

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: KARU KOZIOŁ Email/Phone: kkozioł@ualberta.ca

Speaker's Name: JAROD WEINSTEIN

Talk Title: MODULI OF SHUKAS II

Date: 4 / 8 / 19 Time: 2:00 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: THE SPEAKER CONTINUED TO DISCUSS SHUKAS AND THEIR MODULI SPACES. HE ALSO DISCUSSED HOW COHOMOLOGY OF THESE SPACES REALIZE THE LANGUAGES CORRESPONDENCE AND EXPLAINED DRINFELD'S LEMMA

CHECK LIST

(This is **NOT** optional, we will **not** pay for incomplete forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
 - **Overhead:** Obtain a copy or use the originals and scan them
 - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
 - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

MODULI OF SHUKAS II - WEINSTEIN

(1)

QUESTION F GLOBAL FIELD, G/F SPLIT RED GP

$\pi =$ CUSP ART REP OF $G(A)$.

How do we construct $\sigma_\pi : \text{Gal}(\bar{F}/F) \rightarrow \tilde{G}(\bar{\mathbb{Q}}_l)$?

EX
FOR $F = \mathbb{Q}$, $G = \text{GL}_2$, SUPPOSE

$\pi = \pi_f \otimes \pi_\infty$ w/ π_∞ DISCRETE SERIES ON $\text{GL}_2(\mathbb{R})$

$$Y(N) = \left\{ (E, \alpha) : E \text{ ELL CURVE, } \alpha : (\mathbb{Z}/N)^2 \xrightarrow{\sim} E[N] \right\}$$

\downarrow
 $\text{SPEC}(\mathbb{Z})$

$$H_{\text{ET}}^1(Y(N)_{\bar{\mathbb{Q}}}, \bar{\mathbb{Q}}_l)_{\text{CUSP}} \quad \text{GIVES WT 2 FORMS.}$$

IN GENERAL

$$\lim_{\substack{\leftarrow \\ N}} H_{\text{ET}}^1(Y(N)_{\bar{\mathbb{Q}}}, \bigoplus_{\pi_\infty} \pi_f)_{\text{CUSP}} \cong \bigoplus_{\substack{\pi \text{ CUSP} \\ (\pi)_\infty = \pi_\infty}} \sigma_\pi \otimes \pi_f$$

\updownarrow

$$\text{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \times \text{GL}_2(A_f)$$

SO WE FIND σ_π IN COHOMOLOGY

$Y(N)$ IS LIKE "SHUKAS OVER $\text{SPEC}(\mathbb{Z})$ " WITH
 \downarrow
 $\text{SPEC}(\mathbb{Z})$ ONE SIMPLE POLE AT ∞ AND ONE SIMPLE ZERO

$$F = k(X) \quad \text{AS BEFORE}$$

$$G = GL_2$$

$$I = \{1, 2\} \quad \longleftrightarrow \quad \text{SHTUKAS w/ 2 LEGS}$$

$$W = \{ \text{STD}, \text{STD}^v \}$$

$$\begin{array}{cc} \updownarrow & \updownarrow \\ (1, 0) & (0, -1) \end{array}$$

WTS

$$\Sigma_1 \dashrightarrow \Sigma_2 / X \times S \quad \text{BOUNDED AT } v \text{ BY } (1, 0) \quad \text{MEANS WE HAVE}$$

$$0 \rightarrow \Sigma_1 \rightarrow \Sigma_2 \rightarrow i_{v*} V \rightarrow 0 \quad \text{IN A NEIGHBORHOOD OF } v$$

WHERE $i_v: \{v\} \rightarrow X$

$$V = \text{RK } 1 \text{ } \mathcal{O}_S\text{-MOD}$$

(FOR $(0, -1)$, REVERSE ORDER OF MAPS IE $\Sigma_2 \rightarrow \Sigma_1$)

IN THIS CASE,

$$\text{Sht}_{I, W}(S) = \{ (\Sigma, Z, P, \phi) \}, \quad \text{WHERE}$$

- $\Sigma \in \text{BUN}_{GL_2}(S)$

- $Z, P \in X(S)$

- $\phi: \Sigma \dashrightarrow (1 \times \text{FR}_S)^* \Sigma$ BDD BY $(1, 0), (0, -1)$ AT Z, P

