Subgaussian and lacunary uniformly bounded orthonormal systems

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Abstract

Following recent work by Bourgain and Lewko, we will study lacunarity for uniformly bounded orthonormal systems, in analogy with systems of characters on $\mathbb{T}$ or any other compact Abelian group. A system of random variables (f_n) is called σ-subgaussian (or a "ψ_2-system") if for any x in the unit ball of ℓ_2 we have an estimate

$$\mathbb{E}(\exp(|\sum x_n f_n/\sigma|^2) \leq e.$$

The system is called randomly Sidon if there is a constant C such that

$$\sum |x_n| \leq C\mathbb{E}_{\pm 1}\|\sum \pm x_n f_n\|_\infty$$

for all finitely supported scalar valued $n \mapsto x_n$. It will be called Sidon if there is C such that $\sum |x_n| \leq C\|\sum x_n f_n\|_\infty$. Let $\mathbb{N} = \Lambda_1 \cup \Lambda_2$ be a partition of the integers. We will describe an example based on martingale theory where $\{f_n \mid n \in \Lambda_1\}$ and $\{f_n \mid n \in \Lambda_2\}$ are both Sidon but their union (f_n) (which is subgaussian) fails to be Sidon in an extreme sort of way. Then we will explain how Talagrand's majorizing measure Theorem for Gaussian processes implies that the latter union is such that the system $\{f_n(t_1)f_n(t_2)\}$ is Sidon on the square of the underlying probability space. In fact the same conclusion holds whenever (f_n) is subgaussian. The notion of sequence "dominated by Gaussians" plays a key role. Analogous results for random matrices will be described.