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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Karl Kozar Email/Phone: kkozar@ualberta.ca

Speaker's Name: Xinwen Zhu

Talk Title: Local Langlands parametrization and Local – Global compatibility II

Date: 4 / 12 / 19 Time: 11 : 00 (am) / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: The speaker continued with the proofs of parts 1) and 2) of the main theorem of his previous lecture.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

☒ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☒ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
- **Overhead**: Obtain a copy or use the originals and scan them
- **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- **Handouts**: Obtain copies of and scan all handouts

☒ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

☒ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.
(*YYYY.MM.DD.TIME.SpeakerLastName*)

☒ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

LOCAL LANGLANDS PARAMETRIZATION AND LOCAL / GLOBAL COMPATIBILITY II – ZHU

FROM LAST TALK:

HAVE A MAP OF STACKS

$$\psi_{\Delta(\bar\eta)}(\mathbb{1}, n) \longrightarrow \psi_{\Delta(\bar\eta)}(W, n)$$

ON ITERATED MODULI OF SHTUKAS

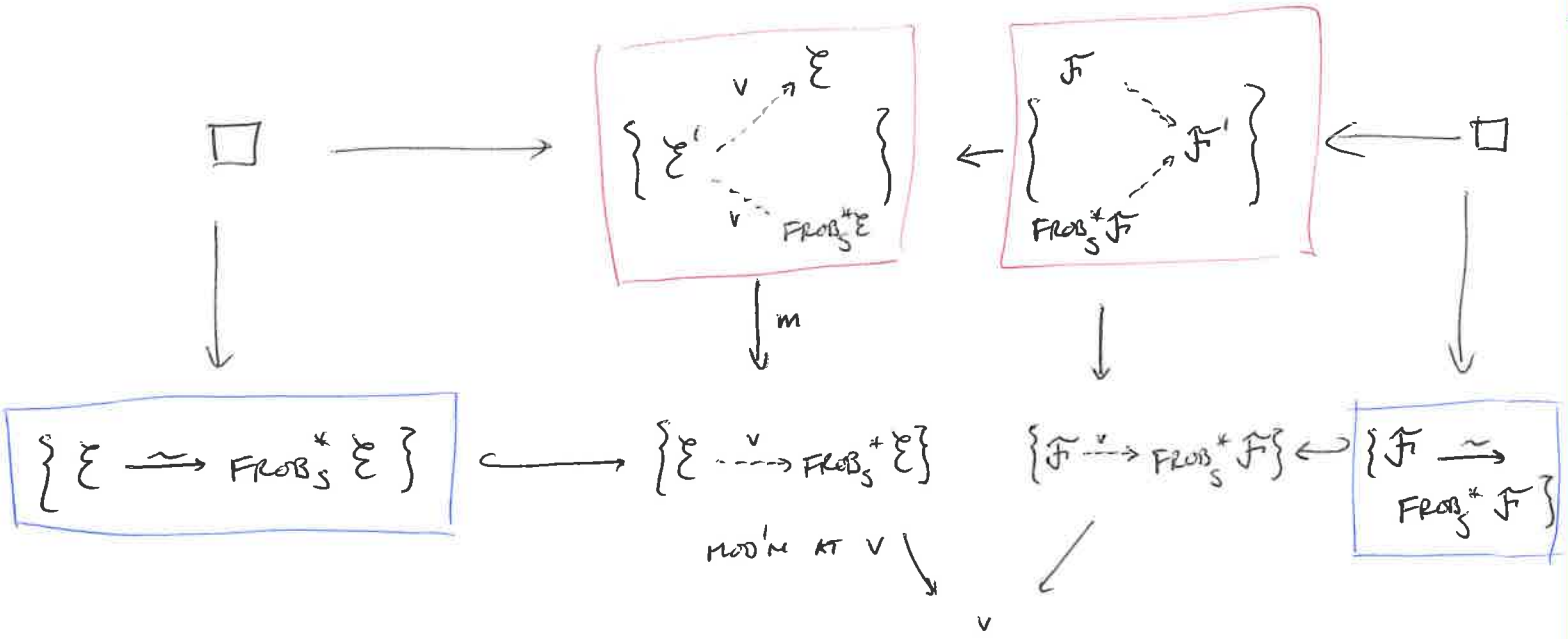
$$\mathrm{FROB}_1^{\times}\, \widetilde{\psi}_{\eta_v^{\bar a}}(W) \xrightarrow{\ \delta\ } \widetilde{\psi}_{\eta_v^{\bar a}}(W)$$

COHOMOLOGY OF THIS MAP IS COMPATIBLE w/ MAP δ ON $\mathcal{H}_I(W)_{\eta_v^{\bar a}}$

THIS FITS INTO

$$\begin{array}{ccccc}
\psi_{\Delta(\bar\eta)}(\mathbb{1}, n) & \xrightarrow{\times} & \psi_{\Delta(\bar\eta)}(W, n) & \dashrightarrow & \mathrm{FROB}_1^{\times}\, \widetilde{\psi}_{\eta_v^{\bar a}}(W, n) \\
 & & \simeq m_1 \mathrm{FROB}_1^{*}\, \widetilde{\psi}_{\Delta(\bar\eta)}(W) & & \downarrow \delta \\
\psi_{\Delta(\bar\eta)}(\mathbb{1}, n) & \xleftarrow{\ \delta\ } & \psi_{\Delta(\bar\eta)}(W, n) & \dashleftarrow & \widetilde{\psi}_{\eta_v^{\bar a}}(W, n) \\
 & & \simeq m_1(--) & &
\end{array}$$

