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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Rebekah Palmer **Email/Phone:** palmer@temple.edu (443) 621-7023

Speaker's Name: Amanda Schaeffer-Fry

Talk Title: On the Action of Galois Automorphisms on Characters and Navarro's Sylow 2-Normalizer Conjectures

Date: 01 / Feb / 18 **Time:** 03 : 30 am / pm (circle one)

List 6-12 key words for the talk: representation theory, groups, Navarro Conjecture, McKay Conjecture, group actions, Sylow, blocks, normalizers, reduction theorems

Please summarize the lecture in 5 or fewer sentences: We consider two conjectures: a necessary and sufficient condition for a finite group G to have a (1, Navarro) self-normalizing Sylow 2-subgroup, which is given in terms of the behavior of the ordinary irreducible characters of G under a specific Galois automorphism, and (2, Navarro-Tiep-Vallejo) Sylow 2-normalizers contain a single irreducible 2-Brauer character. A large part of the proofs is to understand the action of this Galois automorphism on characters of groups of Lie type.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
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  - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
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Amanda Schaeffer-Fry

On the action of Galois automorphisms on characters and Navarro's Sylow 2-normalizer conjectures

02 Feb 18

1 Preliminaries

G finite groups, $\ell|\ |G|$ prime, $P \in \mathrm{Syl}_\ell(G)$, $\mathrm{Irr}_{\ell'}(G) = \{\chi \in \mathrm{Irr}(G) \mid \ell \nmid \chi(1)\}$, $\chi^\times(g) = \chi(g)^\sigma$ for $\sigma \in \mathcal{G} = \mathrm{Gal}\left(\mathbb{Q}(\mathcal{S}_{|G|})\big/\mathbb{Q}\right)$

Navarro's Galois-McKay Conjecture, '04. *For certain $\sigma \in \mathcal{G}$,*

$$|\mathrm{Irr}_{\ell'}(G)^\sigma| = |\mathrm{Irr}_{\ell'}(N_G(P))^\sigma|$$

where $(G)^\sigma$ denotes the fixed points under σ.

Note that the case $\sigma = 1$ is the usual McKay conjecture.

2 The Main Theorems (weak forms)

Fix $\ell = 2$.

SN2S Theorem. *$P = N_G(P)$ if and only if $\chi^{\sigma_0} = \chi, \forall\chi \in \mathrm{Irr}_{2'}(G)$ where*

$$\begin{matrix} \sigma : \mathcal{S}_2 & \mapsto & \mathcal{S}_2 \\ \mathcal{S}_{2'} & \mapsto & \mathcal{S}_{2'} \end{matrix} \in \mathcal{G}$$

Proven by SF ('16), SF-Taylor ('18), SF ('18)

Blockwise SN2S. *$N_G(P) = P \times V$ if and only if $\chi^{\sigma_0} = \chi, \forall\chi \in \mathrm{Irr}_{2'}(B_0)$, where we called B_0 the principal block.*

Conjectured in Carolina Vallejo's talk by Navarro-Tiep-Vallejo. Further handled by NV ('17) and SF-Taylor ('18, hopefully).

Reduction (done by SF, '16, announced in '14): Every nonabelian simple group satisfies SN2S ("inductive conditions"), so SN2S cconjecture holds $\forall G$. The inductive conditions also keep track of $\mathrm{Aut}(G)_2 \curvearrowright \mathrm{Irr}(G)$ (where $\mathrm{Aut}(G)_2$ is a subgroup of automorphism group that is a 2-group).

NV, '17. *Reduced "blockwise SN2S" and blockwise implies original SN2S.*

3 SN2S-Goodness for simple groups

We have our groups

Alt_n (alternating)
Sporadic

Lie type

SF, '16
several low rank cases
Lie(2) (except $E_6^{\pm}(2^a)$)
most notably, $P\mathrm{SL}_n^{\pm}(2^a)$

SF, '18a
Type A
$P\mathrm{SL}_n(q), q$ odd
$E_6^{\pm}(2^a)$

SF, '18
rest of $\mathrm{Lie}(p), p$ odd
$(\underbrace{B_n, C_n, D_n^I}_{\text{symplectic/orthogonal}}, E_6^{\pm}, E_8)$

Clifford Theory
Deligne-Lusztig Theory
GGGRS

Remark: SF-Taylor, '18b, gives us that all this work provides blockwise conditions.

