3.3

MSRI

Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: ORI KATZ **Email/Phone:** ORIKATZ.OK@gmail.com

Speaker's Name: Natalia Tronko

Talk Title: Noëther theorem for magnetized plasmas

Date: 10/10/18 **Time:** 11:30 (am) pm (circle one)

Please summarize the lecture in 5 or fewer sentences: In order to control quality of plasma predictions for the costly experiments, numerical simulations must be used. Using gyrokinetic models (GK) computational time is greatly reduced. Confidence in predictions requires a rigorous & systematic framework. Here, a new & generic theoretical framework to test validity & domain of existing GK codes is presented, emphasizing the role of energy invariants from Noëther theorem

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☒ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☒ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - → __Computer Presentations__: Obtain a copy of their presentation
  - __Overhead__: Obtain a copy or use the originals and scan them
  - __Blackboard__: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - __Handouts__: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack __with this completed sheet on top__ and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to __notes@msri.org__ with the workshop name and your name in the subject line.

N.Tronko [1] in collaboration with A.Bottino [1] C. Chandre [3],E.Lanti[2]
E.Sonnendrücker [1] and L.Villard [2]

EUROfusion

Noether theorem for magnetized plasmas

MSRI, Berkeley, USA
[1] **Max Planck Institute für Plasmaphysik**
[2] **EPFL-SPC** Lausanne, Swiss
[3] **CNRS and Aix-Marseille University**, France

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

Strongly magnetized plasmas

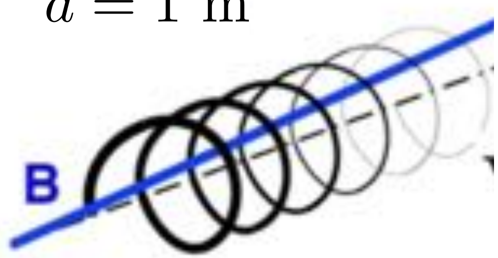
- **Plasma: 4th state of the matter:** hot gas, in which thermic motion is strong enough to separate ions and electrons interacting via EM fields

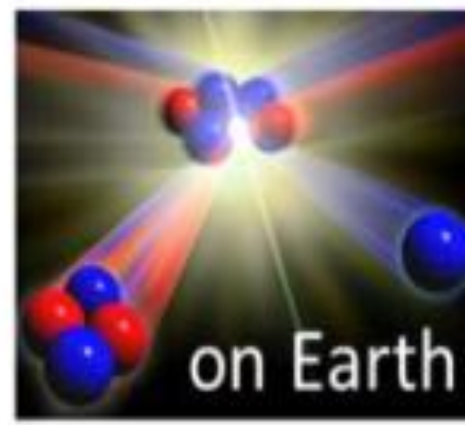
$p + p$

- **Strongly magnetized plasma:** charged particles rotates very fast around magnetic field lines: *cyclotronic motion*
- **Magnetically confined plasmas:** the gyration radius (ρ_L) is much smaller than the size of the system (a)

$\omega \approx 1\text{KHz}$ $\quad\quad$ $\Omega_{ci} = 95.7\text{MHz}$

$\rho_{Li} \approx 1\text{cm}$ $\quad\quad$ $a = 1\ \text{m}$





- **Fusion reaction:** release energy by creating from light Hydrogen isotops heavier elements

$D - T : He^4(3.5MeV) + n(14MeV)$

Natalia Tronko nataliat@ipp.mpg.de

Laboratory devices

Challenge: bring energy from the Sun to the Laboratory

- **New source of energy**
- **Goal:** self-sustained controlled fusion reaction

- **Variety of magnetic configurations**
  - Tokamak (toroidal geometry)
  - Stellerator (twisted magnetic field lines)

