

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

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Speaker's Name: Britta Späth

Talk Title: An overview over the inductive conditions for the global-local conjectures

Date: 01 / Feb / 18 Time: 11 : 00 (am) / pm (circle one)

List 6-12 key words for the talk representation theory, groups, McKay Conjecture,
inductive conditions, induction, reduction theorems, simple groups

Please summarize the lecture in 5 or fewer sentences: _____
We give a short survey on the reduction theorems of global-local conjectures
and their verification so far. We focus on the Clifford-theoretic methods involved
and the local structures of simple groups considered. In the end, we report on
first steps towards the verification of the Alperin-McKay conditions. (In joint work
with J. Brough)

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
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(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

(An overview over) the inductive conditions for the global/local conj. ①

§1. Brauer

What is meant by that? Background?

math

representation theory of finite groups: G finite group, p prime

$\text{Irr}(G) \leftrightarrow$ simple $\mathbb{C}G$ -modules simple $\overline{\mathbb{F}}_p G$ -modules $\leftrightarrow \text{IBr}(G)$

Global-local principles:

(Idea behind Brauer's abelian defect gr. conj.)

* (Tool in classification of finite simple groups): local 2-structures $\xrightarrow{\text{CFSG}}$

* Brauer correspondence: p -blocks of $G \rightarrow p$ -blocks of $N_G(P)$ $\xrightarrow{\text{Ratke}}$
already present in Huppert talk *Ratke*

* Green correspondence: bij. for certain indecomp. modules

* If G group/Lie type: Deligne-Lusztig induction, Harish-Chandra induction \rightarrow

Goal: Global/local principle for $\text{Irr}(G)$ or $\text{IBr}(G)$ [2] done

McKay Conjecture '42: $|\text{Irr}_{p'}(G)| = |\text{Irr}_{p'}(N_G(P))|$

for $P \in \text{Syl}_p(G)$ and $\text{Irr}_{p'}(H) := \{\chi \in \text{Irr}(H) \mid p \nmid \chi(1)\}$
true for p -solvable groups (Isaacs, Okuyama-Wajima) ⊗

Red. theorem (Isaacs-Malle-Navarro '07): McKay Conjecture

holds for G , if the inductive McKay conditions hold for all simple groups involved in G .

HERE the inductive cond. \otimes What is missing from simple groups? \triangle
show up

PATIENCE

inductive conditions are technical \leadsto later

§ 2. More Conjectures

Notation
Blockwise:

determines

p -block of G \rightsquigarrow defect group D , $\text{Irr}(B)$, $\text{Br}(B)$, b

p -subgroup of G

$\text{Irr}(G)$ $\text{Br}(G)$ p -block of $N_G(D)$

p -modular system

$$\text{Irr}_0(B) = \{x \in \text{Irr}(B) \mid x(1)_p | D| = |G|_p\}$$

p -Part of degrees minimal

Alperin-McKay (AM) Conjecture: $|\text{Irr}_0(B)| = |\text{Irr}_0(b)|$

Alperin's weight (AW) Conjecture $|\text{Br}(B)| = |\text{Br}(b)|$ if D is abelian
alternatives...

Brauer's height 0 Conjecture: Disabelian $\Leftrightarrow \text{Irr}(B) = \text{Irr}_0(B)$

again all known for p -solvable groups

And refinement's by Gabriel - *Carolina Mandy*
they are implied by Brauer's Conjecture and Dade's Conj.

(both are more technical; \uparrow only for D abelian) *Gabriel's lecture series*

§ 3. Reduction theorems

(3)

$\frac{AM}{AW}$ Conjecture holds, if the inductive $\frac{AM}{AW}$ conditions ^{are} true for all simples
 (S., Navarro-Tiep, Duig)

BHZ: " \Rightarrow " true (Berger-Knörr ('88), Kessar-Malle ('12))
 " \Leftarrow " true, if inductive AM conditions hold
 (Navarro-S., Kessar-Malle)

