# Picard Group of $E_{(2,2)}^{hC_n}$-modules. Irina Bobkova, 1/30/19.

j/w Beaudry, Hill, Stojanoska.

## I. Crash course on chromatic homotopy theory.

Stable homotopy groups of spheres: $\pi_i^{st} := \lim_n \pi_{i+n} \underset{\substack{\uparrow \\ \text{spaces}}}{S^n} = \pi_i \underset{\substack{\uparrow \\ \text{sphere spectrum}}}{\mathbb{S}}$

To understand $\pi_i \mathbb{S}$: work one prime at a time.

(spectrum that captures p-local information)

$$\begin{tikzcd}
 & \mathbb{S} \arrow[dl] \arrow[d] & \cdots \arrow[from=l, phantom] & \\
\mathbb{S}_{(2)} & \mathbb{S}_{(3)} & \cdots & \mathbb{S}_{(p)} \arrow[from=ull] \arrow[dl] \arrow[d] \arrow[dr] & \cdots \\
 & \mathbb{S}_{(p,1)} & \mathbb{S}_{(p,2)} \quad \cdots & \mathbb{S}_{(p,n)} & \cdots
\end{tikzcd}$$

and then break apart further:

- $\pi_i \mathbb{S}_{(p,n)}$ are building blocks of $\pi_i \mathbb{S}$.
- $\underset{\substack{\uparrow \\ \text{spectra}}}{\mathbb{S}_{(p,n)}} \approx E_{(p,n)}^{hG(p,n)}$ ← group acting on $E_{(p,n)}$

Note: can think about spaces in place of spectra if more comfortable

- $E_{(p,n)}$ is a good spectrum, related to the Lubin-Tate space of deformations of a formal group law $\Gamma_{(p,n)}$.
- $G_{(p,n)}$ is related to automorphisms of $\Gamma_{(p,n)}$.
- we need to take homotopy fixed points, not ordinary ones.
- Thinking about a spectrum as fixed pts gives us additional computational tools.

$G_{(p,n)}$ is a profinite group – easier to study $E_{(p,n)}^{hF}$ for $F \subseteq G_{(p,n)}$ finite.

Moreover, $S_{(p,n)}$ can be decomposed in terms of $E_{(p,n)}^{hF_i}$ for "good" $F_i \subseteq \mathbb{G}_{(p,n)}$, in <u>some</u> cases.

This motivates our study of $R := E_{(p,n)}^{hF}$. <u>Note</u>: all $R$ are ring spectra, so we can consider modules over $R$. Moreover, all $R$ are periodic: $\exists\, m_R$ s.t. $\Sigma^{m_R} R \simeq R$.

II. <u>Picard groups</u>.

$(\mathcal{C}, \otimes, I)$ symmetric monoidal.

$Pic(\mathcal{C}) = \{\text{inv. objects}\}/\text{isoms}$.

Consider $(R\text{-mod}, \wedge_R, R)$. <u>Question</u>: what is $Pic(R) := Pic(R\text{-mod})$?

$\Sigma R$ is an invertible module over $R$, since $\Sigma R \wedge \Sigma^{-1} R \cong R$.

In fact, $\Sigma^n R \in Pic(R)$ $\forall n$. Since $R$ are periodic, we always have $\mathbb{Z}/m_R \subseteq Pic(R)$

<u>Q</u>: Is there anything else in $Pic(R)$? This depends on $R$.

$R = E_{(p,n)}^{hF}$. Some cases are known:

$Pic(R)$ generated by $\Sigma R$. $\left\{ \begin{array}{ll} * \ n=1;\ p, F \text{ anything} & \text{(Hopkins-Mahowald-Sadofsky)} \\ * \ p\text{-odd},\ n=p-1,\ \text{any } F & \text{(Heard-Mathew-Stojanoska)} \\ * \ p=2,\ \text{all } n,\ F=C_2 & \text{(Heard-Li-Shi)} \end{array} \right.$

$n=2,\ p=2,\ F \subseteq \mathbb{G}_{(2,2)}$

| $F$ | $C_2$ | $C_4$ | $C_6$ | $Q_8$ | $G_{24}$ | $G_{48}$ |
|---|---|---|---|---|---|---|
| periodicity of $R$ | 16 | 32 | 48 | 64 | 192 | 192 |
| $Pic(R)$ | $\mathbb{Z}/16$ | $\mathbb{Z}/32 \oplus \mathbb{Z}/2$ | $\mathbb{Z}/48$ | $\mathbb{Z}/64$ | $\mathbb{Z}/192$ | $\mathbb{Z}/192$ |
| | + Stojanoska | Beaudry+B+Hill | | Mathew-Stojanoska | | |