4 Working with groups of Lie type

$q = p^a$, $G = \mathbb{G}^F$ fixed points of $\mathbb{G}$ (connected reductive algebraic group over $\overline{\mathbb{F}_p}$) under Frobenius morphism F.

Example.

$$\begin{aligned} GL_n(q) &= GL_n(\overline{\mathbb{F}_q})^F \\ SL_n(q) &= SL_n(\overline{\mathbb{F}_q})^F \\ \mathrm{SU}(q) &= SL_n(\overline{\mathbb{F}_q})^{F'} \end{aligned}$$

where

$$\begin{aligned} F &: (a.j) \mapsto (a.j^q) \\ F' &: (a.j) \mapsto (a.j^q)^{-T} \end{aligned}$$

Local side When does $G \rtimes Q$ have self-normal Sylows? ($Q \cong \mathrm{Aut}(G)$ 2-group)
Global side Somehow understand $\mathcal{G} \curvearrowright \mathrm{Irr}(G)$???

Malle-Späth, '16. *If G is not type A, then $\chi \in \mathrm{Irr}_{2'}(G)$ lies in "nice" Harish-Chandra series (denoted H-C).*

Strategy (SF, '18): Analyze the action of $\mathcal{G} \curvearrowright$ H-C.

H-C Induction: Take

$$\begin{array}{ll} G = \mathbb{G}^F & G = \mathrm{GL}_n(q) \\ P = U \rtimes L & P = \text{upper-triangular} \\ L \text{ Levi} & \\ \lambda \in \mathrm{Irr}(L) \text{ “cuspoidal”} & L = \begin{pmatrix} * & & 0 \\ & * & \\ 0 & & * \end{pmatrix} \\ R_L^G = \mathrm{Ind}_P^G(\mathrm{Inf}_L^P(\lambda)) & \end{array}$$

Note: every $\chi \in \mathrm{Irr}(G)$ is a constant of some $R_L^G(\lambda)$. $\mathrm{Irr}(G|R_L^G(\lambda))$ is called the "$(L\lambda)$ H-C series".

Howlett-Lehrer Theory:

$$\mathrm{Irr}(G|R_L^G(\lambda)) \longleftrightarrow \mathrm{Irr}(\mathrm{End}_G(R_L^G(\lambda))) \longleftrightarrow \mathrm{Irr}(W(\lambda))$$

$$R_L^G(\lambda)_\eta = \chi \longleftrightarrow \mathrm{Hom}_G\left(R_L^G(\lambda), \chi\right) \longleftrightarrow \eta$$

$$\mathrm{Irr}(G|R_L^G(\lambda^\sigma)) \longleftrightarrow \mathrm{Irr}(\mathrm{End}_G(R_L^G(\lambda^\sigma))) \longleftrightarrow \mathrm{Irr}(W(\lambda^\sigma))$$

where $\sigma \in \mathcal{G}$. We want there to be some kind of projection to make the diagram commute. Here's what we get instead.

Theorem (SF, '18).

$$(R_L^G(\lambda)_\eta) = R_L^G(\lambda^\sigma)_{\eta'} \text{ where } \eta' = \nu_\sigma \delta_\sigma \eta(\sigma)$$

where $\begin{cases} \left.\begin{array}{l} \nu_\sigma \\ \delta_\sigma \end{array}\right\} \text{ a 2-linear character} \\ \eta(\sigma) \end{cases}$ *depends on* $\begin{cases} \sqrt{q}^\sigma \\ \text{choice of extension of } L \text{ to } N(G)_\lambda \\ \text{values of End characters} \end{cases}$