Workshop Sessions:2017.03.17.1100.Adiredja

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Speaker's Name:

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Talk Title:

Workshop Session 3a: Interaction of Power and Cognition in Undergraduate Mathematics

Date:	03/17/2017	**Time**:	11:00 - 12:00	am

Materials:

- Presentation slides (pdf)
- Detailed notes from notetaker (pdf)
- Handout of student stories (pdf)

List 6-12 key words for the talk:

Equity, Cognitive theoretical framework, mathematical thinking

Please summarize the lecture in 5 or fewer sentences:

The speaker presented the conceptual and theoretical framework of power, identity and knowledge in viewing the cognitive work of power in creating mathematics for intersectional groups, most especially women of color. The talk included a shared student work experience and participant discussion. Key themes include that mathematical thinking/learning/cognition are situated within identity discourses/narratives. In addition, our perspective as math educators on students' knowledge and reasoning carry weight and power, we decide what successful sense making look like, and who can be positioned as successful in mathematics. Thus, it is not about optimism (vs pessimism) (or half full/half empty), but accuracy (if we're more accurate we can be more humanizing).

Intersection of Power and Cognition in Undergraduate Mathematics

Aditya P. Adiredja
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MSRI Critical Issues in Mathematics Education
17 March 2017

STATE OF OUR WORK

Q: If you were to guess, what is the percentage of full-time mathematics faculty members are women? What about women of color?

- 29% of full-time mathematics faculty members are women
- 5% of full-time mathematics faculty members are women of color (Asian, Black, Latinx and Native Alaskan, Hawaiian and Native American women)

(CBMS Survey 2010)

HIGHLIGHTS FROM EQUITY RESEARCH IN UNDERGRADUATE MATHEMATICS EDUCATION

"I was like, wow! I guess it's good to see how other Hispanic people are so good at doing math…. It's not what people usually think of. I think it makes me proud that there's a chunk of us, I'll put myself in that group, that are willing to do whatever to be good at math or to excel in math" – Vanessa, Latinx (Oppland-Cordell, 2014, p. 20)

Adiredja, A. P. & Andrews-Larson, C. (2017). Taking the sociopolitical turn in postsecondary mathematics education research. International Journal for Research in Undergraduate Mathematics Education.

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- "Visible and collaborative" way of engaging with authentic mathematical activity (Hsu, Murphy, Treisman, 2008)
- Allows for recalibration of discourses around hierarchy of intelligence associated with different social identities (Oppland-Cordell, 2014)
- Counter-narratives have the potential of repairing "identity infiltration" (Larnell et al. 2014)

HIGHLIGHTS FROM EQUITY RESEARCH IN UNDERGRADUATE MATHEMATICS EDUCATION

F: [gives example of her work] “I actually thought, I can do this stuff instead of just looking at notes from a professor and doing the same thing. I can actually think of stuff for myself and use the tools that they gave me to accomplish something. /…/ So even if you are doing the same basic thing [in your proof], you still have [the feeling that] that’s my work that I can actually say is mine” (Hassi & Laursen, 2015, p. 325).

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- Higher frequency in women in the study reporting increased self-esteem and pride for their mathematical accomplishments compared to the men after IBL class (40% vs. 24%)
- Women also more frequently reported increased confidence to work on mathematical problems after an IBL class compared to the men (74% vs. 59%)

POWER

"[K]nowledge and power are inextricably linked. That is, because the production of knowledge reflects the society in which it is created, it brings with it the power relations that are part of society. What counts as knowledge, how we come to "know" things, and who is privileged in the process are all part and parcel of issues of power" (Gutiérrez, 2013).

