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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Karl Koziol Email/Phone: kkoziol@ualberta.ca

Speaker's Name: Jared Weinstein

Talk Title: Moduli of Shtukas II

Date: 4 / 8 / 19 Time: 2 : 00 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: THE SPEAKER CONTINUED TO DISCUSS SHTUKAS AND THEIR MODULI SPACES. HE ALSO DISCUSSED HOW COHOMOLOGY OF THESE SPACES REALIZE THE LANGLANDS CORRESPONDENCE, AND EXPLAINED DRINFELD'S LEMMA

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

☒ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☒ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
- **Overhead**: Obtain a copy or use the originals and scan them
- **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- **Handouts**: Obtain copies of and scan all handouts

☒ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

☒ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)

☒ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

MODULI OF SHTUKAS II — WEINSTEIN

QUESTION F GLOBAL FIELD, G/F SPLIT RED GP

π = CUSP AUT REP OF $G(\mathbb{A})$.

HOW DO WE CONSTRUCT $\sigma_\pi : \mathrm{Gal}(\overline{F}/F) \longrightarrow \widehat{G}(\overline{\mathbb{Q}}_\ell)$?

EX

FOR $F = \mathbb{Q}$, $G = GL_2$, SUPPOSE

$\pi = \pi_f \otimes \pi_\infty$ w/ π_∞ DISCRETE SERIES ON $GL_2(\mathbb{R})$

$$Y(N) = \{ (E, \alpha) : E \text{ ELL CURVE}, \ \alpha : (\mathbb{Z}/N)^2 \xrightarrow{\sim} E[N] \}$$

$$\downarrow$$

$$\mathrm{Spec}(\mathbb{Z})$$

$H^1_{\text{ét}}(Y(N)_{\overline{\mathbb{Q}}}, \overline{\mathbb{Q}}_\ell)_{\text{cusp}}$ GIVES WT 2 FORMS.

IN GENERAL

$$\lim_{\overrightarrow{N}} H^1_{\text{ét}}(Y(N)_{\overline{\mathbb{Q}}}, \mathcal{F}_{\pi_\infty})_{\text{cusp}} \cong \bigoplus_{\substack{\pi \text{ cusp} \\ (\pi)_\infty = \pi_\infty}} \sigma_\pi \otimes \pi_f$$

$$\circlearrowleft$$

$$\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \times GL_2(\mathbb{A}_f)$$

SO WE FIND σ_π IN COHOMOLOGY

//

$Y(N)$ IS LIKE "SHTUKAS OVER $\mathrm{Spec}(\mathbb{Z})$" WITH ONE SIMPLE POLE AT ∞ AND ONE SIMPLE ZERO

$\downarrow$

$\mathrm{Spec}(\mathbb{Z})$

$F = k(X)$ AS BEFORE

$G = GL_2$

$I = \{1, 2\} \longleftrightarrow$ SHTUKAS W/ 2 LEGS

$$W = \{ \mathrm{STD}, \mathrm{STD}^{\vee} \}$$

$$\begin{matrix} \updownarrow & \updownarrow \\ (1,0) & (0,-1) \end{matrix}$$

WTS

$\mathcal{E}_1 \dashrightarrow \mathcal{E}_2 \,/X\times S$ BOUNDED AT v BY $(1,0)$ MEANS WE HAVE

$$0 \longrightarrow \mathcal{E}_1 \longrightarrow \mathcal{E}_2 \longrightarrow i_{v*} V \longrightarrow 0$$ IN A NHOOD OF v

WHERE $i_v : \{v\} \longrightarrow X$

V = RK 1 $\mathcal{O}_S$-MOD

(FOR $(0,-1)$, REVERSE ORDER OF MAPS IE $\mathcal{E}_2 \longrightarrow \mathcal{E}_1$)

IN THIS CASE,

$Sht_{I,W}(S) = \{ (\mathcal{E}, z, p, \phi) \}$, WHERE

- $\mathcal{E} \in \mathrm{BUN}_{GL_2}(S)$
- $z, p \in X(S)$
- $\phi : \mathcal{E} \dashrightarrow (1 \times \mathrm{FR}_S)^* \mathcal{E}$ BDD BY $(1,0)$, $(0,-1)$ AT z, p