- **Challenge:** Multi-scaled, Multi-species dynamics in space and time governed by **turbulence: space-time chaos**

$$\frac{m_i}{m_e} = 2 * 1.83 * 10^3$$

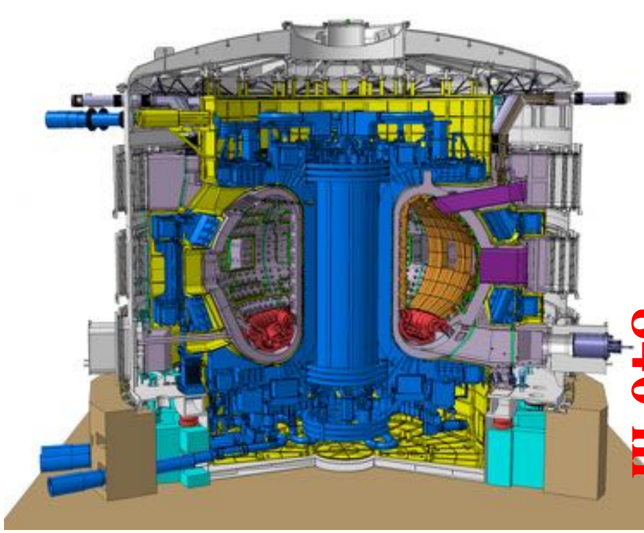
$$\frac{\rho_{Li}}{a} \approx 10^{-3}$$

$$\frac{\omega}{\Omega_{ci}} \approx 10^{-3}$$

$$\frac{\omega}{\Omega_{ce}} \approx 10^{-6}$$

Wendelstein 7 X, Greifswald, Germany

Plasma volume ~30 m^3



ITER, Cadarache, France

Plasma volume ~840 m^3

Natalia Tronko nataliat@ipp.mpg.de

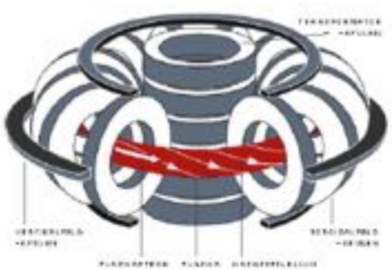
Fusion plasma technical challenge

- Magnetic field 10.000 stronger than on the Earth
- Plasma temperature 100 Millions degrees Celsius
- Requires ultra-robust costly materials

Ignition criterion: no external heating needed to maintain fusion reaction

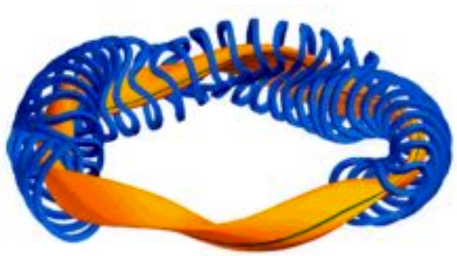
Temperature	10^8 degrees Celsius
Density	10^{20} m^{-3}
Energy Confinement time	2 sec

$$n\tau_E T \geq 2 \times 10^{28} m^{-3} s \ ^\circ\mathrm{C}$$



Toroidal magnetic configuration:
Tokamak JET in Culham UK

$$n\tau_E T \approx 0.4 \times 10^{28} m^{-3} s^\circ\mathrm{C}$$



Stellerator configuration:
Wendelstein 7X Greifswald,
Germany: **new record June 2018, Nature**