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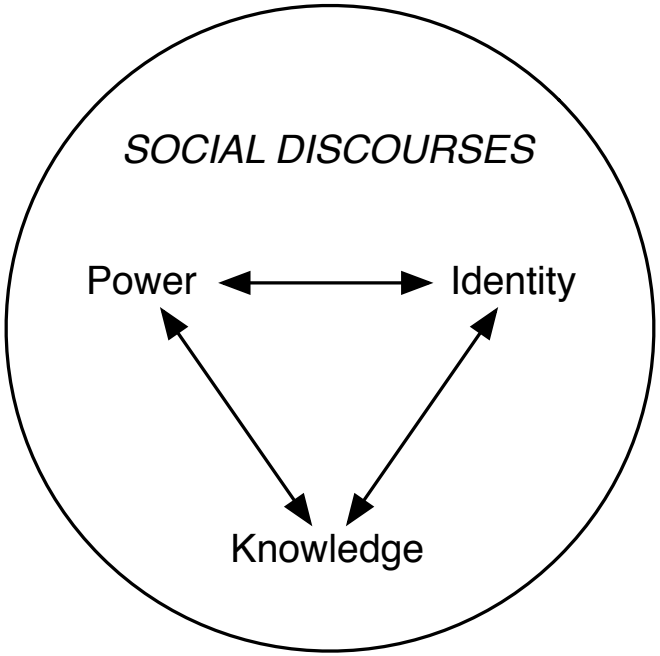
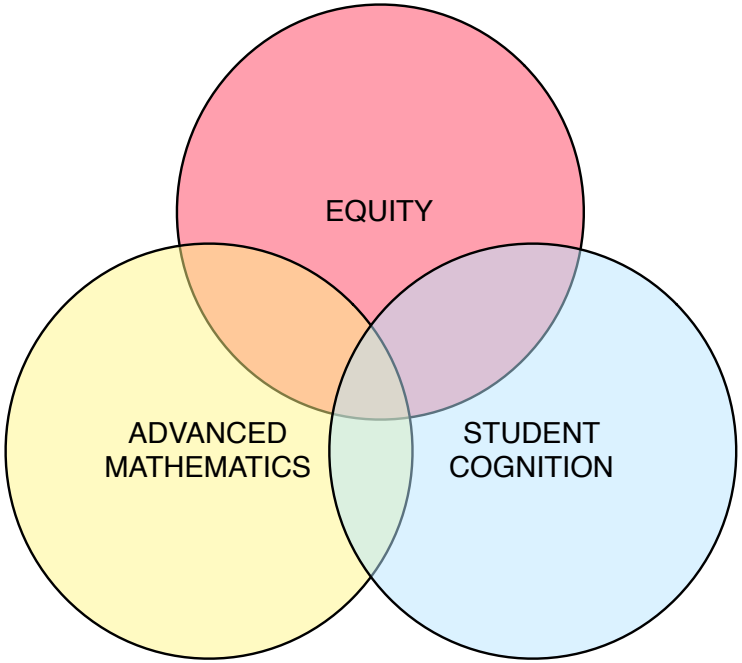
CONCEPTUAL FRAMEWORK

- Sociopolitical Lens (Gutiérrez, 2013; Valero, 2004)



MY RESEARCH FOCUS



- *The role of narratives in selecting from which students we elicit mathematical thinking, and how we view their knowledge*
  - *Women of color's sense making of advanced mathematics*

WHY WOMEN OF COLOR?

- Underrepresentation of women and people of color in STEM and mathematics
- Studied in terms of women's experiences with sexism and students' of color's experiences with racism
- Women of color experience both!
- Call for more *intersectional* research about women of color (WOC) in STEM (Ong, Wright, Espinosa, & Orfield, 2011)
- Sexism and racism as manifested through narratives (Nasir and Shah, 2011; Nasir, Snyder, Shah, & Ross, 2013)

HOW SHOULD WE FOCUS ON WOC?

- Historical focus on deficit narratives about students of color in STEM (Harper, 2010)
- Achievement gap and comparison studies involve deficit narratives (Gutiérrez, 2013; Gutiérrez & Dixon-Roman, 2011)
- Focus instead on understanding particular groups learning experiences, and offer counter narratives (e.g., Berry III, Thunder, and McClain, 2011)
- Studies of successful Black students in undergraduate mathematics and STEM (Ellington & Frederick, 2011; Martin & McGee, 2009)

INTERSECTION WITH MATHEMATICAL SENSE MAKING

- General deficit narrative about students' mathematical sense making (e.g., misconceptions, what students don't know or are not learning)
- Intersects with broader deficit narratives about WOC's participation in mathematics
- Offer a counter narrative about WOC by documenting their successful sense making (e.g., Hidden Figures)

TWO COMPLIMENTARY RESEARCH GOALS

- Construct a counter narrative about WOC by investigating their sense making of mathematics from an anti-deficit perspective
- Investigate details about ways to understand a mathematical concept

THEORETICAL FRAMEWORK

Knowledge in Pieces (diSessa, 1993; diSessa, Sherin and Levin, 2016)

- Explicit anti-deficit perspective on knowledge and students (Smith, diSessa & Roschelle, 1993)
  - Prior knowledge is foundational to learning (constructivism)
  - Misconceptions are faulty extension of productive prior knowledge
- Understand a subject's own way of reasoning about a topic, not to assess its correctness with respect to a normative standard
- Forms of naïve knowledge are diverse, rich, and generative
- Intuitive knowledge (elements) is a target of study

FORMAL DEFINITION OF A LIMIT

Definition. Let f be a function defined on some open interval that contains the number 1, except possibly at 1 itself. Then we say that the **limit of** $f(x)$ **as** x **approaches** 1 **is** 5, and we write

$$\lim_{x \to 1} f(x) = 5$$

if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if} \quad 0 < |x - 1| < \delta \quad \text{then} \quad |f(x) - 5| < \epsilon$$

STORY	FORMAL DEFINITION
The boss gives the error bound on the size relative to 5 inches	For any given $\varepsilon > 0$,
You determine the error bound for the amount of batter relative to 1 cup	there exists a $\delta > 0$ such that
So that if you keep your error in the amount of batter to be less than the discovered error bound	if $0 < \lvert x-a \rvert < \delta$
Then the error in the size will be less than the error bound given by the boss	then $\lvert f(x)-L \rvert < \varepsilon$.

THE CASE OF ADRIANA

- Chicana student
- Mathematics & Chicanx Studies major on her second year at a large public research university
- Answered question about the relationship between delta and epsilon before and after discussing the Pancake Story

Adiredja, A. P. (2017). Anti-deficit Narratives: Politics of Mathematical Sense Making. Manuscript in preparation.

