

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: KAROL KOZUL Email/Phone: kkozul@ualberta.ca

Speaker's Name: WEIZHE ZHONG

Talk Title: NEARBY CYCLES OVER GENERAL BASES

Date: 4 / 11 / 19 Time: 3:50 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: THE SPEAKER DISCUSSED TWO FUNCTORS OF NEARBY CYCLES, ~~AND~~ WHICH ARE USED IN THE CONSTRUCTION OF T_S OPERATORS. HE THEN STATED SEVERAL USEFUL PROPERTIES (FUNCTORIALITY, BASE CHANGE).

CHECK LIST

(This is NOT optional, we will not pay for incomplete forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
 - **Overhead:** Obtain a copy or use the originals and scan them
 - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
 - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

NEARBY CYCLES OVER GENERAL BASES

- ZHENG

①

$$RSht_{X^F} \xrightarrow{\Delta_I} RSht \quad (= L \circ Sht^{MER}) \quad \text{ARTIN STACK OF FINITE PRES}$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \bar{x} \rightarrow X & \xrightarrow{\Delta_I} & X^I \end{array}$$

$$L \in D_c^b(RSht, \Lambda)$$

$$\mathcal{F}_d = \psi(\Delta_d^* L) \in D_c^b(RSht|_{\Delta(\bar{x})}, \Lambda)$$

NEARBY CYCLES

$I = \{1, \dots, k\}$

WANT TO GET

$$T_\delta : \mathcal{F}_d \longrightarrow \mathcal{F}_{d'}$$

$$\delta \in \text{FWEL}(K^{\pm k}, \overline{K^{\pm k}})$$

OF DEG $d' - d$

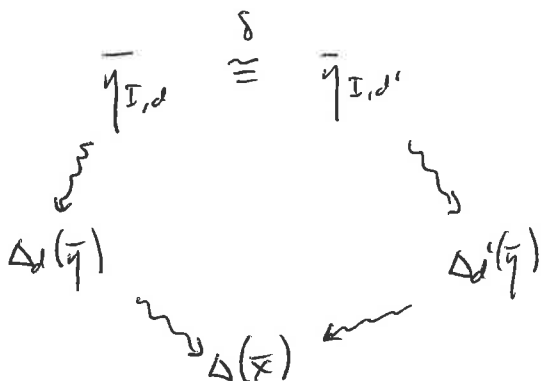
$$\mathcal{F}_d = \psi_{\Delta(\bar{x})}^{\text{NATIVE}} \leftarrow \Delta_d(\bar{y}) L$$

HAVE TWO SPECIALIZATIONS

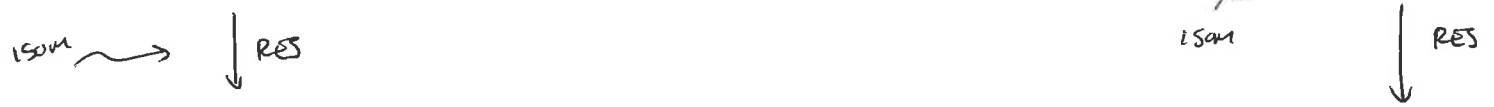


COMPLETE THIS TO

CONSIDER IMAGE OF δ
IN $\text{FWEL}(\eta_I, \bar{\eta}_I)$



$$\Psi_{\Delta(\bar{X}) \leftarrow \Delta_d(\bar{Y})} \xrightarrow{SP^*} \Psi_{\Delta(\bar{X}) \leftarrow \bar{\Gamma}_{Id}} \cong \Psi_{\Delta(\bar{X}) \leftarrow \bar{\Gamma}_{Id'}} \xleftarrow{SP^*} \Psi_{\Delta(\bar{X}) \leftarrow \Delta_d'(\bar{Y})}$$



