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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Rosa Vargas **Email/Phone:** rmvargas@ciencias.unam.mx / 5104243513

Speaker's Name: Emmy Murphy

Talk Title: Hamiltonian dynamics and extension of symplectic/contact forms.

Date: 08 / 17 / 18 **Time:** 11 : 00 (am) / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: In this talk E. Murphy presented results in symplectic and Contact geometry. She showed that in both geometries, the suspension of a Hamiltonian flow defines a germ of a symplectic/contact structure near hypersurface. Additionally, she introduced questions about whether this germ extends to a compact set are related to questions about which flows are generated by positive Hamiltonians. She explained how in the contact case this can be used to prove a general extension result.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - Computer Presentations: Obtain a copy of their presentation
  - Overhead: Obtain a copy or use the originals and scan them
  - Blackboard: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - Handouts: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Given by: Emmy Murphy

Interesting questions

1. Given a smooth manifold M, does there exist a symplectic/ contact structure in M?
2. Given a symplectic/contact structure near $\partial(B^{2n})$, does it extend to B^{2n}?

Hamiltonian suspension

Fix a symplectic manifold $\left((M,\omega)=D^{2n-2},\omega=\omega_{std}=\sum dy\wedge dx\right)$

Let $H : M\times\mathbb{R}\to\mathbb{R}$ be a Hamiltonian,

$$X_H^t \rightsquigarrow \varphi_H^t : M\to M.$$

Assume $H>0$ on ∂M. Look for a larger manifold $M\times\mathbb{R}^2, \tilde{\omega}=\omega+dp\wedge dq$
Consider an hyersurface and get a foliation: $\mathscr{F}=her(\omega\mid_Y)=\text{span}(X_{p-H_{q(x)}})$

Moser/Weinstein

$\mathscr{F}$ determines the germ of $\tilde{\omega}$ near Y. ($\mathscr{F}$ with transverse symplectic structure).

$$\mathscr{F}=\text{span}\left\{\partial q+X_H^q+\dot{H}_q\partial p\right\}$$

To make this more pictoral,

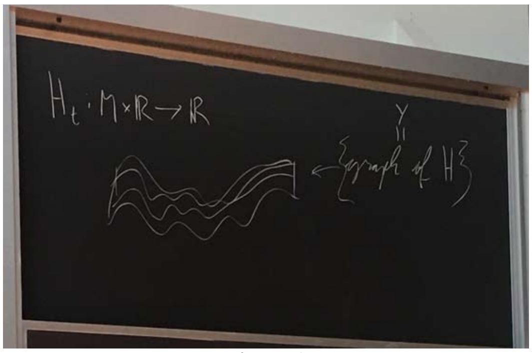

Figure 1

Better: $H : M\times S^1\to\mathbb{R}$

$$Y\subseteq M\times T^*S^1\left\{p=H_q(x)\right\}$$

$$Y\cong_{C^\infty} M\times S^1 \tag{0.1}$$

Definition 0.1 *Let $\partial M\neq\emptyset$,*
$\circ\{p>0\}\subseteq T^*S^1$
$\circ\{p>0\}\cong\mathbb{C}^*\subseteq\mathbb{C}$
$\circ p=r^2\omega\mid_{\mathbb{C}}=dr^2d\theta$ *(the simplectic form of $\mathbb{C}$) So define $\tilde{Y}=\left\{r^2\leq H_\theta(x)\right\}$, $\tilde{Y}\subseteq\partial M\times\mathbb{C}$.*

$\tilde{Y}$ is well defined since $H>0$ near ∂M.

Take $\overline{Y}=Y\cup\tilde{Y}$ (is well defined since $\{p>0\}$)

$$\left\{p=r^2=H_\theta(x);x\in\partial M\right\}$$

$$\overline{Y}\cong_{C^\infty}(M\times S^1)\cup\partial M\times D^2\cong\partial(M\times D^2)$$

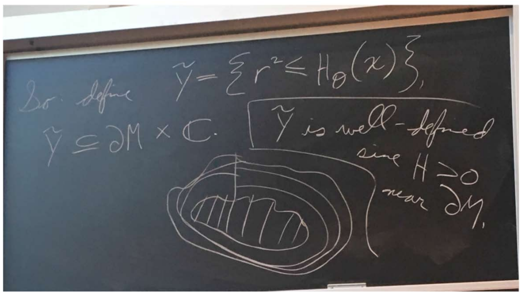

Figure 2

NOTE: Define $H > 0$ everywhere, then $Y \subseteq M \times \{p > 0\} \subseteq M \times T^*S^1$

- o $Y \subseteq M \times \mathbb{C}^*$
- o $\overline{Y} = \partial\left(\{r^2 \leq H_{\theta)}\}\right)$

So, in particular the symplectic form extends both.
For the following I will restrict myself to the disc.