RESULT: EPISODE 1

554	A	[They kinda depend on each other] yeah in a sense because, but more **whatever you're getting, f of x is always gonna depend on what x you're inputting it.** But then, if you want to get something that's within delta [*marks a small interval on the x axis*] you need to see if /.../ for example here [*points to the pancake story*] our epsilon here was already set, then that [*points to* 4.5 *and* 5.5 *in the inequality* $4.5 < f(x) - L < 5.5$] kind of depended on what we were putting in for *x* [*points at the same interval around x on the graph*] but.. but mostly **whatever you putting in for x will determine what you get for f of x [*f* (*x*)]**. So I still say the same thing like delta depends on epsilon but
554	A	\...\ **So I still say the same thing** like delta depends on epsilon but,
555	I	Delta depends on epsilon? Or epsilon depends..
556	A	No, yeah, **epsilon depends on delta.** But, **if epsilon's already set then you'll manipulate your delta so it's within an error bound** and then continue to manipu- wait [long pause] wait, so you're… hm.

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RESULT: EPISODE 4

566	A	Um, see cus I was looking at it like /…/ **the f of x [*f* (*x*)] depends on the *x*** and that's how I was like saying that epsilon depends on delta because **epsilon is related to the f of x [*f* (*x*)]**/…/. But that's just saying **the error of the *L* and the f of x [*f* (*x*)] depends on the *a* and *x*** but **that's not to say that epsilon depends on delta.**
567	I	Ok, so?
568	A	So, I think that **delta depends on epsilon now** [*laughs*]. Just cus if it's given like this [*reference unclear*] and **you're trying to aim at getting /…/ within a certain error bound, then you're gonna try to manipulate your entries /…/ to be within a certain error bound** [*gestures a small horizontal interval*]

RESULT: EPISODE 4

569	I	Ok. Alright, so and so you changed your mind it seems? Um, so how did that happen? Why did you change your mind?
570	A	Because I was **given an epsilon** [*points at the inequality* $0.5<f(x)-L<0.5$] and that's kinda like the main goal. **The main goal is to get the pancake, /…/ and they gave me a constraint.** And then they didn't give me an error bound for the batter or for the *a* or *x*. But I know I want to make it *small* so that it's within the error bound, the epsilon. So then **I would kinda base my delta on what was epsilon**.

RESULT: EPISODE 4

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DISCUSSION: ANTI-DEFICIT FRAMEWORK WITH KIP

- A story about successful mathematical sense making by WOC
- Sensitivity of analysis detected awareness of productive mathematical ideas despite uses of non-normative language
- Adriana did not simply "accept" ideas from the intervention
- She took time to align her prior knowledge and the ideas from the story
- Result: an appropriate mathematical claim grounded in her prior knowledge and the resources leveraged in the story.

DISCUSSION: DEFICIT FRAMEWORK

- Adriana fits with other students who struggled with the formal definition
- Adriana's reliance of functional dependence was incorrect and persisted
- Even with so much support, Adriana still struggled
- Adriana is likely to only be able to reason in the everyday context and not in the "real" mathematical context

HALF FULL v. HALF EMPTY?

- The deficit interpretation produces inaccurate claims about Adriana and her sense making
- The perspective refuses to acknowledge Adriana's contribution in making sense of the mathematics
- It accepts Adriana's non-normative language use as an indicator of a lack in understanding
- It prioritizes immediate change as evidence for learning
- It also undermines the progress made as a result of utilizing non-formal mathematical resources in reasoning, which in turn continues to privilege reasoning solely in formal mathematical context

LET'S CREATE OTHER COUNTER STORIES! (10 minutes!)

- Women of colors' explanation about basis (in Linear Algebra) using everyday ideas (Adiredja & Zandieh, 2017)
- In pairs!
- Read the students' explanation
- Select one explanation and discuss the following questions:
  - What aspects of basis did the student capture in their explanation?
  - What didn't it capture?
  - Can you modify the explanation to express other aspects about basis that you feel is also important?

EIGHT WOMEN OF COLOR

Student	Racial/Ethnic Background	Linear Algebra Completion	Grade	Other Mathematics Courses
Leonie	African American	Spring 2016	A	Calculus I, II, and III
Morgan	Asian/Asian American	Spring 2016	A	Calculus I, II, and III, and Differential Equations
Annissa	Hispanic/Latinx	Fall 2014	B	Calculus I and II
Eliana	Hispanic/Latinx	Spring 2014	C	Calculus I and II
Nadia	Hispanic/Latinx	Fall 2015	A	Calculus I, II, and III
Jocelyn	Hispanic/Latinx	Spring 2015	B	Calculus I, II, and III
Stacie	Hispanic/Latinx	Spring 2016	C	Calculus I, II, and III
Liliane	Hispanic/Latinx/White	Fall 2015	B/C	Calculus I, II, and III

Adiredja, A. P. & Zandieh, M. (2017). Using women of color's intuitive examples to reveal nuances about basis. Paper presented at the 20th Annual Conference on Research in Undergraduate Mathematics Education, San Diego, California.