DEFINE $Sht_{I,W,N}(S)$ FOR A DIVISOR $N \subset X$, DISJOINT FROM Z, P

EFFECTIVE

BY $\{ (\mathcal{E}, Z, P, \phi, \alpha) \}$

WHERE $(\mathcal{E}, Z, P, \phi)$ AS BEFORE, AND

$$\alpha : \mathcal{O}_N^2 \xrightarrow{\sim} \mathcal{E}_N$$

ST THE FOLLOWING COMMUTES

$$\begin{tikzcd} \mathcal{O}_N^2 \arrow[r, "\sim", "\alpha"'] \arrow[d, "\wr"'] \arrow[dr, phantom, "\circlearrowright"] & \mathcal{E}_N \arrow[d, "\phi_N"] \\ (1 \times \mathrm{FROB}_S)^* \mathcal{O}_N^2 \arrow[r] & (1 \times \mathrm{FROB}_S)^* \mathcal{E}_N \end{tikzcd}$$

SO, WE HAVE

$$\pi_N : Sht_{I,W,N} \xrightarrow{\text{DIM } 2} (X \setminus N)^2$$

WANT TO TAKE COHOMOLOGY

$X \times_k X$ HAS GENERIC PT η_2

CONSIDER $R^2 \pi_{N*} \overline{\mathbb{Q}}_\ell$, A CONSTRUCTIBLE SHEAF ON $(X \setminus N)^2$

$$(R^2 \pi_{N*} \overline{\mathbb{Q}}_\ell)_{\overline{\eta}_2} \circlearrowleft \mathrm{Gal}(\overline{\eta}_2 / \eta_2)$$

GEOM GENERIC FIBER

AND FORM

$$(*) \quad \varinjlim_N (R^2 \pi_{N*} \overline{\mathbb{Q}}_\ell)_{\overline{\eta}_2}$$

$$\circlearrowright$$

$$GL_2(\mathbb{A}) \times \mathrm{Gal}(\overline{\eta}_2 / \eta_2)$$

THM (DRINFELD)

$$\circledast_{CUSP} = \bigoplus_{\pi} \pi \otimes \sigma_\pi \otimes \sigma_\pi^\vee$$

WHAT DOES RHS MEAN? THERE IS A MAP

$$\mathrm{Gal}(\bar{\eta}_2/\eta_2) \longrightarrow \mathrm{Gal}(\bar{F}/F) \times \mathrm{Gal}(\bar{F}/F)$$

(NEITHER INJ NOR SURJ)

G GENERAL, I FINITE, $|I|=n$, $W \in \mathrm{Rep}_{\hat{G}^I}$ ($\Rightarrow W = \boxtimes W_i$, $\rho_i : \hat{G} \to \mathrm{Aut}(W_i)$)

EXPECTATION:

$$\circledast_{CUSP} = \bigoplus_{\pi} \pi \otimes (\rho_1 \circ \sigma_\pi) \otimes \dots \otimes (\rho_n \circ \sigma_\pi)$$

DRINFELD SHTUKAS

TAKE $G = GL_2$, $W = \{(1,0), (0,-1)\}$

DEFINE

$$\mathrm{Sht}^{LEFT}_{I,W}(S) = \{ \mathcal{E}_0 \longrightarrow \mathcal{E}_1 \longrightarrow (1 \times \mathrm{FR}_S)^* \mathcal{E}_0 \}$$

TYPE (1,0) AT Z

TYPE (0,-1) AT P

$$\mathrm{Sht}^{RIGHT}_{I,W}(S) = \{ \mathcal{E}_0 \longrightarrow \mathcal{E}_1 \longrightarrow (1 \times \mathrm{FR}_S)^* \mathcal{E}_0 \}$$

TYPE (0,-1) AT P

TYPE (1,0) AT Z

THERE ARE MAPS

$$\begin{tikzcd} \mathrm{Sht}^{\mathrm{LEFT}}_{I,w} \arrow[dr] & & \mathrm{Sht}^{\mathrm{RIGHT}}_{I,w} \arrow[dl] \\ & \mathrm{Sht}_{I,w} \arrow[r] & X \times X \end{tikzcd} \quad (**)$$

SUBTLETY: COULD HAVE $Z = P$.

