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- Analysis & PDE's on uniformly rectifiable sets
- Scale-invariant estimates
- co-dimension 1

TFAE: Geometry: (G1) E uniformly rectifiable
(G2) β-coefficients give Carleson measure
Analysis: (H1) all singular integral operators are bounded in $L^2(E)$
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: (see slides)
"All in red is new" - Svitlana

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on blackboard:

Geometry
uniform rectifiability

T.F.A.E.

PDE
- harmonic measure $\omega \in A^{\infty}$
- Carleson measure estimate
- ε-appx of bounded soln.s.

Analysis:
- all SIO bdd in $L^2(E)$
- Riesz transf. bdd $L^2(E)$

Key issue (Harmonic measure)
- is ω absolutely continuous w.r.t. Lebesgue measure?
- is it quantitatively abs. cont. - $\omega \in A^{\infty}$?

"We want Brownian travellers to see portions of the boundary"

Dimension of harmonic measure:
Carleson 1973: in $\mathbb{R}^2$, for a simply connected domain bounded by a continuum, $\dim \omega > \frac{1}{2} + \varepsilon$.
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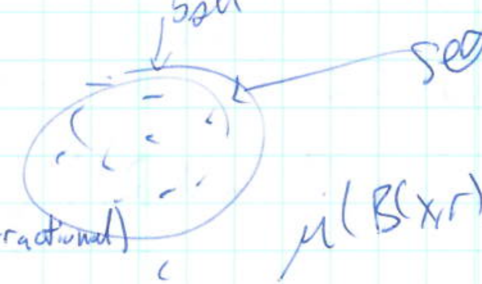
Bourgain 1987, in $\mathbb{R}^n$ $n \geq 2$, $\dim \omega < n$

Looking towards a.e.

Talk I A.D regularity: $d = n-1$
Talk II : $d < n-1$ (possibly fractional)



$\mu(B(x,r) \cap E) \sim r^{n-1}$

When ω is a.c. w.r.t. H^{n-1}?

$n=2$

F.&M. Riesz 1916 $\Omega \subseteq \mathbb{R}^2$, simply connected, rectifiable

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Some connectivity & uniform rectifiability (U.R.) needed to get $\omega \in A^\infty$.

~~example: angle in plane.~~

PDE $\Rightarrow$ ~~smoother angle, smoother solution~~ geometry.

$\partial\Omega$ A.D. corkscrew $\underline{\underline{\omega \in A^\infty}} \Longrightarrow \underline{\underline{\partial\Omega \text{ u.r.}}}$

David - Semmes (now Nazarov, Tolsa, Volberg 2014)

boundedness on $L^2(E)$ & U.r. on E

1996 Mattila & co. boundedness Cauchy transform $(n=2)$ $\longleftrightarrow$ u.r. on E

2014 boundedness Riesz transform $(n \geq 2)$ $\longrightarrow$ u.r. on E

Connect to Harmonic measure.

$$\omega << H^{n-1} \longrightarrow \omega|_E \text{ rectifiable}$$

Back to PDEs: $\left(\omega \in A^\infty \text{ on } \partial(\Omega)\right) \Longleftrightarrow$ (Every bounded soln. u of $Lu = 0$ in Ω satisfies Carleson measure estimate)

In a Lipshitz domain

$\Longleftrightarrow$ (Every bounded soln. is ε - approximable) $\Longleftrightarrow$ (Square fnct. estimates hold)