$$n\tau_E T \approx 0.03 \times 10^{28} m^{-3} s^\circ\mathrm{C}$$

Natalia Tronko nataliat@ipp.mpg.de

Sources of deconfinement

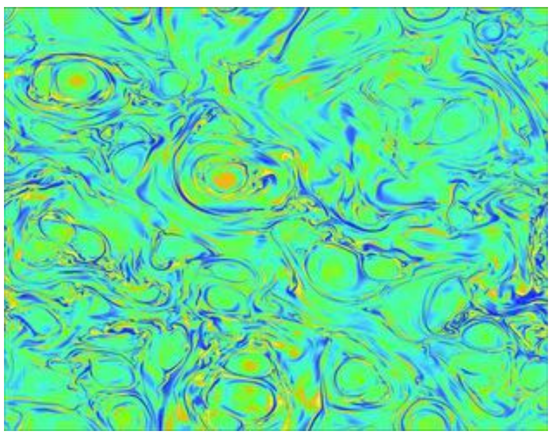
Free sources of energy
∇T
∇n

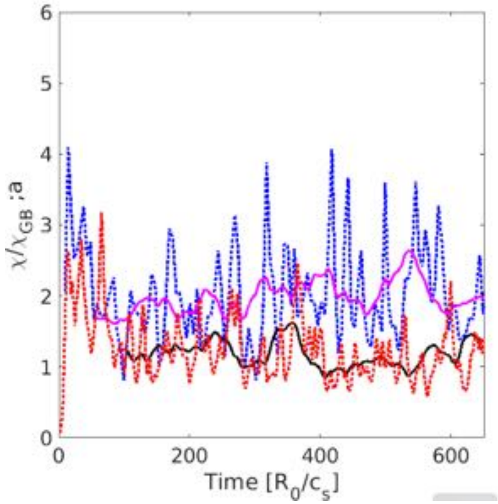
Microinstabilities (exponential growth of EM field fluctuations)

$\delta B \sim \delta B_0 \; e^{\gamma t} \qquad \delta E \sim \delta E_0 \; e^{\gamma t}$

Plasma turbulence: Low-frequency

$$\omega \ll \Omega_{ci} = \frac{q_i B}{m_i} = 9.571 \times 10^7 \; s^{-1}$$





Turbulent (anomaleous) transport

Plasma deconfinement

Improvement of experimental set up requires high quality numerical modeling

Natalia Tronko nataliat@ipp.mpg.de

Computational challenges

- Direct approach:

- Simulating 10^{23} particles interacting by mean of electromagnetic field

Technical requirements: 500 Milliards of TB of data storage=5* 10^{21} Bytes= 5 Million PetaByte:

  - 10 days of calculation on SUMMIT Top 500 of Supercomputers in the world (Oak Ridge National Lab)

Modeling Plasma Turbulence: realistic scenario

A model

- containing essential physical mechanisms driving turbulence
- **robust mathematical structure and conservation properties**

↓

Hamiltonian and Lagrangian description in order to control quality of numerical simulations **are essential**

Natalia Tronko nataliat@ipp.mpg.de

Vlasov-Maxwell Hamiltonian system

Replace a particle (**x**,**v**) by a probability density on the phase space f(**x**,**v**):

Kinetic description: essential for resonant field/particles interactions

Morrison 1980
Marsden Weinstein 1982

- **Phase space**

$$f(\mathbf{x}, \mathbf{v}, t)$$

$$\mathbf{E}(\mathbf{x}, t), \ \mathbf{B}(\mathbf{x}, t)$$

with constraints $\nabla \cdot \mathbf{B} = 0$

- Poisson equation $\nabla \cdot \mathbf{E} = 4\pi \sum_{\mathrm{sp}} \int \mathrm{d}^3\mathbf{v} f(\mathbf{x}, \mathbf{v}, t)$

- **Hamiltonian**

$$H[\mathbf{E}, \mathbf{B}, f] = \frac{1}{2} \sum_{\mathrm{sp}} \int \mathrm{d}^3\mathbf{x}\ \mathrm{d}^3\mathbf{v}\ f(\mathbf{x}, \mathbf{v}, t)\ m_{\mathrm{sp}} v_{\mathrm{sp}}^2 + \frac{1}{8\pi} \int \mathrm{d}^3\mathbf{x} \left(\mathbf{E}^2 + \mathbf{B}^2\right)$$

- **Non-canonical Poisson bracket**

$$[\mathsf{F}, \mathsf{G}] =$$

1) Particle bracket

$$\int \mathrm{d}^3\mathbf{x}\ \mathrm{d}^3\mathbf{v}\ f \left(\frac{\partial}{\partial \mathbf{x}} \frac{\delta \mathsf{F}}{\delta f} \cdot \frac{\partial}{\partial \mathbf{v}} \frac{\delta \mathsf{G}}{\delta f} - \frac{\partial}{\partial \mathbf{x}} \frac{\delta \mathsf{G}}{\delta f} \cdot \frac{\partial}{\partial \mathbf{v}} \frac{\delta \mathsf{F}}{\delta f} \right)$$

