

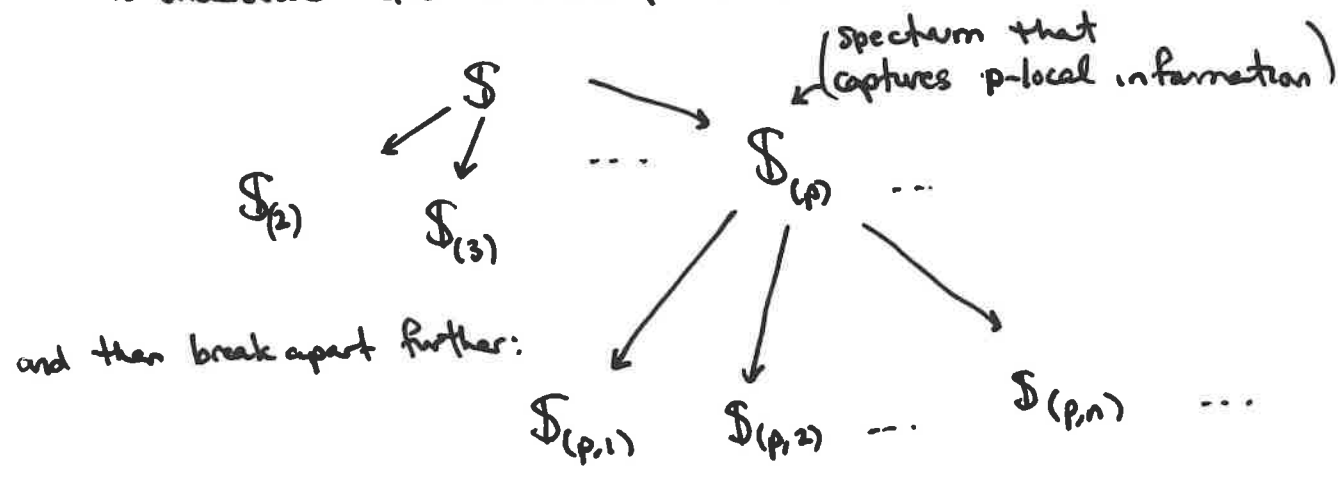
d/w Beaudry, Hill, Stojanoska.

I. Crash course on chromatic homotopy theory.

Stable homotopy groups of spheres: $\pi_i^{st} := \lim_n \pi_{i+n} S^n = \pi_i \mathcal{S}$

\uparrow spaces \uparrow sphere spectrum

To understand $\pi_i \mathcal{S}$: work one prime at a time.



and then break apart further:

- $\pi_i \mathcal{S}_{(p,n)}$ are building blocks of $\pi_i \mathcal{S}$.
- $\mathcal{S}_{(p,n)} \approx E_{(p,n)}^{hG_{(p,n)}}$ ← groupacting on $E_{(p,n)}$
- \uparrow Spectra

Note: can think about spaces in place of spectra if more comfortable

- $E_{(p,n)}$ is a good spectrum, related to the Lubin-Tate space of deformations of a formal group law $\Gamma_{(p,n)}$.
- $G_{(p,n)}$ is related to automorphisms of $\Gamma_{(p,n)}$.
- we need to take homotopy fixed points, not ordinary ones.
- Thinking about a spectrum as fixed pts gives us additional computational tools.

$G_{(p,n)}$ is a profinite group - easier to study $E_{(p,n)}^{hF}$ for $F \subseteq G_{(p,n)}$ finite.

Moreover, $S_{(p,n)}$ can be decomposed in terms of hF_i for "good" $F_i \subseteq G_{(p,n)}$, in some cases.

This motivates our study of $R := E_{(p,n)}^{hF}$. Note: all R are ring spectra, so we can consider modules over R . Moreover, all R are periodic: $\exists m_R$ s.t. $\Sigma^{m_R} R \simeq R$.

II. Picard groups.

$(\mathcal{C}, \otimes, I)$ symmetric monoidal.

$Pic(\mathcal{C}) = \{inv. objects\} / isoms.$

Consider $(R\text{-mod}, \wedge_R, R)$. Question: what is $Pic(R) := Pic(R\text{-mod})$?

ΣR is an invertible module over R , since $\Sigma R \wedge \Sigma^{-1} R \cong R$.

In fact, $\Sigma^n R \in Pic(R) \forall n$. Since R are periodic, we always

have $\mathbb{Z}/m_R \subseteq Pic(R)$

Q: Is there anything else in $Pic(R)$? This depends on R .

$R = E_{(p,n)}^{hF}$. Some cases are known:

- $Pic(R)$ generated by ΣR .
- * $n=1$; p, F anything (Hopkins-Mahowald-Sadovskiy)
 - * p -odd, $n=p-1$, any F (Heard-Matthew-Stojanoska)
 - * $p=2$, all n , $F=C_2$ (Heard-Li-Shu)

$n=2, p=2, F \subseteq G_{(2,2)}$

F	C_2	C_4	C_6	Q_8	G_{24}	G_{48}
periodicity of R	16	32	48	64	192	192
$Pic(R)$	$\mathbb{Z}/16$	$\mathbb{Z}/32$ \oplus $\mathbb{Z}/2$	$\mathbb{Z}/48$	$\mathbb{Z}/64$	$\mathbb{Z}/192$	$\mathbb{Z}/192$

+ Stojanoska \rightarrow Beaudry+B-Hill Mathew-Stojanoska