – establishing connection between P and dP/dt (if you know P you can find dP/dt)
- C_{pit} – population iteration (given P and dP/dt at a moment in time allows one to find P at a later time)
- C_{rit} – rate of change iteration (applying Csy at that later time one can find the corresponding dP/dt)

C_{it} – Cpit and Crit can be combined into a repeating loop

C_{pit}

C_{sy}

Liz: What I understand is that we found our rate of change initially at time zero and that we are using that to find out what our population is after half a year. If we are expected **to grow by 30 rabbits in a year** then, **in a half a year we grow by 15 rabbits. So we'll have 15, I mean 25 because 15 plus 10 is 25**. **Then you start over again**, so it's kind of like our new initial population. We could label it time equals zero if we wanted to.

C_{it}

Individual meanings

Collective progress

Individual roles

Discipline practices

Research goals: Coordinate collective and individual analyses to gain greater explanatory and descriptive power; Better understand the individual and collective meaning making processes

Small Group Collective Progress

Finding: Three ideas functioned as if shared

- dP/dt can be determined from P values (C_{sy})
- A value for dP/dt refers to the amount of change over 1 year
- C_{pit} and C_{rit} can be combined into a repeating loop

Claim: dP/dt can be determined from P values functioned as if shared

Small group discourse satisfied Criterion 2: What was originally a Claim in one argument (Arg 1) functions as Data in a subsequent argument (Arg 5)

Data: This is where 10 rabbits at zero (Jeff)

—

Claim: The initial instantaneous rate of change is 30 (Liz)

Warrant: I would plug in the population of rabbits for P to determine the rate of change (Liz)

Backing: If we had a graph, its kind of like what we were just talking about, we are trying to determine the rate of change when this time is equal to zero (Liz)

Claim in Arg 1 shifts to Data in Arg 5

Data: We have the 30. Three times ten would give us our rate of change. Say 0.5 years passes (Deb)

Claim: which will give us what, the new amount of rabbits plus the old amount of rabbits (Deb)

Warrant: This is our rate of change. Then we'll take that 0.5 times the rate of change (Deb)

Similar type of evidence for the other two ideas that function as if shared

- A value for dP/dt refers to the amount of change over 1 year
- C_{pit} and C_{rit} can be combined into a repeating loop

Full consideration of the data indicate that Liz, Deb, and Jeff made individual progress compatible with the collective mathematical progress.

Toulmin Analysis Overview

- Talk turns: Liz 26; Deb 18; Jeff 13; Joe 8
- There were 14 different arguments (à la Toulmin) that consisted of at least Data and Claim
- The following table shows the distribution of contributions (some contributions co-constructed)

	Liz	**Deb**	**Jeff**	**Joe**
Data	6	5	1	4
Claim	5	5	5	2
Warrant	2	5	1	0
Backing	2	1	0	0

In relation to the collective mathematical progress,

- What meanings for dP/dt emerged and who expressed these meanings?
- What part did these meanings play in the collective mathematical progress?
- What roles did Liz, Deb, Joe, and Jeff play in all of this?
- In what ways did students' mathematical work reflect disciplinary practices?

What meanings for dP/dt emerged and who expressed these meanings?

- Steepness - Liz
- Ratio – Liz, Jeff
- Population length – Liz, Deb
- Tool - Liz
- Function - Deb
- Proportion – Deb
- Fraction- Jeff

Meanings for dP/dt

1 Liz: I would plug in the population of rabbits for P to determine the rate of change, when, initially, just at **the instance, like initially, what's the rate of change when time equals zero**. So if we had a graph, its kind of like what we were just talking about, we are trying to determine the rate of change when this time is equal to zero.

4 Liz: So I get the rate of change at time, initially, the **instantaneous rate of change would be 30**.

Rate as steepness

6 Joe: Are we trying to figure out what P is?

7 Liz: Okay, well this points to [dP/dt] is **the change in the population over the change in time**.

Rate as ratio

Meanings for dP/dt

Huh?

15 Joe: **Well, wouldn't 10 = 3P(t)**? If at time zero we have 10 rabbits.

16 Liz: Well 10 is actually the population so I'm taking that that has to actually be the population at time t. I don't think it's telling us **how the population is changing, which would be dP/dt.**

Rate as steepness

17 Liz: So if we have that [initial rate of change is 30], the question is **how can we use that** to help us figure out the population after a half unit elapsed?

How can rate be a tool?

22 Deb: You said the population is 10 right [Liz: Um hmm]. So three times ten would give us our rate of change. Say 0.5 years passes, this is our rate of change. **Then we'll take that 0.5 times the rate of change** which will give us what [slight pause looks up to Jeff and Joe], the new amount of rabbits plus the old amount of rabbits.