2) Field bracket

$$+ \int \mathrm{d}^3\mathbf{x} \left(\frac{\delta \mathsf{F}}{\delta \mathbf{E}} \cdot \nabla \times \frac{\delta \mathsf{G}}{\delta \mathbf{B}} - \frac{\delta \mathsf{G}}{\delta \mathbf{E}} \cdot \nabla \times \frac{\delta \mathsf{F}}{\delta \mathbf{B}} \right)$$

3) Coupling bracket

$$\int \mathrm{d}^3\mathbf{x}\ \mathrm{d}^3\mathbf{v} \left(\frac{\delta \mathsf{F}}{\delta \mathbf{E}} \cdot \frac{\partial f}{\partial \mathbf{v}} \frac{\delta \mathsf{G}}{\delta f} - \frac{\delta \mathsf{G}}{\delta \mathbf{E}} \cdot \frac{\partial f}{\partial \mathbf{v}} \frac{\delta \mathsf{F}}{\delta f} \right) + \int \mathrm{d}^3\mathbf{x}\ \mathrm{d}^3\mathbf{v}\ f\ \mathbf{B} \cdot \left(\frac{\partial}{\partial \mathbf{v}} \frac{\delta \mathsf{F}}{\delta f} \times \frac{\partial}{\partial \mathbf{v}} \frac{\delta \mathsf{G}}{\delta f} \right)$$

Natalia Tronko nataliat@ipp.mpg.de

Vlasov-Maxwell Hamiltonian system

- **Equations of motion (for one of the species)**

$$\frac{\mathrm{d}\mathbf{E}}{\mathrm{d}t} = [H, \mathbf{E}] = \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{E}$$

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = [H, \mathbf{B}] = -\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{B} - \frac{4\pi e}{c}\int \mathrm{d}^3\mathbf{v}\ \mathbf{v}\ f(\mathbf{x},\mathbf{v},t)$$

$$\frac{\mathrm{d}f}{\mathrm{d}t} = [H, f] = \frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{v}} + \left(\mathbf{E} + \frac{e}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f}{\partial \mathbf{x}}$$

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\boldsymbol{\nabla} \times \mathbf{E}$$

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \boldsymbol{\nabla} \times \mathbf{B} - \frac{4\pi e}{c}\int \mathrm{d}^3\mathbf{v}\ \mathbf{v}\ f(\mathbf{x},\mathbf{v},t)$$

$$\frac{\partial f}{\partial t} = -\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{v}} - \left(\mathbf{E} + \frac{e}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f}{\partial \mathbf{x}}$$

- **Multi-species challenge**

Electrons are 1.83×10^3 lighter than ions!

→ Adiabatic limit $m_i \to \infty$

Electrons modelized by fluid

$$dt_{\mathrm{kin}} \sim \sqrt{\frac{m_e}{m_i}} \sim \frac{1}{60}\ dt_{\mathrm{adiab}}$$

Removes imporant physics from the system

Natalia Tronko nataliat@ipp.mpg.de

Eulerian and Lagrangian approaches for kinetic simulations

Lagrangian code Particle-In-Cell

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{v}$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{E} + \frac{e}{c}\mathbf{v} \times \mathbf{B}$$

Eulerian code, grid-based

$$\frac{\partial f}{\partial t} + \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

- Reconstruct Vlasov dynamics from the particle characteristics
- Fields: treated on the grid: final elements

- Macro-Particles in the phase-space

- **Noise issue: need 10^6 markers at least!**

- Grid approach: direct discretisation of the distribution function f together with fields

- **CFL limit of the time step and space resolutions** : limiting numerical configurations

$$C = \frac{u\Delta t}{\Delta x} \leq 1$$

Natalia Tronko nataliat@ipp.mpg.de

Difficulties of kinetic simulations

- **The Vlasov-Maxwell model is well known but still be unsuitable for realistic numerical simulations**

- Storage problem for 6D distribution function:

  - 1 point in time 2,5 GB in **6D** (**x**,**v**): (150x64x16)x(16x64x16)
  - Realistic simulation with kinetic electrons: ω_{ei}=1.75x10^{11} sec^{-1}
  - TCV energy confinement time τ_E= 2x10^{-2} sec will require Ntimes_steps= 3,5 x10^9
  - 800*10^6 TB of storage
  - Space available on Supercomputer Marconi: 1TB pro Project!

