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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Karen Kozar Email/Phone: kkozar@ualberta.ca

Speaker's Name: SAM RASKIN

Talk Title: MODULI OF RESTRICTED SHTUKAS AND THE CATEGORY OF SHEAVES ON IT I

Date: 4 / 11 / 19 Time: 11 : 00 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: THE SPEAKER DISCUSSED LOCAL SHTUKAS THE LOCAL COUNTERPART TO ~~the~~ GLOBAL OBJECTS PREVIOUSLY CONSIDERED

CHECK LIST

(This is **NOT** optional, we will **not** pay for **incomplete** forms)

- ☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - <u>Computer Presentations</u>: Obtain a copy of their presentation
  - <u>Overhead</u>: Obtain a copy or use the originals and scan them
  - <u>Blackboard</u>: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - <u>Handouts</u>: Obtain copies of and scan all handouts
- ☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack <u>with this completed sheet on top</u> and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☑ Email the re-named files to <u>notes@msri.org</u> with the workshop name and your name in the subject line.

MODULI OF RESTRICTED SHTUKAS + SHEAVES ON IT I

— RASKIN

RECAP IN HARMONIC ANALYSIS:

HECKE ALG $\circlearrowright$ {AUT. FORMS}

IN GEOMETRY:

HAVE MORE SYMMETRY B/C GEOM. SATAKE WORKS OVER X^I, $|I|>1$

YOGA (BEILINSON, DRINFELD, V. LAFFORGUE)

EXTRA SYMMETRIES ACCOUNT FOR LANGLANDS DECOMP

GOAL ADAPT TO SETTING OF LOCAL LANGLANDS (FOLLOWING GENESTIER - LAFFORGUE)

- $k = \mathbb{F}_q$
- "FROB" := q - FROB

IDEA (WEIL, LANG) TO STUDY $Y(k)$ FOR Y/k SOME GEOM OBJECT, LOOK AT

$$\begin{tikzcd} Y^{FR=id} \arrow[r] \arrow[d] \arrow[dr, phantom, "\square"] & Y \arrow[d, "(id, FR)"] \\ Y \arrow[r, "\Delta"] & Y \times Y \end{tikzcd}$$

EX $Y = BH$, H ALG GP (CONNECTED) $/k$

$$(BH)^{FR=id} = B(H(k))$$

PF LHS IS ÉTALE ARTIN STACK, CONNECTED, W/ k PT

$\Longrightarrow$ LHS $= BG$ FOR SOME FINITE G

N.B. $(BH)^{FR=id} = H/{}^{\sigma}H$ $\Leftarrow$ FROB $\qquad h \cdot^{\sigma} g = h g \sigma(h)^{-1}$

$\square$

$\Longrightarrow$ $G = H(k)$

IDEA APPLY THIS TO $\mathcal{L}G$ = LOOP GP FOR G RED, SPLIT$/k$

$(\mathcal{L}G = LG)$ FROM BEFORE

$$\boxed{\begin{array}{l} \textbf{DEF} \;\bullet\; \mathrm{LocSht}^{MER} = \mathcal{L}G/{}^{\sigma}\mathcal{L}G \qquad \text{"MER" = "MEROMORPHIC"} \\ \bullet\; \mathrm{LocSht} = \mathrm{LocSht}^{REG} = \mathcal{L}G/{}^{\sigma}\mathcal{L}^{+}G \end{array}}$$

NOTATION:

- X SMOOTH CURVE $/k$ (MAYBE AFFINE) } FIXED FOREVER
- $x \in X(k)$ } (EG $X = \mathbb{A}^1$, $x = 0$)
- $\mathcal{L}G$ IS BASED AT x (IE t IN DEF OF $\mathcal{L}G$ IS t_x)

$S = \cancel{\mathrm{Spec}}(A) \xrightarrow{\ y\ } X$
Spec

$$\begin{array}{lcl} \hat{D}_y & = & \text{FORMAL COMPLETION OF } X \times S \text{ ALONG } \Gamma_y \\ \downarrow & = & \mathrm{SPF}(\cdots) \\ & & \ \ \Updownarrow \text{ SAME RING} \\ D_y & = & \mathrm{SPEC}(\cdots) \\ \uparrow & & \end{array}$$

$\mathring{D}_y = D_y \setminus \Gamma_y = D_y \setminus y$ FORMAL PUNCTURED DISC AT y

$\hat{D}_y$ AND D_y HAVE SAME G-BUNDLES

$$\hat{D}_{x,S} = \hat{D}_{x \circ (S \to \mathrm{Spec}(k))}$$

RMK
$$\mathrm{LocSht}^{\mathrm{mer}}(S) = \left\{ \begin{array}{l} \mathcal{P}_G \text{ on } \mathring{D}_{x,S} \text{ a } G \text{ bundle,} \\ + \ \Theta : \mathrm{Fr}_S^* \mathcal{P}_G \xrightarrow{\sim} \mathcal{P}_G \end{array} \right\}$$

$$\mathrm{LocSht}^{\mathrm{reg}}(S) = \left\{ \begin{array}{l} \mathcal{P}_G \text{ on } \hat{D}_{x,S} \text{ a } G\text{-bundle,} \\ + \ \Theta : \mathrm{Fr}_S^* \mathcal{P}_G|_{\mathring{D}_{x,S}} \xrightarrow{\sim} \mathcal{P}_G|_{\mathring{D}_{x,S}} \end{array} \right\}$$