TAKING COHOMOLOGY GIVES DIAG FROM FIRST LECTURE



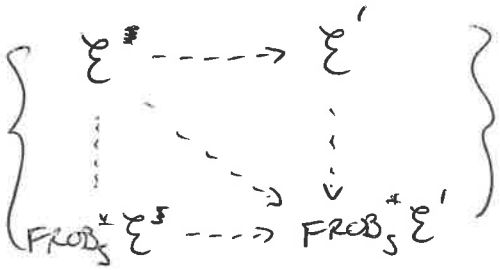
HAVE

$$\psi_{\Delta(\bar{\eta})}(W, n) \cong m! \, \mathrm{FROB}_1^* \, \widetilde{\psi}_{\Delta(\bar{\eta})}(W)$$

SINCE NEARBY CYCLES COMMUTE W/ PROPER PUSHFORWARD

CAN ADD TO PREVIOUS DIAGRAM (P. ①)

CONSIDER THE SPACE



WHICH MAPS TO B THE 4 VERTICAL SETS ABOVE, ISOM'CALLY TO $\square$ $\square$

FURTHER,

$$\left\{\begin{array}{ccc} \mathcal{E} & \dashrightarrow & \mathcal{E}' \\ \downarrow s & & \downarrow s \\ \mathrm{FROB}_S^* \mathcal{E} & \dashrightarrow & \mathrm{FROB}_S^* \mathcal{E}' \end{array}\right\}$$

MAPS TO PREVIOUS SET, AND ALSO TO $\square$ $\square$ AND FACTORS THROUGH FIBER PRODUCTS

WHEN $n = 0$, CAN ALWAYS EXTEND DIAG ON P ①

"CLAIM" $\widetilde{\psi}_{\Delta(\bar{\eta})}(W, n) \xrightarrow{\sim} \widetilde{\psi}_{\bar{\eta}_v^{\alpha}}(W, n)$

$$\mathrm{FROB}_1^* \psi_{\mathrm{FROB}(\bar{\eta}_v^{\alpha}) \to \Delta(v)} \left(\mathcal{F}_{V_1 \tilde{\times} V_2}\right) = \psi_{\bar{\eta}_v^{\alpha} \to \Delta(v)} \left(\mathrm{FROB}^* \mathcal{F}_{V_1 \tilde{\times} V_2}\right)$$

ACTION OF δ

$$\mathrm{FROB}_1^* \psi_{\bar{\eta}_v^{\alpha} \to \Delta(\bar{v})} \left(\mathcal{F}_{V_2 \tilde{\times} V_2}\right) \xrightarrow{\delta} \psi_{\bar{\eta}_v^{\alpha} \to \Delta(v)} \left(\mathcal{F}_{V_2 \tilde{\times} V_1}\right)$$

$$\delta \longmapsto (1, 0)$$

$$\psi_{\bar{\eta}_v^{\alpha} \to \Delta(\bar{v})} \; \mathrm{FROB}_1^* \; \mathcal{F}_{V_1 \tilde{\times} V_2} \cong \psi_{\bar{\eta}_v^{\alpha} \to \Delta(\bar{v})} \; \mathcal{F}_{V_2 \tilde{\times} V_1}$$

$$\psi_{\Delta(\bar{\eta}) \to \Delta(v)} \; \mathrm{FROB}_1^* \; \mathcal{F}_{V_1 \tilde{\times} V_2} \cong \psi_{\Delta(\bar{\eta}) \to \Delta(\bar{v})} \; \mathcal{F}_{V_2 \tilde{\times} V_1}$$

SO IT SUFFICES TO PROVE

$$\psi_{\Delta_d(\bar{\eta}) \rightsquigarrow \Delta(\bar{v})}(-) \xrightarrow{\sim} \psi_{\bar{\eta}^{\alpha} \rightsquigarrow \Delta(\bar{v})}(-) \qquad \text{FOR } (d, 0)$$

$d \gg 0$

TO PROVE THIS USE STRAT'M THM FROM YESTERDAY $\square$

(HAVE TO CONTROL BLOW UP)

THIS IDENTIFIES EXCURSION OPERATOR AS A

COH. CORR $\leftrightsquigarrow$ HECKE OPERATOR

ALL THE ABOVE CONSTRUCTIONS:

$S_{n,I,x,\xi,\delta}$ IS GIVEN BY SOME HECKE OPERATOR

AT v ACTING ON $C_c(\Xi G(F)\backslash G(\mathbb{A})/K^v K_n)$

THIS EXCURSION OPERATOR COMMUTES W/ ALL

HECKE OPERATORS

MORE DETAILS: WRITE OUT MODULI SPACES AS COSETS

$$G(F)\backslash G(\mathbb{A})/K^v \overset{G(\mathcal{O}_v)}{\times} G(F_v)/G(\mathcal{O}_v)$$

$$G(F)\backslash G(\mathbb{A})/K^v G(\mathcal{O}_v) \qquad G(F)\backslash G(\mathbb{A})/K^v G(\mathcal{O}_v)$$

CORR HERE SHOULD BE A HECKE OPERATOR