Proportional reasoning?

Meanings for dP/dt

25 Liz: So the old amount of rabbits is 10.

26 Deb: Am I making sense?

27 Jeff: **I think so, so that would be 25**, is that what you're saying?

28: Liz: Okay I think I get what you're saying. Ok, so like we're at tim zero and we have 10 rabbits, and supposedly the rate change, well not supposedly, we're saying that the rate of change is 30 [Jeff: yeah for the] at time zero. **So its going to grow at a rate of, I don't know if I'm going to say this right, at 30 rabbits per year?** [looks up at Deb]

29 Deb: Right. [Liz: Ok] So we'll have 30 more rabbits.

32 Liz: And so we're really not figuring out the rate of change we figuring…**Well this is the rate of change and we're using the rate of change to figure out the number of rabbits we are going to increase by in half a year**.

Computes change in P from dP/dt

Rate as "pop length"

Rate as tool

Meanings for dP/dt

38 Deb: This is what I did. First I looked at the fact that this is a **rate of change equation. So this is telling me how many rabbits are being produced every year.** So If I know 3 times the original population is produced every year, **then I know 3 times 10 is produced every year**. But I want to know how many is produced in 0.5 years. **So I know how many rabbits are produced per year, so if I multiply that by 0.5 then I'll know how many more rabbits have been produced**. So I take that new number that I get and add it to the old population.

43 Jeff: I think you can go **dp/dt=30, actually your dt will be 0.5**, and then you add that to the old and then you do it again for the next one.

45 Deb: …. **And once I know the new population I know the new rate of change** because I know the rate of change is right here.

Rate as function

Rate as "pop length"

Prop. reasoning

Treats dP/dt as fraction

C_{rit}

Meanings for dP/dt

Rate as function

46 Liz: And the reason for putting in the new population would be what?

47 Deb: Because **now my population is larger** [pulls hands apart] and I know the **population changes at a constant of 3 times whatever that population is at that moment in time**.

48 Liz: Okay, so basically, I get you up into the point where you say you want to put in, what I understand is that *we found our rate of change initially at time zero and I understand using that to find out what our population is after half a year*. ***If we are expected to grow by 30 rabbits in a year then, in a half a year we grow by 15 rabbits***. *So we'll have 15*

Rate as proportion

Rate as pop length

What part did these meanings play in the collective mathematical progress?

- The principle of a form-function-shift (Saxe, 2002) of notations in use is particularly suitable for analyzing the interplay between tool use (in this case $dP/dt = 3P$) and conceptual development.
- This shift describes the interplay between cultural forms (external representations) and the meanings that develop for structuring and accomplishing specific goals.
- As we saw, there is a shift in the meaning of dP/dt - from steepness to a “population length” (clearly for Liz and likely for Deb)
- This shift coincided with “a value for dP/dt refers to the amount of change over 1 year” functioning as if shared AND the initial articulation of how to find the estimate for the population at $t = 0.5$.

What roles did Liz, Deb, Joe, and Jeff play in the collective mathematical progress?

Production Design
Author: responsible for content and formulation
Relayer: not responsible for either content or formulation
Ghostee: Reformulates previous content as a new idea
Spokesman: Rephrases content in his/her own words

- Author (co-author): operationalize as one who contributes to any part of an argument (Data, Claim, Warrant, or Backing)

Production Design Roles

- Talk turns: Liz 26; Deb 18; Jeff 13; Joe 8
- Raw count of co-author shows that there was fairly even distribution (Liz 6/14; Deb 5/14; Jeff 6/14; Joe 4/14)
- More nuanced look however reveals important differences
  - Joe offered 2 incorrect arguments
  - Jeff often Revoiced (with and without reformulation)
  - Liz and Deb did the main intellectual lifting (as you saw in the excerpts)
    - For example, Liz was primarily Author (core of argument) for C_{sy} and as Spokesman for meaning of dP/dt as population length
    - Deb, on the other hand, was the primary author for C_{it}

Production Design Roles

26 Deb: [articulates the main iteration idea but wit
numerical result] Am I making sense?

Relayer

27 Jeff: **I think so, so that would be 25**, is that what you're
saying?

28: Liz: Okay I think I get what you're saying. Ok, so like we're at
time zero and we have 10 rabbits, and supposedly the
rate of change, well not supposedly, we're saying that
the rate of change is 30 [Jeff: yeah for the] at time zero.
**So its going to grow at a rate of, I don't know if I'm
going to say this right, at 30 rabbits per year?** [looks up
at Deb]