$$\Psi_{\Delta(\bar{X}) \leftarrow \Delta_d(\bar{Y})}^{NAIVE}$$

T_S



$$\Psi_{\Delta(\bar{X}) \leftarrow \Delta_d'(\bar{Y})}^{NAIVE}$$



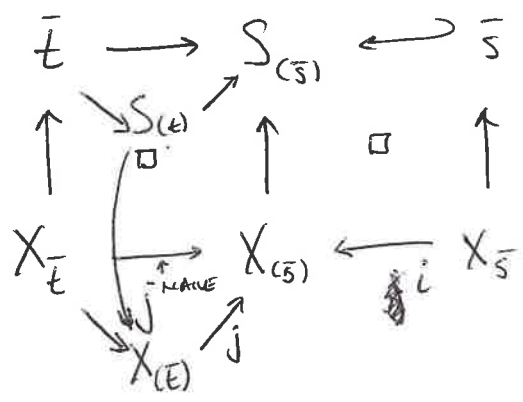
Ψ, Ψ^{NAIVE} WILL BE EXPLAINED LATER



$X \rightarrow S$ MORPHISM OF SCHEMES, Λ FINITE COMM RING, $n\Lambda = 0$ FOR n INV'BLE ON S

$\bar{S} \leftarrow \bar{E}$
ON S

MEANS



GIVEN $L \in D^+(X, \Lambda)$, CAN DEFINE

$$\Psi_{\bar{S} \leftarrow \bar{E}}^{NAIVE} L := i^* j^* (L|_{X_{\bar{E}}}) \in D^+(X_{\bar{S}}, \Lambda)$$

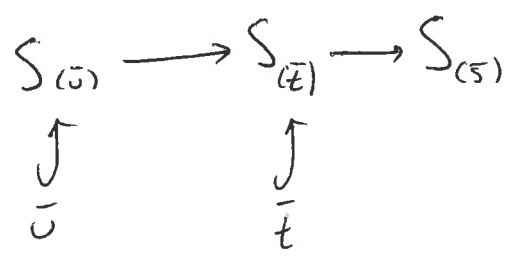
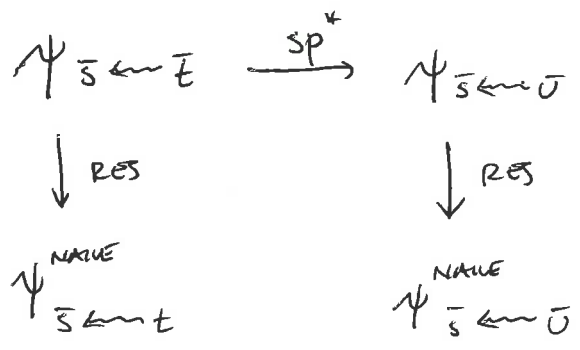
$$\Psi_{\bar{S} \leftarrow \bar{E}} L := i^* j^* (L|_{X_{(E)}})$$

RMK • t GENERIC PT $\Rightarrow \Psi_{\bar{S} \leftarrow \bar{E}} = \Psi_{\bar{S} \leftarrow \bar{E}}^{\text{NAIVE}}$

• IN GENERAL $T = \text{CLOSURE OF } \bar{E} \rightarrow S_{(S)}$

$$\Psi_{\bar{S} \leftarrow \bar{E}}^{\text{NAIVE}} L = \Psi_{\bar{S} \leftarrow \bar{E}} (L|_T)$$

FUNC TORIALITY $\bar{S} \leftarrow \bar{E} \xrightarrow{SP} \bar{U}$



"SOPHISTICATED" NEARBY CYCLES ACTUALLY GIVES A MAP SP^*

"MILNOR FIBER"

$$\left(\Psi_{\bar{S} \leftarrow \bar{E}}^{\text{NAIVE}} L \right)_{\bar{x}} \cong \text{RT} \left(X_{(S)} \times_{S_{(S)}} \bar{E}, L \right)$$

"MILNOR TUBE"

$$\left(\Psi_{\bar{S} \leftarrow \bar{E}} L \right)_{\bar{x}} \cong \text{RT} \left(X_{(\bar{E})} \times_{S_{(S)}} S_{(\bar{E})}, L \right)$$

