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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: KAROL KOZIOL Email/Phone: kkoziol@ualberta.ca

Speaker's Name: ANA CARAIANI

Talk Title: MODULI OF REPRESENTATIONS AND GLOBAL LANGLANDS PARAMETRIZATION II

Date: 4/10/19 Time: 11:45 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: THE SPEAKER CONTINUED DESCRIBING PSEUDOREPRESENTATIONS, GIVING A MORE ABSTRACT APPROACH AND DESCRIBING THEIR MODULI

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- [x] Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- [x] Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - <u>Computer Presentations</u>: Obtain a copy of their presentation
  - <u>Overhead</u>: Obtain a copy or use the originals and scan them
  - <u>Blackboard</u>: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - <u>Handouts</u>: Obtain copies of and scan all handouts
- [x] For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack <u>with this completed sheet on top</u> and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- [x] When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- [x] Email the re-named files to <u>notes@msri.org</u> with the workshop name and your name in the subject line.

MODULI OF REPRESENTATIONS + GLOBAL LANGLANDS PARAMETRIZATION II — CARAIANI

§3 MODULI STACKS OF REPS

Gp = CAT OF GRPS

$\cup$

FFG = FREE FINITELY GEN'D GPS

I FINITE $\rightsquigarrow$ $FG(I) \in FFG$

IF M IS AN ABSTRACT GP, HAVE A BIJECTION OF SETS

$$\mathrm{Hom}_{Gp}(FG(I), M) = M^I$$

+ $FG(I) \longrightarrow FG(J) \rightsquigarrow M^J \longrightarrow M^I$

EX 1) $\{1,2\} \longrightarrow \{1\} \rightsquigarrow \Delta : M \longrightarrow M^2$ DIAG'L

$FG(\{1,2\}) \longrightarrow FG(\{1\})$

2) $FG(\{x\}) \longrightarrow FG(\{y_1, y_2\}) \rightsquigarrow m : M^2 \longrightarrow M$ MULT

$x \longmapsto y_1 y_2$

DOES NOT COME FROM MAP OF SETS

ANY MORPHISM IN FFG IS A COMPOSITION OF

1) $FG(I) \longrightarrow FG(J)$

COMING FROM MAP OF SETS

2) $FG(\{x_1, \ldots, x_n\}) \longrightarrow FG(\{y_1, \ldots, y_{n+1}\})$

$$x_i \longmapsto y_i \qquad i < n$$

$$x_n \longmapsto y_n y_{n+1}$$

3) $FG(\{x_1, \ldots, x_n\}) \longrightarrow FG(\{y_1, \ldots, y_n\})$

$$x_i \longmapsto y_i \qquad i < n$$

$$x_n \longmapsto y_n^{-1}$$

Γ ABSTRACT GP. CONSIDER SLICE CAT FFG/Γ W/ OBJECTS $FG(I) \longrightarrow \Gamma$

IF M IS AN ABSTRACT GP, HAVE BIJECTION

$$\mathrm{Hom}_{Gp}(\Gamma, M) \longrightarrow \lim_{(FFG/\Gamma)^{op}} (\mathrm{Hom}(FG(I), M))$$

$$= \lim_{(FFG/\Gamma)^{op}} M^I$$

NOW LET M/E BE AN AFFINE GP SCHEME, Γ ABSTRACT GP

FUNCTOR:

$$\mathrm{Hom}(\Gamma, M) : R \longmapsto \mathrm{Hom}_{Gp}(\Gamma, M(R))$$

"FRAMED M-VALUED REP'N VARIETY"

THIS IS REP'D BY AN AFFINE SCHEME

$$\mathrm{Hom}(\Gamma, M) = \lim_{(FFG/\Gamma)^{op}} (M^I)$$

$$E[\mathrm{Hom}(\Gamma, M)] = \mathrm{colim}_{FFG/\Gamma} E[M^I]$$

EX

1) $\Gamma = FG(I)$ $\rightsquigarrow$ GET $\mathrm{Spec}(E[M^I])$

2) Γ FINITELY GEN'D $\rightsquigarrow$ CLOSED SUBSCHEME OF ↑

3) IN GENERAL

$$E[M^\Gamma] = \bigotimes_{x \in \Gamma} E[M] = \mathrm{colim}_{I \subset \Gamma} E[M^I]$$