$\mathrm{Sht}_{I,w}\big|_{(X \times X) \setminus \Delta}$ IS DEF'D UNAMBIGUOUSLY

Sht_I —— " ——

DEFINE

$$\mathrm{Sht}_{I,w} := \overline{\mathrm{Sht}_{I,w}\big|_{(X \times X)\setminus\Delta}} \quad \text{CLOSURE IN } \mathrm{Sht}_I$$

GIVEN $\mathcal{E}_0 \longrightarrow (1 \times \mathrm{FR}_S)^* \mathcal{E}_0$ W/ Z, P DISJOINT,

$$\mathcal{E}_1 = \begin{cases} (1 \times \mathrm{FR}_S)^* \mathcal{E}_0 & \text{AWAY FROM } Z \\ \mathcal{E}_0 & \text{AWAY FROM } P \end{cases}$$

$(**)$ ARE ISOMS AWAY FROM $\Delta \subseteq X \times X$

<u>DRINFELD</u>: $(**)$ INDUCES ISOMS ON COHOMOLOGY

HOW TO UNDERSTAND MAP TO $\mathrm{GAL}(\overline{F}/F)^2$?

PARTIAL FROB

$$\begin{array}{ccc} Sht^{LEFT}_{I,W} & \xrightarrow{FR_{\{1\}}} & Sht^{RIGHT}_{I,W} \\ \downarrow & & \downarrow \\ X\times X & \xrightarrow{FR_X \times 1} & X \times X \end{array}$$

$FR_{\{1\}}$ CARRIES

$$\mathcal{E}_0 \xrightarrow[\substack{(1,0)\text{ AT}\\ z}]{} \mathcal{E}_1 \xrightarrow[\substack{(0,-1)\\ \text{AT } P}]{} (1\times FR_S)^*\mathcal{E}_0$$

TO

$$\mathcal{E}_1 \xrightarrow[\substack{(0,-1)\\ \text{AT } P}]{} (1\times FR_S)^*\mathcal{E}_0 \xrightarrow[\substack{(1,0)\text{ AT}\\ FR(z)}]{} (1\times FR_S)^*\mathcal{E}_1$$

OUR $\mathcal{H} = R^2\pi_{N,*}\overline{\mathbb{Q}}_\ell$ IS A CONSTR'BLE $\overline{\mathbb{Q}}_\ell$ SHEAF ON

$(X-N)\times_k (X\setminus N)$ TOGETHER WITH

$$FR_{\{1\}} : (FR_X\times 1)^*\mathcal{H} \xrightarrow{\sim} \mathcal{H}$$

$$FR_{\{2\}} : (1\times FR_X)^*\mathcal{H} \xrightarrow{\sim} \mathcal{H}$$

WHICH "COMMUTE" AND "$FR_{\{1\}}\circ FR_{\{2\}} = FR_{X\times X}$"

> DRINFELD'S LEMMA THE ACTION
>
> $\mathcal{H}_{\overline{\eta}_2} \circlearrowleft \mathrm{GAL}(\overline{\eta}_2/\eta_2)$ FACTORS THROUGH $\mathrm{GAL}(\overline{F}/F)^2$

LAFFORGUE DEFINED ITERATED SHTUKAS:

$$\mathrm{Sht}^{I_1,\dots,I_k}_{I,W} \longrightarrow \mathrm{Sht}_{I,W} \qquad \left(I = \coprod_{i=1}^{k} I_i \right)$$

CLASSIFIES

$$\mathcal{E}_0 \longrightarrow \mathcal{E}_1 \longrightarrow \cdots \longrightarrow \mathcal{E}_{k-1} \longrightarrow (1 \times \mathrm{Fr}_S)^* \mathcal{E}_0$$

OF V.B.'S = # OF I_i, NOT # OF LEGS

ISOM AWAY FROM Γ_{x_i} FOR ALL $i \in I_1$, BDD BY W_i

SAME FOR I_2

SIMILARLY, GET PARTIAL FROBS