MSRI
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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: KARA KOZIOL Email/Phone: kkoziol@ualberta.ca

Speaker's Name: ZHIWEI YUN

Talk Title: SPECIALIZATION TO THE DIAGONAL I

Date: 4 / 9 / 19 Time: 2 : 00 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: THE SPEAKER DISCUSSED A TECHNICAL RESULT REGARDING CERTAIN IND-CONSTRUCTIBLE SHEAVES AND CERTAIN SPECIALIZATIONS

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- [x] Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- [x] Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - Computer Presentations: Obtain a copy of their presentation
    - Overhead: Obtain a copy or use the originals and scan them
    - Blackboard: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
    - Handouts: Obtain copies of and scan all handouts
- [x] For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- [x] When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- [x] Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

SPECIALIZATION TO THE DIAGONAL I - YUN

I = FINITE SET

$\boxed{\text{IGNORE LEVEL STRUCTURE}}$

$W \in \mathrm{REP}(\widehat{G}^I)$

$\rightsquigarrow \mathcal{H}_I(W)$ IND-CONSTRUCTIBLE ℓ-ADIC SHEAF ON X^I

$H_I(W) = \mathcal{H}_I(W)|_{\overline{\eta^I}} \leftarrow$ GEOM GENERIC PT

Q HOW CLOSE IS $\mathcal{H}_I(W)$ TO BEING A LOCAL SYSTEM?

EXPECTATION

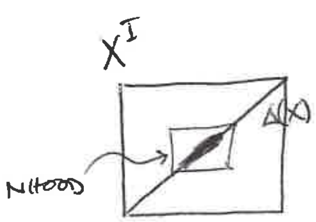
$$\underset{\boxtimes_{i \in I} W_i}{\mathcal{H}_I(W)} = \bigoplus_{\pi} m(\pi)\, \sigma_\pi^{W_1} \boxtimes \dots \boxtimes \sigma_\pi^{W_n} \qquad (I = \{1, \dots, n\})$$

$$\pi \leftrightsquigarrow \sigma_\pi : \pi_1(X) \longrightarrow \widehat{G}$$

SUGGESTS $\mathcal{H}_I(W)$ SHOULD BE A LOCAL SYSTEM

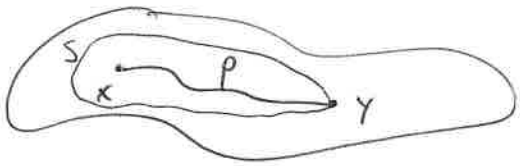
WILL SHOW $\mathcal{H}_I(W)$ IS A LOCAL SYSTEM IN A NHOOD OF $\Delta\eta$, η GENERIC PT OF X

$\Delta : X \hookrightarrow X^I$



<u>RECALL</u> SPECIALIZATION (ALG GEOM ANALOG OF A PATH)

TOP SP.



M

- IF $\mathcal{F}$ LOCAL SYSTEM ON M

$$p : \mathcal{F}_y \xrightarrow{\sim} \mathcal{F}_x$$

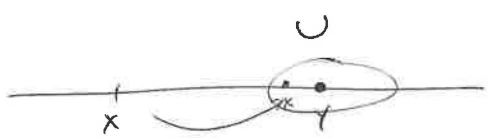
- IF $\mathcal{F}$ IS A SHEAF ON M, CONSTRUCTIBLE WRT A STRAT'N

$x \in S$, $y \in \{y\}$ (STRATA)

GET MAP

$$p : \mathcal{F}_y \longrightarrow \mathcal{F}_x$$

<u>EX</u>



$\mathcal{F}$ LOC CONST ON $\{z < y\}$

$$\mathcal{F}_y = \varinjlim_{U} \mathcal{F}(U) \xrightarrow[\text{RES}]{} \mathcal{F}_x$$