RESULTS: STUDENTS' EVERYDAY CONTEXTS

Student	Context (for basis and vector space)
Leonie	friendship
Morgan	driving in a city (on a grid), Legos, cooking, groups of pens
Annissa	set of solutions (no actual everyday example)
Eliana	least amount of myself I need to cover the space of the room, storage room, dimension, skeleton, outline of a paper
Nadia	floor, universe and earth, syntax in programming
Jocelyn	fashion, recipe, art sculpture, collage
Stacie	walking to places in a room, floor as a plane, marching band
Liliane	religious teachings

ASPECTS OF BASIS

Roles:

- ***Generating***: To create the larger space from the basis vectors
- ***Describing***: To describe the space using the basis vectors

Characteristics:

- ***Minimal*** focuses on the required number or amount of vectors needed for the basis. A basis has the least amount of vectors necessary.
- ***Essential*** focuses on the identities of the vectors that make them core and necessary for the larger space.
- ***Representation*** focuses on identifying and/or naming the smaller set of objects as providing structure or serve as representative for the larger space.
- ***Non-redundant*** focuses on not wanting extraneous information in a set, and/or the act of reducing or removing the extraneous information.
- ***Different/sameness*** focuses on comparing items (vectors) based on their difference/similarity for the sake of keeping or removing items from the basis.

DISCUSSION

- Results from anti-deficit analysis of these women's explanations contribute to local theories about the learning of basis
- These everyday explanations provide a window into students' sense making and contextualizes basis
- Creativity and breadth of the everyday contexts used to describe basis by these female students of color
- Sophistication in their assessment of everyday contexts they generated and the degree that they fit with the formal definition of basis
- Repositioning these students' as resource for teaching instead of a group of students that need extra support
- Intentional selection of participants and connecting to broader narratives highlight the politics

TAKEAWAYS

- Mathematical thinking/learning/cognition are situated within identity narratives
- Our perspective on students' knowledge and reasoning carry weight and power
- We decide what successful sense making look like, and who can be positioned as successful
- It is not about optimism, but accuracy
- One slice in striving for equity, and you can help!

Thank you!

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Using Women of Color's Intuitive Examples to Reveal Nuances about Basis

Aditya Adiredja & Michelle Zandieh (2017)

"I'm just thinking like the whole universe, and that's like planets and stars and that like describes the universe. But only a planet describes earth. So to get a basis for the universe you'd need planets and stars." (Nadia, Latinx).

Using Women of Color's Intuitive Examples to Reveal Nuances about Basis

Aditya Adiredja & Michelle Zandieh (2017)

"So I'm very religious and so the teachings that I that we share with each other and that we read about and all that stuff. Like, there are a lot of things that you can add to and be like here's an application and here's the things, and this expands to this and this and this. But there's like the most basic teachings and like it all comes back to that. And this is the basic thing like you have the Ten Commandments. You have the Scriptures and you have like the prophets and you have your connection with God and, like all of the decisions and all of things that come from that and you can reach all of the other points with this basis." (Liliane, Latinx).

Using Women of Color's Intuitive Examples to Reveal Nuances about Basis

Aditya Adiredja & Michelle Zandieh (2017)

"I think I could describe it as my different friend group[s]. /…/ They don't really like associate with each other, but I hang out with them, the three of those groups, and I don't know, I get something from all of them. And it makes like a whole. So it's almost like I'm the space and then they're spanning me. They're all like completely different. Like, one of my friends, my best friend is, she's really serious and everything. Then my roommate is really goofy. And then, my other friend, she's just really wild and crazy. So it's, like, they all kinda fill all the gaps"(Leonie, African American).

Using Women of Color's Intuitive Examples to Reveal Nuances about Basis

Aditya Adiredja & Michelle Zandieh (2017)

"I immediately try to think of, I thought of fashion and a wardrobe. And so, say you have like all these different like outfits you want to make. You have a minimum number of like pieces. Like a pair of shoes or a shirt or like a pair of pants that you need that allows you to make all of those outfits. But you don't want to have like two of the same pairs of shoes cause you know that's wasteful. You don't need two of them. You just use one. /…/ When you scale a vector by a constant, it might look different. It might change its length or its direction but its identity isn't really changed, and so it'll be the same vector. And so if you have different pairs of heels, they'd look different but they're still heels, they're still like formal. So they're (inaudible) still the same. (Jocelyn, Latinx)

Detailed Notes of Workshop Sessions: 2017.03.17.1100.Adiredja

Adiredja started by sharing his story from his initial desire to do equity work to teach math better. He moved to the US at 15, not speaking English. From there he shared how his K-12 experiences to community college, his emerging scholars undergraduate work at Berkeley, his graduate school experience in math education and his experience in moving to a small town in Oregon. This move made his aware of the protection that he had had previously as a math person, a person of color. For that reason equity became very real. This shifted his math education research to think about cognition and equity together.

Q: If you were to guess, what is the percentage of full-time mathematics (all US institutions tenure track/postdoc/instructors) faculty members (teaching mathematics content courses) are women? How many are women of color (non-white people, including asians)?

