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①

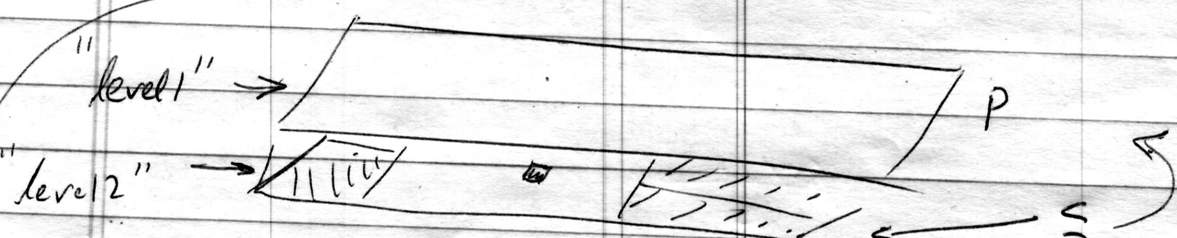
Weighted inequalities.

Bilinear form ass. w/ Hilbert transform:

① $$B_H(f,g) = \iint f(x-y)\, g(x)\, \frac{dx\,dy}{x-y}$$

② A collection of cubes $\mathcal{P}$ is sparse (in fact, η-sparse; $0<\eta<1$) if $\forall\, P \in \mathcal{P}$, $\exists \{E_p\} \subseteq P$ s.t. $|E_p| \geq \eta |P|$, $\{E_p\}$ disjoint.

$\mathbb{R}^2$



$\eta = 1/2$ in this talk.

? maybe this is a typo... $|E_p| < \eta |P|$?

Associated w/ a family of cubes is a family of Bi-linear forms.

$$B_{\mathcal{P}}(f,g) = \sum_{P\in\mathcal{P}} \langle f\rangle_P \langle g\rangle_P |P| \qquad \langle f\rangle_P = \frac{1}{|P|}\int_P f\,dx$$

<u>Prop.</u> $B_{\mathcal{P}}(f,g) \lesssim \cancel{\|f\|_2 \|g\|_2} = \|f\|_2 \cdot \|g\|_2$

pf// $$\int \sum_P \langle f\rangle_P \langle g\rangle_P \mathbb{1}_{E_p}\,dx \leq \int Mf\, Mg \leq \|f\|_2 \cdot \|g\|_2$$

Sparse forms are extremal.

f, g bd. & compactly supp.

$$|B_H(f,g)| \le c \, B_P(f,g) \qquad (*)$$

NOT true as stated. but we can add, you give me $f, g \to \exists P$ s.t. $(*)$ holds

defn. $K(x,y)$ is a CZ kernel if

$$|\nabla^j K(x,y)| \le \frac{1}{|x-y|^{n+j}} \qquad j=0,1$$

Ex $\displaystyle\sum_{P\in\mathcal{P}} \frac{\mathbb{1}_{\exp}}{|P|}$ is a (+) CZ kernel

↑ cold. fig.

Sparse T1 Thrm. K is a CZ kernel.

fact $\forall$ cubes Q, $|\phi_Q| \le \mathbb{1}_Q$

$$B_K(\mathbb{1}_Q, \phi_Q) + |B_K(\phi_Q, \mathbb{1}_Q)| \le |Q|.$$

"the dual"

$\to \forall f, g \ \exists \ \mathcal{P}$ sparse s.t. $B_P(|f|,|g|) \ge |B_K(f,g)|$

A_2 Theorem.

Cor.

- $L^p \to L^p$ $\quad 1<p<\infty$
- $L^1 \to L^{1,\infty}$
- $L^2_w \to L^2_w$ w/ norm $[w]_{A_2}$

Muckenhoupt weights.

Bounds in terms of Γ & S have many applications for B in weighted Lebesgue space.

(Bern. Frey, Petermich)

$\| B:(r,s) \|$ is least constant C $\ni \forall f,g$; $r,s \in [1,\infty)$

$\exists$ sparse $\mathcal{P}$ s.t. $\underbrace{|B(f,g)|}_{\text{messy}} \le C \underbrace{B_{\mathcal{P},r,s}(f,g)}_{\text{positive}}$

<u>Thm.</u>: B bounded CZ form

$\| B(1,1) \| < \infty$

Implies <u>all</u> L^p bounds, weak type, sharp weighted L^p, sharpest known for L^1 weights.