CAN ASSUME $x \in U$

$$sp^* : \mathcal{F}_y \longrightarrow \mathcal{F}_x \quad \text{(COSPECIAL'N MAP)}$$

$$x \rightsquigarrow y$$

A.G : Y SCHEME, $\bar{y}, \bar{x}$ GEOM PTS OF Y OVER y, x (PTS IN ZAR TOP)

SUPPOSE $x \rightsquigarrow y$ IE $y \in \overline{\{x\}}$

A SPECIALIZATION $\bar{x} \rightsquigarrow \bar{y}$ IS A MAP

$$\operatorname{Spec}(k(\bar{x})) \longrightarrow Y_{(\bar{y})} = (\mathcal{O}_{Y,y})^{SH}$$ ← STRICT HENSEL'N

$$= \varinjlim_{R \text{ ÉT}/\mathcal{O}_{Y,y}} R$$

w/ $k_R \cong k(\bar{y})$

EX $y = \Delta\eta$ $\qquad I = \{1,2\}$

$x = \eta^2$ GEOM PT OF X^2

CHOOSE $\bar{y}$ $\Leftrightarrow$ CHOOSE ALG CLOSURE $\bar{F}$ OF $F = k(X)$

CHOOSE $\bar{x}$ $\Leftrightarrow$ CHOOSE —— " —— $\bar{F}_I$ OF $F_I = k(X^I)$

$$\begin{tikzcd} \bar{F}_I & \bar{F} \otimes_{\bar{k}} \bar{F} \arrow[l, dashed] \arrow[r] & \bar{F} \\ F_I \arrow[u, phantom, "\cup"] & F \otimes_k F \arrow[l] \arrow[r] \arrow[u, phantom, "\cup"] & F \arrow[u, phantom, "\cup"] \end{tikzcd}$$

$\longleftrightarrow$ CHOICE OF $\bar{x} \rightsquigarrow \bar{y}$

CHOICE OF $\dashleftarrow$

EX X $x = \eta$, $y = v \in |X|$ CLOSED PT

$\bar{x} \leftrightarrow \bar{F}$, $\bar{y} \leftrightarrow \overline{k(v)}$

CHOICE OF $\bar{x} \rightsquigarrow \bar{y}$ $\sim$ CHOICE OF $\bar{F} \hookrightarrow \bar{F}_v$, F_v COMPLETED LOCAL FIELD

$\mathcal{F}$ SHEAF ON Y FOR ÉTALE TOP

$sp : \bar{x} \rightsquigarrow \bar{y} \implies sp^* : \mathcal{F}_{\bar{y}} \longrightarrow \mathcal{F}_{\bar{x}}$

$\boxed{\begin{array}{l}\textbf{THM 1} \quad \forall I \text{ FINITE, CHOOSE } sp : \bar{\eta}^I \rightsquigarrow \Delta\bar{\eta}. \\ \forall\, W \in \mathrm{REP}(\hat{G}^I), \text{ HAVE} \\ \qquad sp^* : \mathcal{H}_I(W)|_{\Delta\bar{\eta}} \xrightarrow{\sim} \mathcal{H}_I(W)|_{\overline{\eta^I}} = H_I(W) \\ \text{AS HECKE MODS}\end{array}}$

SKETCH OF SURJ'Y

BABY CASE PRETEND $\mathcal{H}_I(W)$ IS CONSTRUCTIBLE

PARTIAL FROB : FOR $i \in I$, HAVE

$$F_i : \mathrm{FROB}_i^* \mathcal{H}_I(W) \xrightarrow{\sim} \mathcal{H}_I(W)$$

LET Ω = LARGEST OPEN SUBSET OF X^I ON WHICH $\mathcal{H}_I(W)$ IS A LOCAL SYSTEM

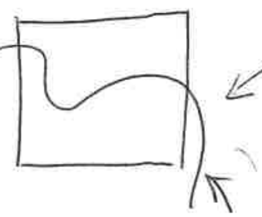
EXISTENCE OF F_i $\Longrightarrow$ Ω IS INVT UNDER FROB_i^{-1}