Rises the questions: Questions of which symplectic structure on $\partial(B^{2n})$ extend to B^{2n} are lied to questions what which Hamiltonian diffeomorphism $\varphi \in Ham(D^{2n-2})$ can be generated by positive H_t

NOTE: Simplectic structure $\overline{Y}$ is different for $\tilde{H} = H + cte$, Since $\tilde{Y}$ is different

Exemple: Say a Hamiltonian of the form
Possible to "scale' values so that volume is positive, but it extends to no symplectic structure

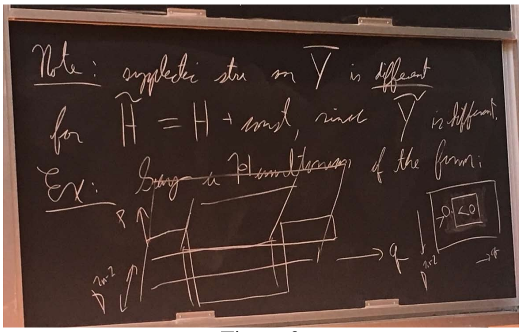

Figure 3

(Similar to Gromov-non-squeezing)

Contact Geometry Let (M, α) contact, $H : M \to \mathbb{R}$ defines X_H by

- o $(X \lrcorner\, d\alpha) |_{\text{her}\alpha} = (-dH) |_{\ker\alpha}$
- o $\alpha(X_H) = H$

If φ_H^t is the flow of X_H $(\varphi_H^t)^*\alpha = \lambda\alpha for \lambda > 0, \lambda \in C^\infty M$

Let $\mathscr{L}_{X_H}\alpha = R_\alpha(H)\alpha$, with $R_\alpha = X_{H=1}$ the Lie derivative of α.

In contact geometry, the Lie Albegra, $\mathfrak{conT}(M, \alpha) \cong C^\infty(M)$.

Exemple in Darboux chart:

$$\mathbb{R}^{2n+1}, \ \ \alpha = dz - \sum Y_i dx_i$$

$$(x,y,z) \mapsto (cx, cy, c^2 z) \quad \text{preserves } \alpha \text{ conformally}$$

$$X_H = \partial_x + \partial_y + 2\partial z$$

is Hamiltonian.

Can also graph $H : M \times \mathbb{R} \to \mathbb{R}$ in $M \times \mathbb{R}^2$, $\tilde{\alpha} = \alpha + pdq$ get $Y = \left\{ H_q(x) = p \right\}$ still has characteristic foliation,

$$\mathscr{F} = \left\{ \partial q + X_H + \partial_p \right\}$$

$\mathscr{F}$ determines contact structure near Y.

NOTE: Is not true , if $n \geq 2$, that exist $\varphi_{min} \in$ [oNT(D^{2n-1}_{std}), such that for any φ, exists ψ,

$$\varphi^{-1}_{min} \psi \varphi \psi^{-1}$$

generated by $H^{>}0$.

Is also not true, if $n \geq 2$, that exist $\varphi^{-1}_{min} \in$ [oNT(D^{2n-1}_{std}) such that for any φ, exists ψ,

$$\varphi^{-1}_{min} \psi \varphi \psi^{-1}$$

generated by $H > 0$.

But,

Theorem (Borman-Eliashberg-Nancy Hingston) Exist φ_{min}, such that for all φ, exists ψ, $k > 0$. Such that

$$\varphi_{min} \sharp ... \sharp \varphi_{min} < \psi \varphi \psi^{-1}$$

with $\sharp ... \sharp \quad k-$ times.

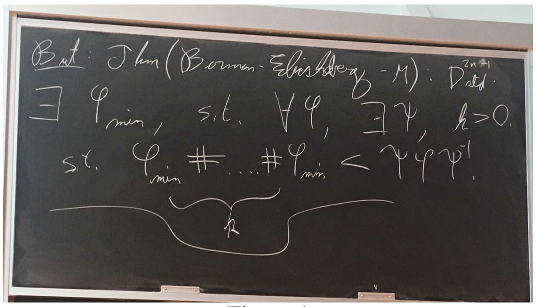

Figure 4

Most essential steps in the proof:

Theorem: Any smooth manifold M, where $TM \oplus \mathbb{R}$ is complex admits a contact structure.

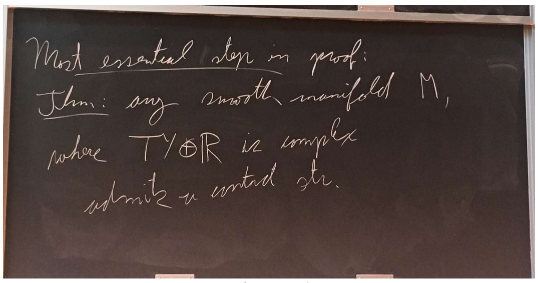

Figure 5