RMK CAN CLASSIFY ISOM CLASSES OF $\bar{k}$-PTS OF $\mathrm{LocSht}^{\mathrm{mer}}$, ANALOGOUS TO DIEUDONNÉ-MANIN CLASS'N

EX $\begin{pmatrix} 0 & t \\ 1 & 0 \end{pmatrix} \in \mathcal{L}GL_2(\bar{k})$ IMAGE = ISOM CLASS $\neq$ id

<u>IDEA</u> STUDY $D(\mathrm{LocSht}^{\mathrm{MER}}) \leftsquigarrow \overline{\mathbb{Q}}_\ell$ SHEAVES

<u>WARNING</u> THIS CAT DOES MAKE SENSE (BUT SOMEWHAT TECHNICAL)
NEED TO WORK w/ IND-CONSTRUCTIBLE SHEAVES, AND
ONLY DERIVED CAT MAKES SENSE (NO PERVERSE SHEAVES)

<u>RMK</u> $D(\mathrm{LocSht}^{\mathrm{MER}}) \overset{??}{=} D(\mathrm{Bun}_G)$

STUDIED BY
FARGUES - SCHOLZE

$$\underbrace{B(G(k((t))))}_{\text{AS A TOP GP}} \xrightarrow{j} \mathrm{LocSht}^{\mathrm{MER}}$$

<u>N.B.</u> j IS FORMALLY ÉTALE

$$j^! : D(\mathrm{LocSht}^{\mathrm{MER}}) \longrightarrow D\big(B(G(k((t))))\big)$$
$$= \mathrm{Rep}(G(k((t))))$$

<u>CLAIM</u> $j^!$ ADMITS A FULLY FAITHFUL LEFT ADJOINT
(LLC SHOULD BE ABOUT $\mathrm{LocSht}^{\mathrm{MER}}$)

WHAT ABOUT X^I?

<u>DEF</u> $\mathrm{LocSht}^{\mathrm{MER}}_{X^I}(S) = \left\{ \begin{array}{l} y_i : S \to X, \ i \in I \\ P_G^1, P_G^2 \text{ ON } (D_{x,S} \cup \bigcup_{i \in I} D_{y_i}) \setminus \{x\} \\ \alpha : P_G^1|_{(D_x \cup \bigcup D_{y_i}) \setminus x \{y_i\}} \xrightarrow{\sim} P_G^2| \ldots \\ \Theta : \mathrm{FR}_S^* P_G^1|_{\mathring{D}_x} \xrightarrow{\sim} P_G^2|_{\mathring{D}_x} \end{array} \right\}$

RMK ALSO HAVE $\mathrm{LocSht}^{\mathrm{REG}}$ DEFINED SIMILARLY,

BUT P_G^1 IS A G-BUNDLE ON $D_x \cup \bigcup_{i\in I} D_{y_i}$

EX 1) IF $I=\emptyset$, RECOVER $\mathrm{LocSht}^{\mathrm{MER}}$

SIMILARLY, IF "$y_i = x \ \forall i$", SAME

IE $\mathrm{LocSht}^{\mathrm{MER}}_{X^I} \times_{X^I} \{(x,\dots,x)\} = \mathrm{LocSht}^{\mathrm{MER}}$

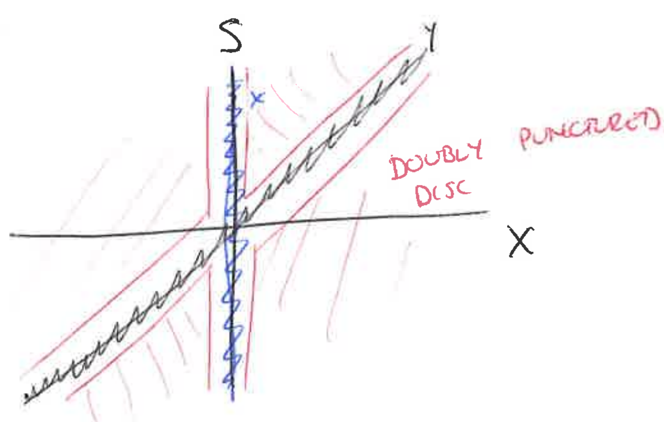
2) $\mathrm{LocSht}^{\mathrm{MER}}_{X^I} \times_{X^I} (X\setminus\{x\})^I = \mathrm{LocSht}^{\mathrm{MER}} \times L^+G_{(X\setminus x)^I} \backslash \mathrm{Gr}_{G,(X\setminus x)^I}$

WARNING THERE IS NO MAP

$$\mathrm{LocSht}^{\mathrm{MER}}_{X^I} \longrightarrow \mathrm{LocSht}^{\mathrm{MER}}$$

ISSUE: THERE IS NO MAP $\mathring{D}_x \longrightarrow (D_x \cup D_y)\setminus\{x,y\}$

PICTURE:



Q: GLOBAL / LOCAL SHTUKAS

A: X SMOOTH PROJ

$$\exists \quad \mathrm{Sht}_x(X) \longrightarrow \mathrm{LocSht}^{\mathrm{REG}} \; : \; \mathcal{F} \longmapsto \mathcal{F}|_{\mathring{D}_x}$$

EXER 1) THIS MAP IS FORMALLY ÉTALE

2) DEDUCE "LOCAL MODEL" STATEMENT FROM SOPHIE'S TALK