X/S SEP FURTHER TYPE

$$R\Gamma_c(\psi_{\bar{S} \leftarrow \bar{E}}) \xrightarrow{SP^*} R\Gamma_c(\psi_{\bar{S} \leftarrow \bar{U}})$$

$$\begin{array}{ccc} \downarrow \text{RES} & & \downarrow \text{RES} \\ R\Gamma_c(\psi_{\bar{S} \leftarrow \bar{E}}^{\text{NAIVE}}) & \circlearrowleft & R\Gamma_c(\psi_{\bar{S} \leftarrow \bar{U}}^{\text{NAIVE}}) \end{array}$$

$$\begin{array}{ccc} \downarrow \text{BC} & & \downarrow \text{BC} \\ R\Gamma_c(X_{\bar{E}}, L) & \xrightarrow{SP^*} & R\Gamma_c(X_{\bar{U}}, L) \end{array}$$

EX $S = (A_k^2)_{(0)}$, $k = \bar{E}$

↑

$$X = \text{BL}_0(S) \quad \bar{x} \in X_0, \quad 0 \in \bar{E}, \quad t \neq 0$$

$$X_{(\bar{x})} \times_S S_{(\bar{E})} \cong X_{(\bar{x})} \times_X X_{(\bar{E})}$$

$$H^i(X_{(\bar{x})} \times_X X_{(\bar{E})}, \Lambda) = 0 \quad \forall i > 0$$

BUT $|\pi_0(X_{(\bar{x})} \times_X X_{(\bar{E})})| = \infty, \quad |H^0(X_{(\bar{x})} \times_X X_{(\bar{E})}, \Lambda)| = \infty$

$\Rightarrow \psi_{0 \leftarrow \bar{E}} \Lambda$ NOT CONSTRUCTIBLE

IF $t = \eta$ $\psi_{0 \leftarrow \bar{E}} \Lambda = \psi_{0 \leftarrow \bar{E}}^{\text{NAIVE}} \Lambda$ NOT CONSTRUCTIBLE

IF $t \neq \eta$ $\psi_{0 \leftarrow \bar{E}} \Lambda \xrightarrow{\text{RES}} \psi_{0 \leftarrow \bar{E}}^{\text{NAIVE}} \Lambda \leftarrow$ IS CONSTRUCTIBLE

$\Rightarrow \text{RES} \neq \cong$

IF X NOT IN CLOSURE OF t IN X , $X_{(t)} \times_S \bar{E} = \emptyset$

SO IN GENERAL ψ / ψ^{NAIVE} IS NOT SO GOOD.

BUT

THM (ORGOGOZO) S QCQS, HAS FINITELY MANY COMM. COMP. (EG NOETHERIAN), \mathcal{F} OF FINITE PRES, $L \in D_c^b(X, \Lambda)$

THEN \exists PROPER BIRAT'L MORPHISM $\tilde{S} \rightarrow S$ SUCH THAT
"MODIFICATION" \tilde{S} DEPS ON L

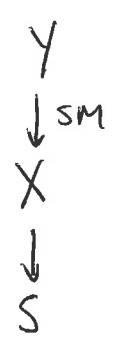
1) $\forall \tilde{S} \leftarrow E$ ON \tilde{S} ,
 $\psi_{\tilde{S} \leftarrow E}(L|_{\tilde{X}}) \in D_c^b(\tilde{X}_{\tilde{S}}, \Lambda)$ $\tilde{X} := X \times_S \tilde{S}$

AND \downarrow_{RES}
 $\psi_{\tilde{S} \leftarrow E}^{NAIVE}(L|_{\tilde{X}})$

2) \exists STRAT'N $\tilde{S} = \cup S_\alpha$ BY LOCALLY CLOSED CONSTRUCTIBLE SUBSET ST FOR $\tilde{S} \leftarrow E \leftarrow \bar{U}$ IN COMPONE S_α

