<u>Chalkboard notes from Olga Maleva's talk</u>

8/18/17

<u>Claim</u>: $\mathcal{N}(E,x)$ is a linear subspace:

$$\underbrace{e_1, e_2}_{\neq 0} \in \mathcal{N}(E,x) \overset{?}{\Rightarrow} e := e_1 + e_2 \in \mathcal{N}(E,x)$$

- "joining the cones"
- Fix $\eta > 0$ and let $\varepsilon := \frac{\eta}{2} \cdot \frac{\|e\|}{\|e_1\| + \|e_2\|}$.
- Let $\gamma \in \Gamma_{e,\eta}$. We can assume that $\gamma \in C^1$ and that $\|\gamma'(t)\| = 1$.
- $\langle \gamma'(t), e \rangle \geq \eta \|e\| \iff \langle \gamma'(t), e_1 \rangle + \langle \gamma'(t), e_2 \rangle = 2\varepsilon(\|e_1\| + \|e_2\|)$.
- Fix $t = t_0$. Then $\langle \gamma'(t_0), e_i \rangle \geq 2\varepsilon \|e_i\|$ for at least one index $i = 1, 2$.
- Since e_1, e_2 are directions of nullness, $\exists r$ s.t. $w_{e_i, r}(E \cap B(x,r)) = 0$.
  Moreover, $\langle \gamma'(t), e_i \rangle \geq \varepsilon \|e_i\|$ for all $|t - t_0| < \tau$.
- Finally, take a union over all neighborhoods.