Answers from the crowd:

- 30% women, 1% women of color
- 8% women, 1% or less women of color
- 20% women, 1% women of color
- 15% women, 3% women of color

In reality… Conference Board of Mathematics Survey, 2010:

- 29% of full-time math faculty of women
- 5% of full-time mathematics faculty women are women of color (Asian, Black, Lantinx, and Native Alaskan, Hawaiian, and Native-American Women).
  Asian narratives, especially pacific islander, is a lumping narrative that we need to also a story.

Why does this matter?
It matters because of the kind of story that it tells about women in mathematics. Whose field is this?

Research on emerging scholars undergraduate education program and focuses on power and identity impact on students of color. Here's the quote:

"I was like, wow! I guess it's good to see how other Hispanic people are so good at doing math.... It's not what people usually think of. I think it makes me proud that there's a chunk of us, I'll put myself in that group, that are willing to do whatever to be good at math or to excel in math" – Vanessa, Latinx (Oppland-Cordell, 2014, p. 20)

Adiredja picked this quote to highlight that fact that our students are aware of the narrative that exists about how people perceive to be their status in the field. It also highlights the work that Hsu, Murphy, Treisman wrote about the main principle of the emerging scholars program: "visible and collaborative" way of engaging with authentic mathematical activity. Seeing other people that look like you do strong mathematical work that has potential to recalibration of discourses around hierarchy of intelligence associated with different social identities. Larnell (Chicago) studies the idea of potential of counter-narratives telling another story of repairing "identity infiltration" (you start believing the stereotypes you hear about your group).

Here is another quote: [gives example of her work] *"I actually thought, I can do this stuff* instead of just looking at notes from a professor and doing the same thing. *I can actually think of stuff for myself and use the tools that they gave me to accomplish something. /.../ So even if you are doing the same basic thing [in your proof], you still have [the feeling that] that's my work that I can actually say is mine"* (Hassi & Laursen, 2015, p. 325).
This is from study of impact on identity…
In one word - can you describe what this student is feeling?
Accomplishments,empowerment, ownership, Belief, ability, feeling, Agency.

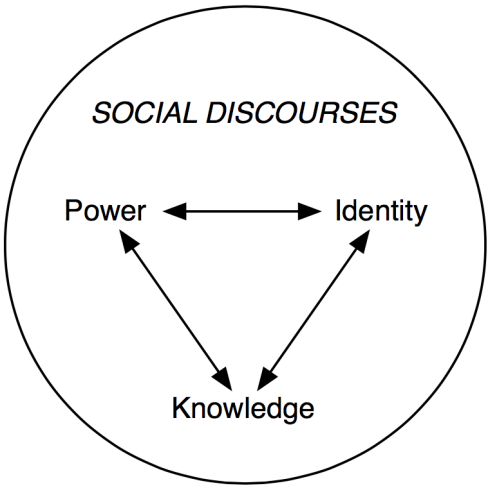
The words that you're using are words of empowerment. That is what they are studying. The impact of identity, agency/believe on empowerment including social empowerment. They did see interesting and encouraging pattern with women in the study. And they came up with overall a narrative of practice. Looking at the highlighted section you see again that this is what they understand to be the practice of mathematics - look at notes to do what the professor is doing vs constructing mathematics.

"[K]nowledge and power are inextricably linked. That is, because the production of knowledge reflects the society in which it is created, it brings with it the power relations that are part of society. What counts as knowledge, how we come to "know" things, and who is privileged in the process are all part and parcel of issues of power" (Gutiérrez, 2013).

Through these narratives we see the power. The narratives communicate particular their part in the creation of mathematics. That is what cognitive work is all about this, we're studying knowledge. Those of us in mathematics do this everyday with our students, we look at how they do mathematics. There is a great deal of power and responsibility in that.

Conceptual Framework

Sociopolitical Lens (Gutiérrez, 2013; Valero, 2004)



Interrelatedness of power, identity, and knowledge. Influences how we position students and how students negotiate themselves and negotiate power. They are situated in, emerge from, and influence social discourse.

Adiredja's research lies in the intersection of equity, advanced mathematics, and student cognition. Lately he's been focusing role of narratives, thinking about from which students do we elicit mathematical thinking? How

do we view that knowledge when we get it out? In particular chose to study women of color and sense making in advanced mathematics. We often think about women's experience with sexism (systemic or institutional) or people of color's experience with racism (systemic or institutional). And women of color experience both these things at the same time. Intersectional research is needed and it is needed for more including looking at other sexual identities. In general we need to be more inclusive.

How should we focus on women of color (WOC)?
Historically there is always discussion of deficits. They think this way, they don't have the required ways of thinking to do mathematics. This is reflected in all the achievement gaps. We love reporting on achievement gaps. What we need to realize is when we talk about achievement gap we center the dominate culture - everyone else is trying to catch up or become more like that dominate culture- they become the norm.

To focus on this Adiredja focused on the narratives - sexism and racism as established in the narratives. Look at what White presented earlier, the preconceptions of pre-service teachers (PSTs) before they meet these students of color, these come from the narratives about those students.