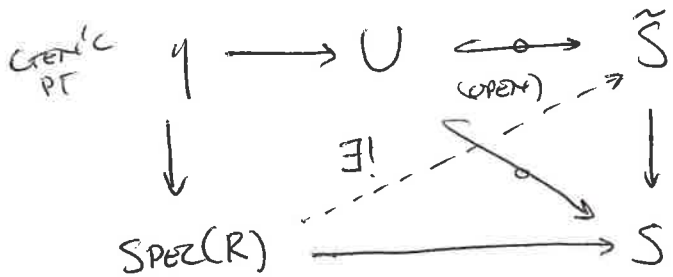
$$\psi_{\tilde{S} \leftarrow E} L \xrightarrow{sp^*} \psi_{\tilde{S} \leftarrow \bar{U}} L$$

RMK • SMOOTH BASE CHANGE



$$\left(\psi_{\tilde{S} \leftarrow E} L \right) \Big|_{Y_{\tilde{S}}} \xrightarrow{\sim} \psi_{\tilde{S} \leftarrow E} (L|_Y)$$

\Rightarrow THM HOLDS FOR $X =$ ARTIN STACK OF FINITE PRES / S



R VALUATION RING

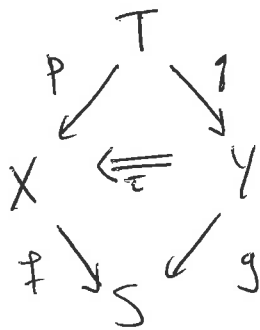
BY VALUATIVE CRITERION

DELIGNE'S VANISHING TOPOS

UPSHOT $R\psi L \in D^+(\underbrace{X \leftarrow_S S}_{\text{VANISHING TOPOS}}, \Lambda)$

$R\psi_{S \leftarrow E} L$ ARE SLICES OF $R\psi L$

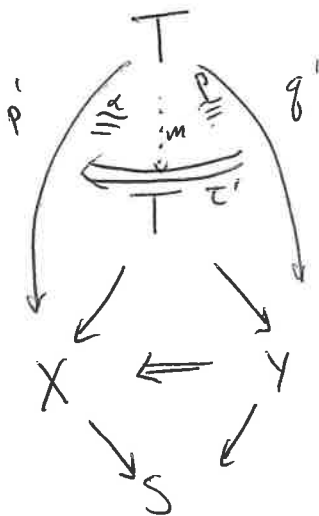
DEF (ORIENTED PRODUCT OF TORI)



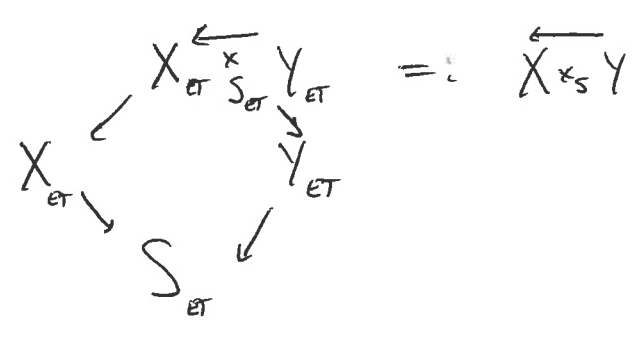
ORIENTED PRODUCT OF X, Y, S IS $(T, p, q, \tau : f \leftarrow g) =: X \leftarrow_S Y$

\leftarrow MORPHISMS OF TOPOI

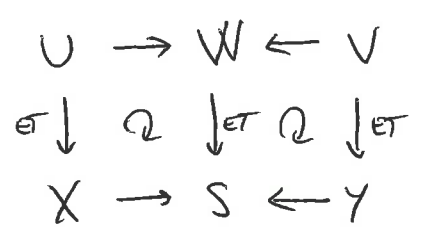
UNIV PROPERTY: $\forall (T', p', q', \tau')$ AS IN PICTURE,