- Computational resources**: time resolution is limited by cyclotron frequency**
  space resolution is limited by Debye length 10^{-4} m!

- **Reduction of kinetic model :**
- Adapting dynamic coordinates with respect to physical properties of turbulence
- Store only energy and other moments of the distribution function

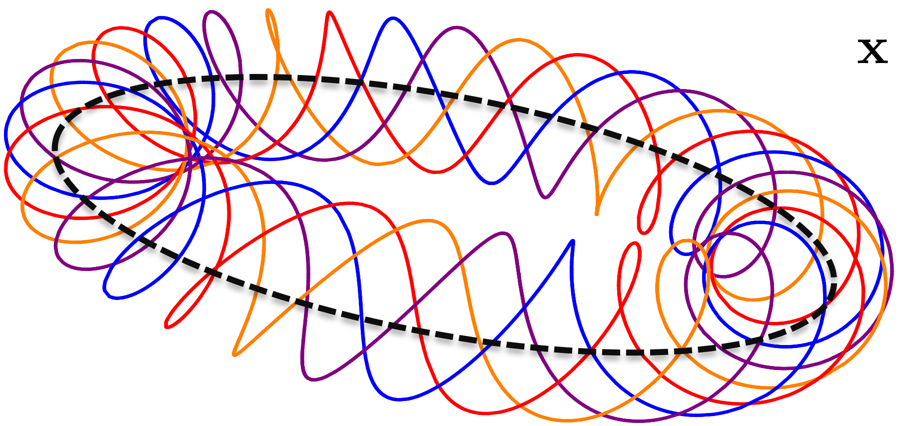
Natalia Tronko nataliat@ipp.mpg.de

What is Gyrokinetic theory?

Idea: Use physics as a guidance for low frequency Maxwell-Vlasov dynamical reduction

$$\epsilon_\omega = \frac{\omega}{\Omega_{ci}} \sim 10^{-3}$$

1. Replacing particle with position x by the guiding-center: instantaneous center of rotation X around magnetic field lines



$$\mathbf{x} = \mathbf{X} + \boldsymbol{\rho}_0$$

6D ⟶ 5D (4D+1)

$$f(\mathbf{x}, \mathbf{v}) \to f(\mathbf{X}, v_\parallel, \mu)$$

2. Scales of motion separation: use existence of fast and slow variables

Systematically eliminate fastest scale of motion irrelevant for turbulent transport: increasing dt by 1000!

- Magnetic Moment: ***adiabatic invariant*** $\mu = \dfrac{m v_\perp^2}{2B}$
- Gyroangle ***: fast angle*** θ

Natalia Tronko nataliat@ipp.mpg.de

Gyrokinetic dynamical reduction

A systematic dynamical reduction procedure such that at each step

$\dot{\mu} = 0$	Has a trivial dynamics	6D → 4D+1
θ	Is uncoupled	$f(\mathbf{x}, \mathbf{v}) \to f(\mathbf{X}, v_{\parallel}, \mu)$

Simple gyroaveraging leads to loss of important information: resonant interaction between fields and particles

Goal: Invertible near identity change of coordinates

Range of small parameters raising from several aspects: geometry, physics of turbulent motion: **multi-scaled asymptotic theory**

Goal: two step

- Systematic asymptotic procedure for dynamical reduction on the particle phase space → **Hamiltonian approach**
- Systematic coupling of the reduced particle dynamics with fields → **Lagrangian approach**

Natalia Tronko nataliat@ipp.mpg.de

Costs of Gyrokinetic simulations

The GK codes require HPC platforms to get results in a reasonable amount of time

1 node-hour ≈ 0.4 CHF ≈ 0.4 USD

- EUROfusion projects: Marconi #18 in the world

- *HPC Budgets*
- **2018 "GKICK"** 850 000 node hours

- **2015-2017 "VeriGyro"** 1 280 000 node hours

Type of simulation	Node-hours pro run	Restarts (every 24 hours)	Time step in $1/\Omega_{ci}$	Required Storage
Adiabatic electrons	200	0	dt =50	1 GB
Linear with kinetic electrons	780	1	dt= 1	5 GB
Nonlinear with kinetic electrons	14400	2	dt =0.25	300 GB