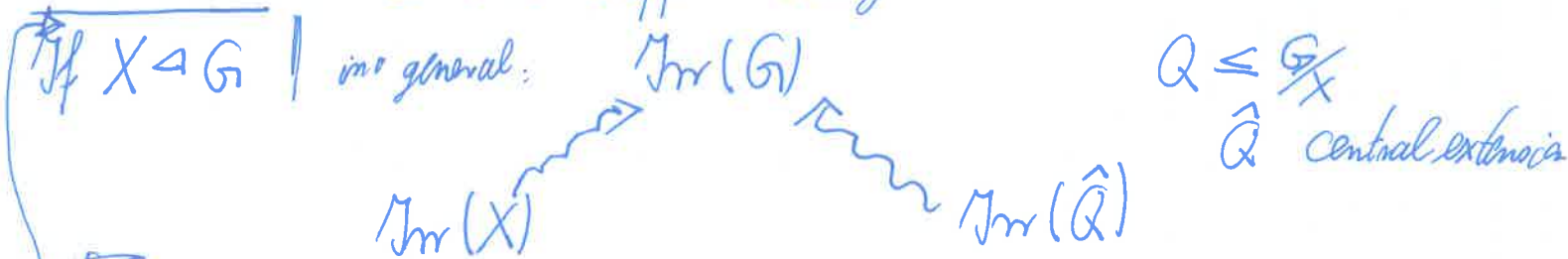
analogous result for Dade Conjecture (S.), pre-reduction BADC (Crawley-Read)

§ 4 The inductive conditions

lin 60

Reduction theorems are proven by induction on $|Z(G)|$

Main idea: Control Clifford theory



G acts on X and $\text{Irr}(X)$

Example: If every $\psi \in \text{Irr}(X)$ extends to some $\tilde{\psi} \in \text{Irr}(G_\psi)$, then

$$\text{Irr}(G) = \left\{ \text{Ind}_{G_\psi}^G(\tilde{\psi}\eta) \mid \psi \in \text{Irr}(X), \eta \in \text{Irr}(G_\psi/X) \right\}$$

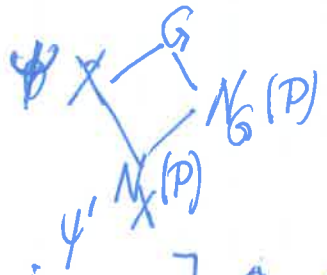
Inductive McKay conditions for a simple S:



Let X be the universal covering group of S and $P \in \text{Syl}_p(S)$.

Then $\exists \text{Aut}(X)_p$ -equiv. $\Omega: \text{Irr}_{P'}(X) \xrightarrow{\sim} \text{Irr}_{P'}(N_X(P)$,
 s.t. $\forall \psi \in \text{Irr}_{P'}(X): \psi \mapsto \psi'$

ψ and ψ' have the same Clifford theory in any $X \triangleleft G$,
 $N_X(P) \triangleleft N_G(P)$



i.e. \exists a group $A: X \triangleleft A$ and $A/C_A(X) \cong \text{Aut}(X)_p$

- ψ extends to some $\tilde{\psi} \in \text{Irr}(A)$
- ψ' " " some $\tilde{\psi}' \in \text{Irr}(N_A(P))$

have analogues with proj rep

$\text{Irr}(\tilde{\psi}|_{C_A(X)}) = \text{Irr}(\tilde{\psi}'|_{C_A(X)})$

Remark: • lead to a better control of Clifford theory

- can be adapted to blocks \rightarrow ind $\frac{AM}{AW}$ conditions
- ideas used in other contexts

$|\text{Irr}_A(G)| = |\text{Irr}_{C_A}(G)|$

Main parts of the proof go from
 $X \triangleleft A$ to $X^{\psi \times \dots \times \psi} \triangleleft G$
 ψ wreath products
 construction butterfly Thm

AM $bl(\tilde{\psi}|_Z) = bl(\tilde{\psi}'|_{N_G(P)}) \forall X \in \mathcal{L}_A$

§ 5. Verification

* for ^{ind.} AM/AW conditions:

A_m , most sporadic, "def. characteristic case, low ranks
(S, Malle) (An-Dietrich, Brewer) (S) : ...g, Feng, Li (Zhang)

nilpotent blocks, cyclic defect (Kö-Sp), some blocks of type A with max defect

* inductive McKay conditions:

• A_m ($m \geq 5$), sporadics, (Malle)

Jordan decompr. of HC Param $\text{Irr}(N_G(P))$

• groups of Lie type p/q : bijection (Malle, S) (if $p \nmid q$)
ind cond. ok, if $p \nmid q$ (S.)

in 10 of 16 types (Cabanes-S.)

Thm (Malle-S.) $|\text{Irr}_2(G)| = |\text{Irr}_2(N_G(P))|$ for any G

§ 6. and next?

if there is ever a natural bijection it should have automatically the Clifford theoretic properties present in inductive conditions