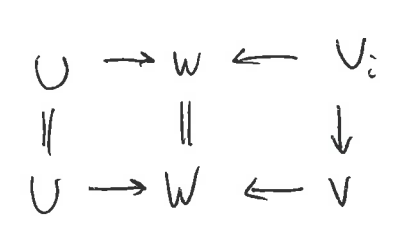
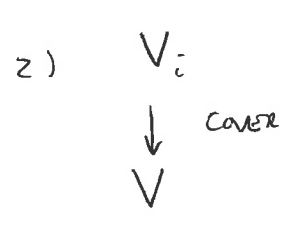
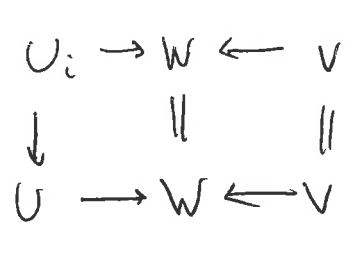
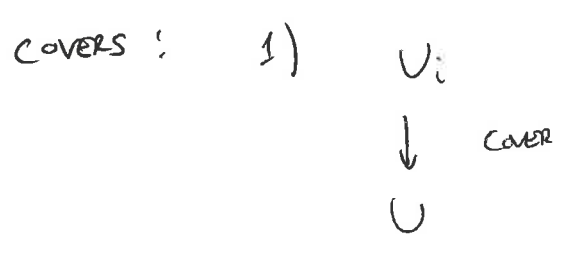
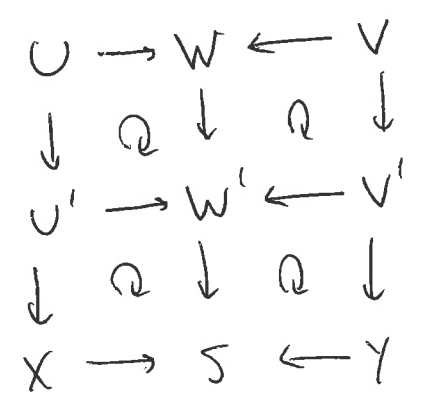
$\exists (m, \alpha, \beta)$ UNIQUE UP TO
UNIQUE ISOM MAKING
DIAG COMMUTE



SITE FOR $X \times_S Y$: OB!

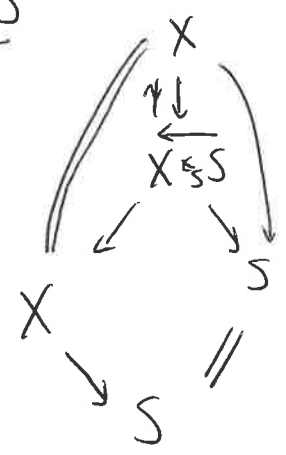


MORPHISMS!



$$\begin{array}{ccccc}
 3) & U & \longrightarrow & W & \longleftarrow & V \\
 & \parallel & & \downarrow & \square & \downarrow \\
 & U & \longrightarrow & W & \longleftarrow & V
 \end{array}$$

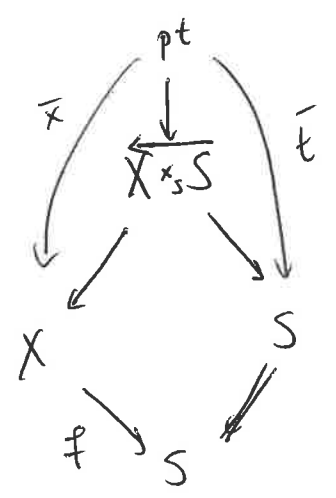
For $Y=S$



$$\psi: X \longrightarrow X \overset{\longleftarrow}{\times}_S S \quad \text{MORPHISM OF TOPOI}$$

\uparrow
 VANISHING TOPOS

$$\psi = R\psi_* = D^+(X, \Lambda) \longrightarrow D^+(X \overset{\longleftarrow}{\times}_S S, \Lambda)$$



$$\text{PT OF } X \overset{\longleftarrow}{\times}_S S = (\bar{x}, \bar{t}, f(\bar{x}) \longleftarrow \bar{t})$$

THM (OROGOROZO) $\exists \tilde{S} \longrightarrow S$ MOD'N ST

1) $\psi(L|_{\tilde{X}})$ COMMUTES w/ BASE CHANGE $T \longrightarrow \tilde{S}$

2) $\psi(L|_{\tilde{X}}) \in D_c^b(\tilde{X} \overset{\longleftarrow}{\times}_S \tilde{S}, \Lambda)$