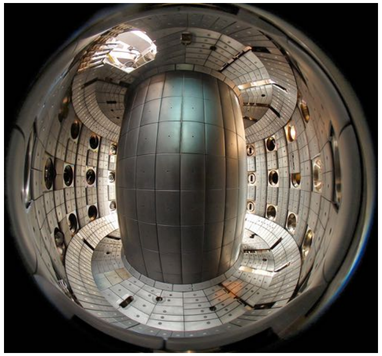
Investing in data storage and backups is important!

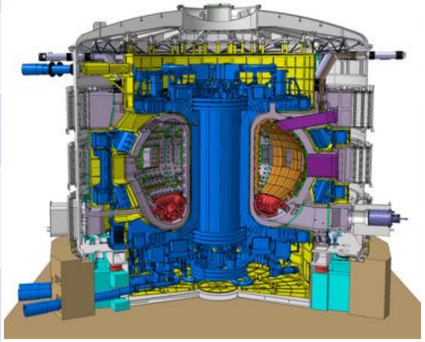
Natalia Tronko nataliat@ipp.mpg.de

Costs of experiments

- **1 shot of TCV Tokamak in SCP Lausanne costs 1000 CHF**
- **1 shot of ITER is estmated 1 000 000 CHF**

	TCV	ITER
Major radius	1.54 m	6.2 m
Minor Radius	0.56 m	2.0 m
B_{Tor}	1.54 T	5 T
n	$20*10^{20}$ m^{-3}	$10*10^{20}$ m^{-3}
T_i	$\leq$ 1KeV	8.0 KeV
T_e		
Discharge time	2.6s	400s
Plasma Heating	1 MW	40 MW
Energy gain	no	yes

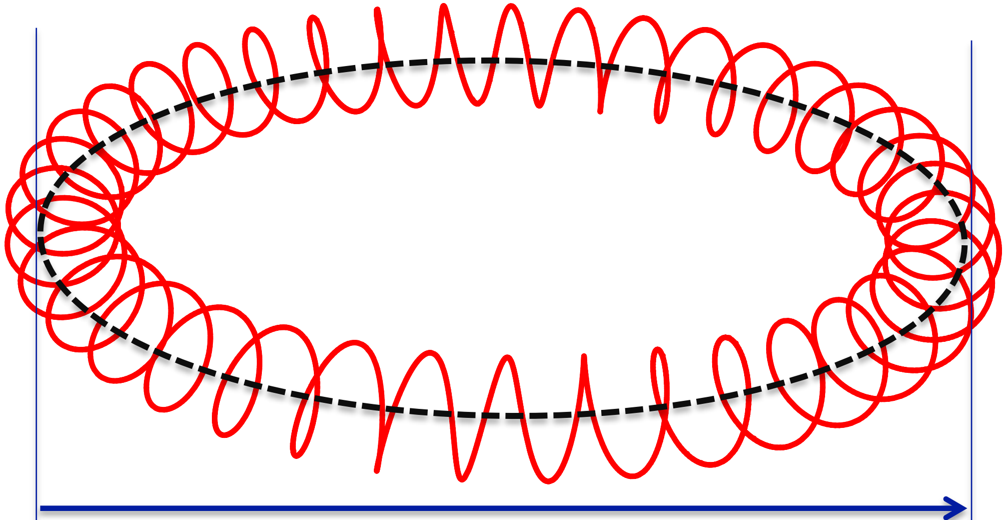




Developing trustable and robust mathematical modeling is essential for success of magnetic fusion