$\Longrightarrow$ $X^I \setminus \Omega$ IS INVT UNDER FROB_i^{-1} $\forall i \in I$

$\Longrightarrow$ $X^I \setminus \Omega = \bigcup$ "COORDINATE SUBSPACES"

PICTURE

$f(x,y)=0$



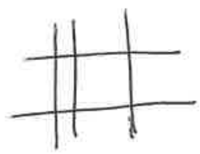
LOCUS OF WHERE $\mathcal{H}_I(W)$ IS NOT A LOC SYS

THIS IS INVT UNDER FROB_i^{-1}

$\Longrightarrow$ $\sqrt{(f(x,y))} = \sqrt{(f(x^q,y))}$

$\Longrightarrow$ $f = f_1(x)\, f_2(y)$

$\Longrightarrow$ DIVISOR LOOKS LIKE



$\Longrightarrow$ $\Omega \supset U^I$ WHERE $U \subset X$ OPEN DENSE

$\Longrightarrow$ sp^* IS $\cong$

ACTUAL ARGUMENT USES FINITENESS OF $H_I(W)$ AS A HECKE MOD (THM OF C. XUE)

SAY $\mathcal{F} \subset \mathcal{H}_I(W)$ IS "GOOD" IF

- $\mathcal{F}$ IS CONSTRUCTIBLE
- $\mathcal{F}_{\bar{\eta}^I} \subset H_I(W)$ GENERATES $H_I(W)$ AS A HECKE MOD

F.D./E

$$\Omega = \bigcup_{\mathcal{F}\ \text{GOOD}} \Omega(\mathcal{F})$$

AS BEFORE

F_i SENDS ~~FROB_i~~ A GOOD $\mathcal{F}$ TO ANOTHER GOOD $\mathcal{F}$

$\Longrightarrow$ $\underline{\Omega}$ IS STABLE UNDER FROB_i^{-1} $\forall i \in I$

$\Longrightarrow$ SAME AS ABOVE $\Longrightarrow$ $\Omega \supset U^I \ni \Delta\eta$

$\Longrightarrow$ $\exists$ GOOD $\mathcal{F}$ ST $\Omega(\mathcal{F}) \ni \Delta\eta$

$\Longrightarrow$

$$
\begin{array}{ccc}
Sp^* : \quad \mathcal{F}\big|_{\Delta\bar{\eta}} & \xrightarrow{\sim} & \mathcal{F}\big|_{\bar{\eta}^I} \\
\cap & & \cap \\
\mathcal{H}_I(W)\big|_{\Delta\bar{\eta}} & \longrightarrow & H_I(W)
\end{array}
$$

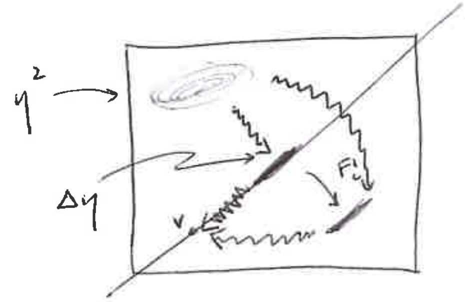
USING HECKE ACTION, SURJ UPSTAIRS GIVES SURJ DOWNSTAIRS.

FOR INJECTIVITY, SEE V.L. ARGUMENT USES EICHLER-SHIMURA RELN AS A REPLACEMENT FOR HECKE FINITENESS

"FAKE" ARGUMENT: ASSUME $\mathcal{H}_I(W)\big|_{\Delta\bar{\eta}}$ IS A F.G. HECKE MOD

$\mathcal{F}$ IS CALLED "NICE" IF MOREOVER

$\mathcal{F}\big|_{\Delta\bar{\eta}}$ GENERATES $\mathcal{H}_I(W)\big|_{\Delta\bar{\eta}}$ AS A HECKE MOD



$V, \mathcal{Y}^2$ IS STABLE BY FROB_i

E-S RELN $\Longrightarrow$ RECURSIVE FORMULA FOR $F_i^N(a)$