DEFINE $\text{Sh}_t_{I,W,N}(S)$

FOR $A \wedge$ DIVISOR
EFFECTIVE

$N \subset X$, \square $\textcircled{3}$
DISJOINT FROM Z, P

BY $\{(\xi, Z, P, \phi, \alpha)\}$

WHERE (ξ, Z, P, ϕ) AS BEFORE, AND

$$\alpha: \mathcal{O}_N^2 \xrightarrow{\sim} \xi_N$$

ST THE FOLLOWING COMMUTES

$$\begin{array}{ccc} \mathcal{O}_N^2 & \xrightarrow{\sim} & \xi_N \\ \downarrow & \circlearrowleft & \downarrow \phi_N \\ (1 \times \text{FROB}_S)^* \mathcal{O}_N^2 & \longrightarrow & (1 \times \text{FROB}_S)^* \xi_N \end{array}$$

SO, WE HAVE

$$\pi_N: \text{Sh}_t_{I,W,N} \xrightarrow{\dim 2} (X \setminus N)^2$$

WANT TO TAKE COHOMOLOGY

$X \times_k X$ HAS GENERIC PT y_2

CONSIDER $R^2 \pi_{N,*} \overline{\mathcal{O}}_2$, A CONSTRUCTIBLE SHEAF ON $(X \setminus N)^2$

$$\left(R^2 \pi_{N,*} \overline{\mathcal{O}}_2 \right)_{\overline{y}_2} \hookrightarrow \text{GL}_2(\overline{y}_2 / y_2)$$

GEOM CONIC
FIBER

AND FORM

$$\textcircled{*} \quad \varinjlim_N \left(R^2 \pi_{N,*} \overline{\mathcal{O}}_2 \right)_{\overline{y}_2} \hookrightarrow \text{GL}_2(\mathbb{A}) \times \text{GL}_2(\overline{y}_2 / y_2)$$

THM (DRINFELD)

$$\otimes_{\text{CUSP}} = \bigoplus_{\pi} \pi \otimes \sigma_{\pi} \otimes \sigma_{\pi}^{\vee}$$

WHAT DOES RHS MEAN? THERE IS A MAP

$$\text{GAL}(\bar{y}_2/y_2) \longrightarrow \text{GAL}(\bar{F}/F) \times \text{GAL}(\bar{F}/F)$$

(NEITHER INJ NOR SURJ)

// G GENERAL, I FINITE, $W \in \text{REP}_{G^I}$ ($\Rightarrow W = \boxtimes W_i$)
 $|I|=n$ $\rho_i: \hat{G} \rightarrow \text{Aut}(W_i)$

EXPECTATION:

$$\otimes_{\text{CUSP}} = \bigoplus_{\pi} \pi \otimes (\rho_1 \circ \sigma_{\pi}) \otimes \dots \otimes (\rho_n \circ \sigma_{\pi})$$

DRINFELD SHUKAS

TAKE $G = \text{GL}_2$, $W = \{(1,0), (0,-1)\}$

DEFINE

$$\text{Sh}_{I,W}^{\text{LEFT}}(S) = \left\{ \begin{array}{c} \Sigma_0 \xrightarrow{\quad} \Sigma_1 \xrightarrow{\quad} (1 \times \text{FR}_S)^* \Sigma_0 \\ \uparrow \qquad \qquad \uparrow \\ \text{TYPE (1,0)} \quad \text{TYPE (0,-1)} \\ \text{AT } Z \qquad \qquad \text{AT } P \end{array} \right\}$$

$$\text{Sh}_{I,W}^{\text{RIGHT}}(S) = \left\{ \begin{array}{c} \Sigma_0 \xrightarrow{\quad} \Sigma_1 \xrightarrow{\quad} (1 \times \text{FR}_S)^* \Sigma_0 \\ \uparrow \qquad \qquad \uparrow \\ \text{TYPE (0,-1)} \quad \text{TYPE (1,0)} \\ \text{AT } P \qquad \qquad \text{AT } Z \end{array} \right\}$$

THERE ARE MAPS

$$\begin{matrix} \text{ShT}_{I,W}^{\text{LEFT}} & & \text{ShT}_{I,W}^{\text{RIGHT}} \\ \searrow & & \swarrow \\ \text{ShT}_{I,W} & \longrightarrow & X \times X \end{matrix}$$

(**)

SUBTLETY: COULD HAVE $Z = P$.