There is hope! We look to a group of scholars, especially black scholars who chose to focus on successful students, where success isn't just defined by achievement but also how are they making sense of their experience and navigating racism. The goal is providing counternarratives from these folks. (Ellington & Frederick, 2011; McGee & Martin, 2009)

Another intersection - all this intersects with our common deficit view of mathematical thinking. When we think about student sense making we focus too much on on misconceptions. On what students aren't learning. How their thinking is deficient in some ways. So that narrative of general mathematical thinking intersects with deficit narrative about particular groups.

Adiredja wants to create a counter narrative of WOC, documenting their successful sense making. Looking for more stories like Hidden Figures which is so popular because it highlights different story. It changes the narrative of the role of black women in the advancement of science in the nation.

Two goals for Adiredja's research-

1. Construct a counter narrative about WOC by investigating their sense making of mathematics from an anti-deficit perspective
2. Investigate details about ways to understand a mathematical concept

Theoretical Framework - Knowledge in Pieces
This theoretical framework was established in physics. What it shows is that it's not just a choice to look at misconceptions differently. There is research that backs up the futility in replacing misconception. This paper (Smith, diSessa & Roschelle, 1993) changed how I looked at things. They found research about misconceptions is the wrong way to do. Instead look how foundational student's prior knowledge is in learning. Understand a subject's own way of reasoning about a topic, not to assess its correctness with respect to a normative standard.

The thing… Formal Definition of a Limit and Pancakes
It's just so difficult. It is true. There is research that show why this is so challenging. Let's look at the conceptual relationship between delta and epsilon.

Definition. Let f be a function defined on some open interval that contains the number 1, except possibly at 1 itself. Then we say that the **limit of $f(x)$ as x approaches 1 is 5**, and we write

$$\lim_{x \to 1} f(x) = 5$$

if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if} \quad 0 < |x - 1| < \delta \quad \text{then} \quad |f(x) - 5| < \epsilon$$

Just look at as a student for the first time. Delta is next to x and epsilon is related to f(x), somehow.
If I was to ask you what the relationship is to epsilon and delta then you'd say epsilon depends on delta because f(x) depends on x. A lot of students believe that. They think that epsilon with f(x) and delta with x so the relationship much go that way.
Adiredja was trying to build on this using student's pre-knowledge. There was a consistent misconception about this from students so he created the pancake story. This seemed to be consistent with students' understanding of quality control….

Story: You work at a pancake restaurant and the pancakes all have to be 5 inches. Your boss is allowing you for the first week to have pancakes 5 inches with a little bit of error (epsilon). So your goal is that for any error bound of the pancake that your boss is going to give you then you need to find the delta on the pancake batter to ensure that your pancake size is within the delta.

Your job is to figure out what the error bound on the 1 cup of batter so that you're within resulting pancakes will be within the pancake error bound that your boss gave you. So we have a notion of quality control.

Adiredja created this story and then did a study: would ask students the relationship between epsilon and delta, provide this intervention, and then ask them again, crossing his fingers that it would change their mind.

Study result:
ADRIANA: Chicana student. Mathematics & Chicanx Studies major on her second year at a large public research university

Episode 1: She thought about it the way he described before: delta depends on x, epsilon depends on y, y depends on x so epsilon depends on delta. So asked after the intervention - she followed the story, answered all the questions, and then he asked her again:
whatever you're getting, f of x is always gonna depend on what x you're inputting it… whatever you putting in for x will determine what you get for f of x [f(x)].So I still say the same thing like delta depends on epsilon but
He was heart broken. Beautiful intervention and it hadn't gotten at that. Yet, she did recognize that delta was some kind of bound: *if epsilon's already set then you'll manipulate your delta so it's within an error bound*. The givenness of epsilon is key so she still connected epsilon and delta as depending on each other but she did understand the quality control.

During following episodes she kept articulating it and kept doing it wrong.

Episode 4: *the f of x [f (x)] depends on the x and that's how I was like saying that epsilon depends on delta because epsilon is related to the f of x [f (x)]/.../. But that's just saying the error of the L and the f of x [f (x)] depends on the a and x but that's not to say that epsilon depends on delta… So, I think that delta depends on epsilon now [laughs]. Just cus if it's given like this [reference unclear] and you're trying to aim at getting /.../*

within a certain error bound, then you're gonna try to manipulate your entries /.../ to be within a certain error bound [gestures a small horizontal interval]

What's critical here isn't that functional misconception. What's cool here is she took that relationship and repurposed it in context that it's appropriate. Once she distinguished between error and error bound she changed her language and started talking about pancakes. Notice the change in language, she's reasoning in context of pancakes.

My conclusions from my analysis: this is a story of successful mathematical sense making by a women of color.

- Productive knowledge, even at the beginning.
- Sensitivity of analysis detected awareness of productive mathematical ideas despite uses of non-normative language
- Adriana did not simply "accept" ideas from the intervention
- Took time to align with prior knowledge, resulted in a more grounded, appropriate claim about epsilon/delta.
- Result: an appropriate mathematical claim grounded in her prior knowledge and the resources leveraged in the story.