(Γ AS A SET)

CONSIDER $f_\gamma \in \bigotimes_{x \in \Gamma} E[M]$ PURE TENSOR

$$f_\gamma = \left(\bigotimes_{x \neq \gamma} 1\right) \otimes \underset{x=\gamma}{f} \qquad f \in E[M]$$

$$f_{\gamma_1 \gamma_2} = \sum_i f^1_{i,\gamma_1} \cdot f^2_{i,\gamma_2}, \quad \text{WHERE}$$

$$f \in E[M] \overset{\text{COMULT}}{\longmapsto} \sum f^1_i \otimes f^2_i \in E[M] \otimes_E E[M]$$

Assume now that $\widehat{G}/E$ is a smooth affine Gp scheme acting on M by auts

$$\rightsquigarrow \quad \widehat{G} \circlearrowright \mathrm{Hom}(\Gamma, M)$$

Def We let the moduli stack of M-valued reps of Γ (up to $\widehat{G}$-conj) to be the Artin stack

$$\mathrm{Hom}(\Gamma, M)/\widehat{G}$$

We have

$$\mathrm{QCoh}\left(\mathrm{Hom}(\Gamma, M)/\widehat{G}\right) = \lim_{(FFG/\Gamma)^{op}} \mathrm{QCoh}\left(M^I/\widehat{G}\right)$$

w/ transition morphisms given by direct image

$\widehat{G}$-equiv $E[\mathrm{Hom}(\Gamma, M)]$-modules

<u>Upshot</u> Want: view $H_\emptyset(\mathbb{1})$ as QCoh sheaf on GIT quotient

$$\mathrm{Hom}(\Gamma_F, \widehat{G}) /\!/ \mathrm{Ad}(\widehat{G})$$

Instead: Look at $\mathrm{Reg} := E[\widehat{G}]$ w/ action of $\widehat{G}$ by left mult

We will show $H_{\{\emptyset\}}(\mathrm{Reg}) \in E\text{-v.s.}$ has two structures:

1) module over $E[\mathrm{Hom}(\Gamma_F, \widehat{G})]$
2) action of $\widehat{G}$, compatible w/ adjoint action of $\widehat{G}$ on $E[\mathrm{Hom}(\Gamma_F, \widehat{G})]$

THIS WILL BE INDUCED FROM $a_R(\hat{G})$ ON REG

$H_{\{*\}}$ PRESERVES COLIMITS $\rightsquigarrow$ $H_{\{*\}}(a_R(\hat{G}))$ $\circlearrowright$ $H_{\{*\}}(\mathrm{REG})$

$$H_{\{*\}}(\mathbb{1}) \cong H_{\{*\}}(\mathrm{REG})^{H_{\{*\}}(a_R(\hat{G}))}$$

$$H_{\{*\}}(\mathbb{1}) \cong H_{\emptyset}(\mathbb{1})$$

KEY INGREDIENT EVALUATION BUNDLES

I FINITE, CAN CONSTRUCT COMPATIBLE FUNCTORS

$$\mathrm{REP}(M^I \times \hat{G}) \longrightarrow \mathrm{REP}(\Gamma^I, \mathrm{QCOH}(\mathrm{HOM}(\Gamma, M)/\hat{G}))$$

$$W \longmapsto \widetilde{W} \circlearrowleft \Gamma^I$$

IF $r : \hat{G} \longrightarrow \mathrm{END}(W)$, THEN $E[\mathrm{HOM}(\Gamma, M)] \otimes W$ HAS A $\hat{G}$-EQUIV. STRUCTURE $\Longrightarrow$ DESCENDS TO A QCOH SHEAF $\widetilde{W}$

$\hat{G} \subset M$ NOW

GIVEN $\tilde{r} : M^I \times \hat{G} \longrightarrow \mathrm{END}(W)$

$$\rightsquigarrow r = \tilde{r} \circ \Delta : \hat{G} \longrightarrow \mathrm{END}(W)$$