Spokesman

Recipient Design
Conversation Partner: listener with next turn
Co-hearer: directly addressed but not treated as next speaker
Over-hearer: tolerated by speaker but do not participate
Eavesdropper: deliberately excluded by speaker

Talk turns: Liz 26; Deb 18; Jeff 13; Joe 8

For the most part, Liz, Deb, and Jeff were conversation partners and co-hearers. Joe was mostly a co-hearer and at times an over-hearer

Production/Recipient Design Insufficient

Facilitator Design
Focuser is assigned when a speaker directs attention to a particular mathematical issue
Elicitor is given when a speaker attempts to bring out another's ideas
Checker is one who seeks agreement or sensibility of an utterance
Summarizer pulls ideas together

Facilitator Design

17 Liz: So if we have that [initial rate of change is 30], **the question is how can we use that to help us figure out the population after a half unit elapsed?** [32 sec pause, everyone looking down at their papers and making marks]

Focuser

18 Jeff: So I was just going to say **how would we work time into the equation** to get the next, uh, population or change in population?

Focuser

40 Liz: Yeah I get it, **do you guys get what Deb is saying**?

Checker

41 Jeff: Yeah you get 25 and then you get 55.

46 Liz: **And the reason for putting in the new population would be what?**

Elicitor

48 Liz: **Okay, so basically, I get you up into the point where you say you want to put in, what I understand is that** …..

Summarizer

53 Deb: **Everybody agree?**

Checker

In what ways did students' mathematical work reflect disciplinary practices?

Algorithmatizing

- Engaging in goal directed activity
- Isolating attributes
- Forming quantities
- Creating relationships between quantities
- Expressing relationships symbolically

$$P_{next} = P_{now} + \left(\frac{dP}{dt}\right)_{now} \times \Delta t.$$

Summary

<u>Implications for Instruction</u>

- Examples contribute to instructor notes about student thinking, about implementation suggestions, video clips of paradigmatic student thinking
- Instructional design – Suggest teacher questions
  - What is the initial rate of change and what does this value mean to you?
  - How can you use the 30 to figure out the population after half unit of time?
- Helping students become better small group facilitators (define roles, illustrate roles, assign roles, assess enactment of roles)

Implications for Research

- A priori analysis did not include unit population change meaning of rate – need to make this an explicit part
- Contribute to local instructional theory
- Offer an approach for coordinating individual and collective mathematical progress (more to do on this)
- Illuminate social and individual processes that contribute to mathematical progress

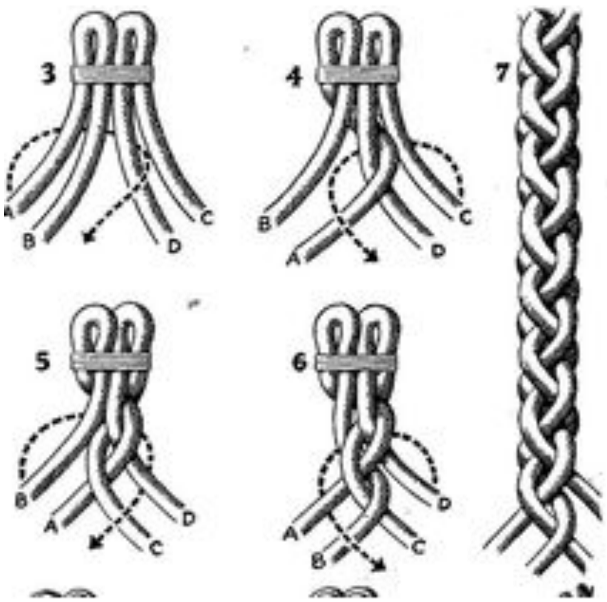
Further Coordination

- Choose an individual and trace his/her utterances for the ways in which they contributed to the emergence of ways of reasoning that function as if shared and/or disciplinary practices
- Characterize the individuals that offer claims, data, warrants, and backing (as related to ways of reasoning that function as if shared)
  - What are their characteristics?
  - What is the instructor's role?
  - How do individual contributions relate to production and recipient design roles?
- How do patterns over time in student participation relate to growth in their mathematical conceptions?
- In what way are different participation patterns correlated with different mathematical growth trajectories?
- In what ways are particular classroom math practices consistent (or inconsistent) with various disciplinary practices?

Individual and Collective Mathematical Progress Metaphor

Strands that make up collective progress

- Meanings
- Production design
- Recipient design
- Facilitator design



COLLECTIVE

The end – thanks for listening

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