Compare to deficit framework of looking at this student:

- She aligns with other students in the literature who struggled with the formal definition.
- Adriana's reliance of functional dependence was incorrect and persisted
- Even with so much support, Adriana still struggled and then continued to struggle
- Maybe I can recognize she did something right but Adriana is likely to continue to struggle …. Likely to only be able to reason in the everyday context and not in the "real" mathematical context.

A key to highlight here is how reasonable this all sounds and how easy it is for us to reach this conclusion if we aren't careful.

<u>Is it just about HALF FULL versus HALF EMPTY?</u>

• The deficit interpretation produces inaccurate claims about Adriana and her sense making.
• The perspective refuses to acknowledge Adriana's contribution in making sense of the mathematics.
• It accepts Adriana's non-normative language use as an indicator of a lack in understanding.

We can think about this implication for our english language learners, especially our reliance on language to demonstrate knowledge.

• It prioritizes immediate change as evidence for learning.
• It also undermines the progress made as a result of utilizing non-formal mathematical resources in reasoning, which in turn continues to privilege reasoning solely in formal mathematical context

Let's create other counter stories… Women of colors' explanation about basis (in Linear Algebra) using everyday ideas

Given a handout of student stories (Aditya Adiredja & Michelle Zandieh (2017)).
Instruction:

- Can you explain basis (from linear algebra)? stand up
- If you are seated, select someone who is standing to work with
- Try to talk to each other about what basis is… Work in pairs or groups of three.
- Once you've met your partner then talk with each other about what basis is - before you look at what the students did.

- After you did that what you have in front of you is a selection of explanation of basis from WOCs using everyday context. Can you use it in explaining basis? Feel free to separate pages - four separate examples.
- As you are reading - what aspects of basis did the students' explanations capture. Focus on an anti-deficit view of this work. How would you modify the explanation so it does reflect all the things you want it to reflect?

There were four explanations of basis:

fashion and a wardrobe. "I think I could describe it as my different friend group[s]. /.../ They don't really like associate with each other, but I hang out with them, the three of those groups, and I don't know, I get something from all of them. And it makes like a whole. So it's almost like I'm the space and then they're spanning me. They're all like completely different. Like, one of my friends, my best friend is, she's really serious and everything. Then my roommate is really goofy. And then, my other friend, she's just really wild and crazy. So it's, like, they all kinda fill all the gaps"(Leonie, African American).

Scaling vector by constant**-** understand the aspect of dimension
Captured idea of dimension - numbers and choices. Hadn't captured vectors

my different friend group[s] "I think I could describe it as my different friend group[s]. /.../ They don't really like associate with each other, but I hang out with them, the three of those groups, and I don't know, I get something from all of them. And it makes like a whole. So it's almost like I'm the space and then they're spanning me. They're all like completely different. Like, one of my friends, my best friend is, she's really serious and everything. Then my roommate is really goofy. And then, my other friend, she's just really wild and crazy. So it's, like, they all kinda fill all the gaps"(Leonie, African American).

Get to linear independence/ spanning - all three of them cover her
Captured idea of dimension - numbers and choices. Hadn't captured vectors

Religion "So I'm very religious and so the teachings that I that we share with each other and that we read about and all that stuff. Like, there are a lot of things that you can add to and be like here's an application and here's the things, and this expands to this and this and this. But there's like the most basic teachings and like it all comes back to that. And this is the basic thing like you have the Ten Commandments. You have the Scriptures and you have like the prophets and you have your connection with God and, like all of the decisions and all of things that come from that and you can reach all of the other points with this basis." (Liliane, Latinx).

Some foundational thing out of which all important things can be generated.

Whole universe "I'm just thinking like the whole universe, and that's like planets and stars and that like describes the universe. But only a planet describes earth. So to get a basis for the universe you'd need planets and stars." (Nadia, Latinx).

Captured idea of dimension - numbers and choices. Hadn't captured vectors

Group responses after we read these explanations...

- *My first instinct was to work hard to not be affected by jarred reaction that didn't conform to my understanding. Fight back against jarring aspects to get there.*

Struggle is real

- *Really impressed by creativity. I thought it was really cool. I had to try to work very hard to understand the conceptual things inside these narratives, helped me try to do work arounds and to place myself in their context.*
- *I liked about it, notice that often that students struggle from nice vector spaces to functional ones. These girls have made that jump, not just do this vector spaces in coordinates. These students have done this well.*
- Grandmother rule - can you explain this to my grandmother. It highlights that people can't hide behind jargon, don't presume other people understand what you mean when you say this thing. Do you really understand what you're talking about and can you communicate that to other people?
- They're thinking about this metaphorically. When you can use metaphors when doing mathematics can be really powerful.
- Question: Where do you go from here? How do you build on these metaphors in a practical way.
- First three capture dimension different quantities- specific something giving numbers and choices with different possibilities. Not quite to vectors. The last one has idea of some foundational aspect out of which all other aspects are built. Still struggled to see what is a metaphor with vectors? Maybe street grid to identify intersection - street with avenue and floor. Still focuses on orthogonality? What if you had angled streets?
- I wish I was teaching their group theory class. Stuff here with group relations, so many nice connections. Also thinking, What's next? These are very valuable stories in teaching mathematics. Showing people how to make the connections could be useful as a next step. Definitely see eigenspace here as well.
- I wish they were all in one class to share these metaphors together - look at the formal definition of vector space at the same time to point out these kinds of aspects and try to bring them together.
- When I look at this I think about how I could use analogies in the classroom but I do feel like it's important for students to come with the analogy and not me imposing. It won't be as honest if I can come up with them, especially because of my eagerness to be precise. I think that the way I would use it as an exercise is I would ask them to create it.
- Explaining to grandma's… think about change the language. It does imply something. Think about changing to explaining to grandmas. Think about it. This is not gendered. Not implied. Check out Grandma got stem Blog