CAN WRITE END OF

$$E[M^I] \otimes W \longrightarrow E[M^I] \otimes W$$

$$f \otimes w \longmapsto (\tilde{f} : m \longmapsto f(m) \otimes \tilde{r}(m, 1).w)$$

EQUIV FOR $E[M^I]$-ACTION (TRIV ON W)

ALSO $(\mathrm{Ad} \otimes r)(\widehat{G})$ - EQUIV

$\rightsquigarrow$ DESCENDS TO END OF QCOH SHEAF CORR TO W ON $M^I/\widehat{G}$

$\gamma_I \in \Gamma^I$ GIVES $ev_{\gamma_I} : \mathrm{Hom}(\Gamma, M) \longrightarrow M^I$

<u>THM</u> (DRINFELD, ZHU) THE FOLLOWING FUNCTOR IS AN EQUIV OF CATS ($\widehat{G}$ CONNECTED, RED'IVE)

$$\mathrm{QCOH}(\mathrm{Hom}(\Gamma, M)/\widehat{G}) \xrightarrow{\sim} \mathrm{FUN}^L_{\mathrm{FIN}_*}(\mathrm{REP}(M^\bullet \times \widehat{G}), \mathrm{REP}(\Gamma^\bullet))$$

(FIN_*: POINTED FINITE SETS)

$$\mathcal{A} \longmapsto H_{\mathcal{A},\bullet}$$

$$H_{\mathcal{A},\bullet} : W \in \mathrm{REP}(M^I \times \widehat{G}) \longmapsto \Gamma(\mathrm{Hom}(\Gamma, M)/\widehat{G}, \widetilde{W} \otimes \mathcal{A}) \circlearrowleft \Gamma^I$$

<u>PF IDEA</u> QUASI INVERSE $H_{\{*\}}(\mathrm{REG}) \rightsquigarrow$ QCOH SHEAF ON $\mathrm{Hom}(\Gamma, M)/\widehat{G}$

IF $\widehat{G}$ IS AN AFFINE GRP SCHEME $\circlearrowleft X = \mathrm{SPEC}(A)$, THEN

$$X /\!\!/ \widehat{G} := \mathrm{SPEC}(A^{\widehat{G}})$$

$\rightsquigarrow \mathrm{Hom}(\Gamma, M) /\!\!/ \widehat{G}$ GIT QUOT

DEF A PSEUDO REP OF Γ VALUED IN $M(R)$ UP TO $\hat{G}$-CONJ IS AN R-VALUED PT OF $\mathrm{Hom}(\Gamma, M) /\!\!/ \hat{G}$

EG $\Gamma = FG(\{*\})$, $\hat{G} = GL_n$, $M = GL_n$

$\rightsquigarrow$ TR : $\mathrm{Hom}(\Gamma, M)/\hat{G} \longrightarrow \mathrm{Hom}(\Gamma, M) /\!\!/ \hat{G}$

$[g] \longmapsto$ CHAR POLY OF g

DEF AN $E[FFG]$-ALG IS A FUNCTOR

$$FFG \longrightarrow E\text{-ALG}$$

IE,

$$FG(I) \longmapsto A_I$$

$$(FG(I) \to FG(J)) \longmapsto (A_I \to A_J)$$

EX 1) Γ ABSTRACT, $R = E$-ALG

$\rightarrow$ $\mathrm{MAPS}(\Gamma, R)$ IS AN $E[FFG]$-ALG

2) M AFFINE GP SCHEME $/E$, $R = E$-ALG

$\Rightarrow$ $R[M^\bullet]$ IS AN $E[FFG]$-ALG

3) $\hat{G} \subset M$, $R[M^\bullet]^{AD(\hat{G})}$ IS AN $E[FFG]$-ALG

LEMMA $\exists$ A NATURAL BIJECTION B/W

$$\left\{ \begin{array}{c} E[FFG]\text{-ALG HOMS} \\ E[M^\bullet]^{\hat{G}} \longrightarrow \mathrm{MAPS}(\Gamma^\bullet, R) \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{c} M(R)\text{-VALUED PSEUDO-REPS} \\ \text{OF } \Gamma \text{ UP TO } \hat{G}\text{-CONJ} \end{array} \right\}$$