$\text{ShT}_{I,W} |_{(X \times X) \setminus \Delta}$ IS DEF'D UNAMBIGUOUSLY

$$\text{ShT}_I \quad \text{-----} \quad \text{-----}$$

DEFINE

$$\text{ShT}_{I,W} := \overline{\text{ShT}_{I,W} |_{(X \times X) \setminus \Delta}} \quad \text{CLOSURE IN } \text{ShT}_I$$

GIVEN $\Sigma_0 \longrightarrow (1 \times \text{FR}_S)^* \Sigma_0$ w/ Z, P DISJOINT,

$$\Sigma_1 = \begin{cases} (1 \times \text{FR}_S)^* \Sigma_0 & \text{AWAY FROM } Z \\ \Sigma_0 & \text{AWAY FROM } P \end{cases}$$

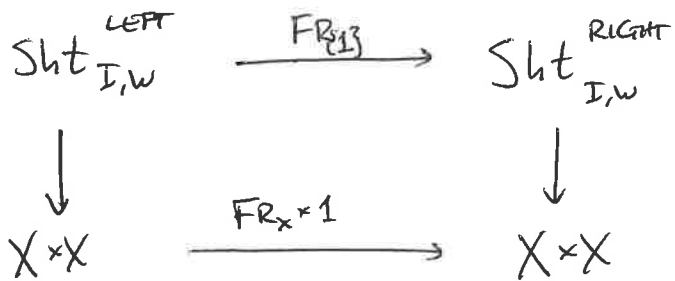
(**) ARE ISOMS AWAY FROM $\Delta \in X \times X$

DRINFELD: (**) INDUCES ISOMS ON COHOMOLOGY

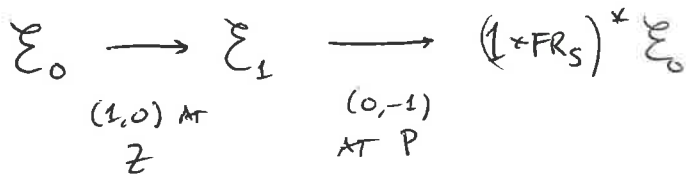
HOW TO UNDERSTAND MAP TO $\text{GAL}(\bar{F}/F)^2$?

(6)

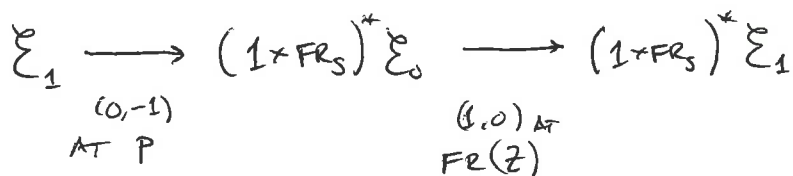
PARTIAL FROB



$\text{FR}_{\{1\}}$ CARRIES



TO



OUR $\mathcal{H} = \mathbb{R}^2 \times_{\mathbb{N},+} \overline{\mathbb{Q}}_2$ IS A CONSTR'BLE $\overline{\mathbb{Q}}_2$ SITEAF ON

$(X-N) \times_k (X \setminus N)$ TOGETHER WITH

$$\text{FR}_{\{1\}} : (\text{FR}_X \times 1)^* \mathcal{H} \xrightarrow{\sim} \mathcal{H}$$

$$\text{FR}_{\{2\}} : (1 \times \text{FR}_X)^* \mathcal{H} \xrightarrow{\sim} \mathcal{H}$$

WHICH "COMMUTE" AND " $\text{FR}_{\{1\}} \circ \text{FR}_{\{2\}} = \text{FR}_{X \times X}$ "

DRINFELD'S LEMMA THE ACTION

$\mathcal{H}_{\bar{\eta}_2} \supset \text{GAL}(\bar{\eta}_2/\eta_2)$ FACTORS THROUGH $\text{GAL}(\bar{F}/F)^2$

LAFFORGUE

DEFINED

ITERATED

SITUATIONS:

⑦

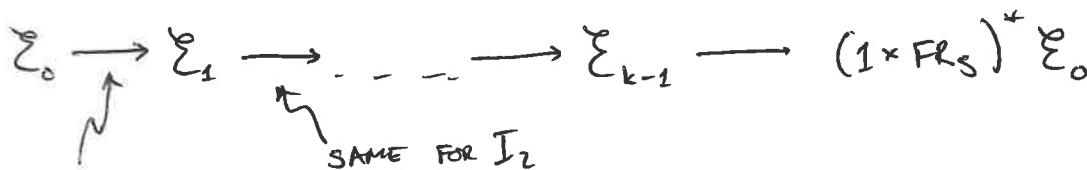
$$\text{Sht}_{I, W}^{I_1, \dots, I_k}$$



$$\text{Sht}_{I, W}$$

$$\left(I = \bigsqcup_{i=1}^k I_i \right)$$

CLASSIFIES



← # OF V.B.S
= # OF I_i , NOT # OF LEGS

SIMILARLY, GET PARTIAL FROBS