Adiredja Response to these thoughts:

- Resisting our own values of precisions and rigor. Those are real values in our field, they are there for a reason. It is a very different thing when we think for students. How do we humanize access for students?
- "Grandma questions" is helpful for some students in the study.

What would you have come up with? Grid and cooking are two conventional ways to think about basis. When Adiredja interviewed students he didn't know what would get. The religion explanation was the 1st interview and he was floored.

<u>Who were the students?</u>
8 students - 1 african american, 1 asian/asian american, 5 Hispanic/Latinx, and 1 Hispanic/Latinx/white.
They had taken linear algebra before and then came to the interview.
They didn't have this explanation ready, they constructed them in the interview.
Adiredja then shared the others, not all of them were as well developed and some were more well developed.

What is the benefit of this beyond cognitive standpoint?

When we think about basis we think of span and independence but then there are all these aspects of basis - different roles and different nuances for characteristics of basis: Minimal, essential, representation, non-redundant, different/sameness….. These are all closely related words and yet different aspects of basis.

Adiredja then color coded with Jocelyn's coding:

Roles:

- ***Generating***: To create the larger space from the basis vectors
- ***Describing***: To describe the space using the basis vectors

Characteristics:

- ***Minimal*** focuses on the required number or amount of vectors needed for the basis. A basis has the least amount of vectors necessary.
- ***Essential*** focuses on the identities of the vectors that make them core and necessary for the larger space.
- ***Representation*** focuses on identifying and/or naming the smaller set of objects as providing structure or serve as representative for the larger space.
- ***Non-redundant*** focuses on not wanting extraneous information in a set, and/or the act of reducing or removing the extraneous information.
- ***Different/sameness*** focuses on comparing items (vectors) based on their difference/similarity for the sake of keeping or removing items from the basis.

CODES ILLUSTRATED

Um it's **minimal**. To pick **one pair of heels and one pair of tennis shoes**. So when I think of my idea of a basis, my mind goes to **minimal**. Um, what doesn't it capture? Well, ok, so it's weird cause I guess you can use one pair of shoes for different outfits. But, like, if I'm trying to make...it's harder to kind of have, like, a casual outfit and in a formal outfit there's **not a whole lot of, like, overlap you end up having each piece in each outfit in the basis.** So it's, like. How do I explain this? I feel like the basis I'm making, all of the pieces aren't as, like, **they're not all the same**. Like, you have shoes, tops and pants. **You can't make an outfit with just shoes.** But if you have a basis, you can pick just some of the vectors, **combine them, and make something,** and leave all the rest out. **Cause you can't just put on shoes and pants.** So that's where it kinda...that's one of the ways that doesn't really [fit the definition].

This example shows a base knowledge. They are very astute in analyzing in what works and what doesn't work. Very important.

Final discussion

- Results from anti-deficit analysis of these women's explanations contribute to local theories about the learning of basis
- These everyday explanations provide a window into how students' sense making and contextualizes basis
- **Creativity** and breadth of the everyday contexts used to describe basis by these female students of color - just phenomenal.
- Sophistication in their assessment of everyday contexts they generated and the degree that they fit with the formal definition of basis - they were quite sophisticated in their assessment.

What were we able to do?

- Repositioning these students' as resource for teaching instead of a group of students that need extra support
- Intentional selection of participants and connecting to broader narratives highlight the politics - *all of this is struggling for power.*

TAKEAWAYS

- Mathematical thinking/learning/cognition are situated within identity discourses/narratives
- Our perspective as math educators on students' knowledge and reasoning carry weight and power
- We decide what successful sense making look like, and who can be positioned as successful in mathematics.

• It is not about optimism (vs pessimism) (or half full/half empty) ,but accuracy (if we're more accurate we can be more humanizing).
• One slice in striving for equity, and you can help!

Final quote: Racism is not merely a simplistic hatred. It is, more often, broad sympathy towards some and broader skepticism towards others. - Ta-Nehisi Coates.
When we're thinking about knowledge, skepticism may be our enemy.

Question: Changing the nature of knowledge. I can understand the attribution of knowledge/successfully doing mathematics. I'm still unresolved about the sense of the nature of knowledge being changed.
A: It's not necessary for my work to change the nature of knowledge but to add the types of knowledge that students have that can enrich the mathematics.

Q: Did the students all articulate a mathematically correct definition of basis prior to interview?
A: They had an opportunity to do that.

Q: By getting them to use their own language, it gives me a resource to see … I can engage with somebody that you're hanging onto that is really important and this is the part we can build from.
A: Take Away from me as a researcher was doing this ...was very fruitful.

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