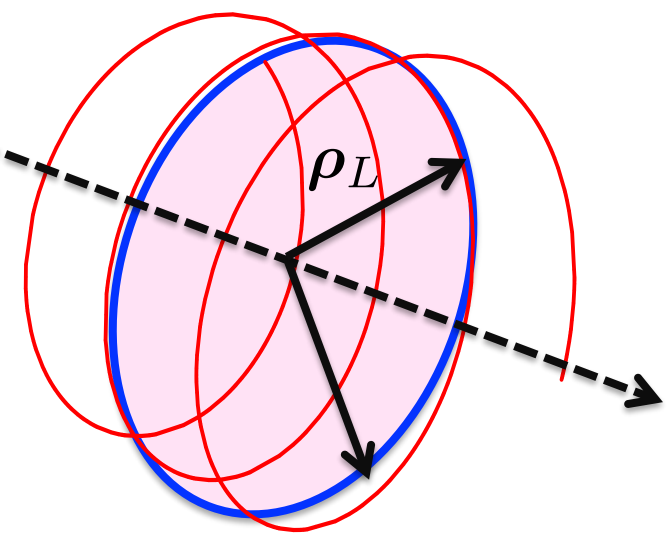
Natalia Tronko nataliat@ipp.mpg.de

Small parameters: 1) Magnetic curvature



$$L_B = \left| \frac{\nabla B}{B} \right| \approx 1m$$

Separation of scales of motion



$$\rho_L \approx 10^{-3} m$$

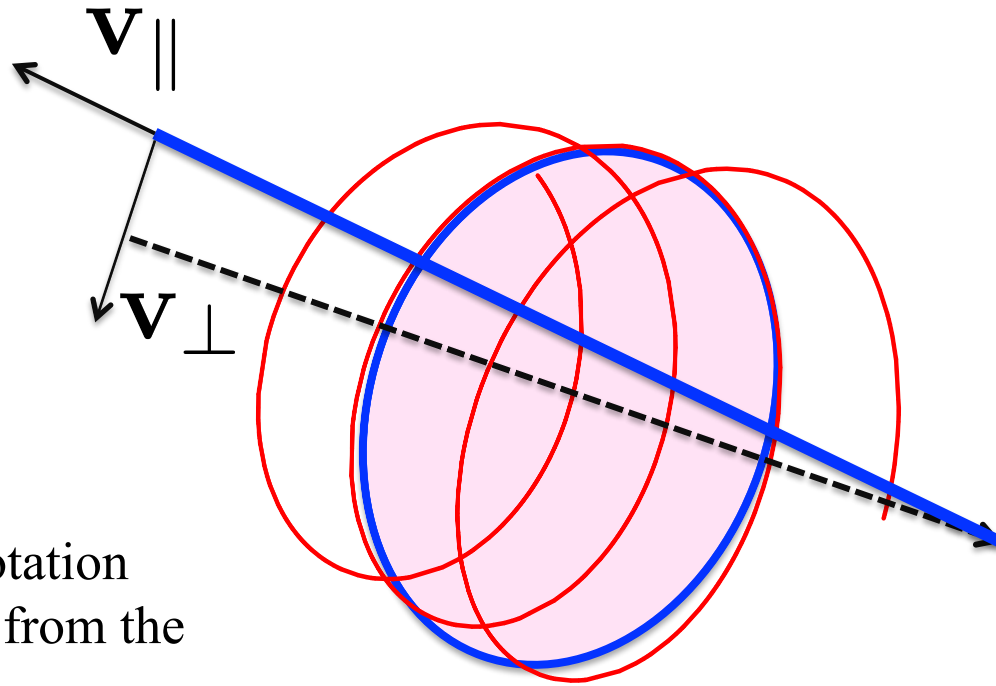
Small parameter

$$\epsilon_B = \rho_L \left| \frac{\boldsymbol{\nabla} B}{B} \right|$$

Natalia Tronko nataliat@ipp.mpg.de

Small parameters : 2) Anisotropy of turbulence

Plasma turbulence : perpendicular to magnetic field lines



$$\frac{E_{1\parallel}}{E_{1\perp}} \ll 1$$

$$\frac{B_{1\parallel}}{B_{1\perp}} \ll 1$$

$$\epsilon_{\parallel} = \frac{k_{\parallel}}{k_{\perp}} \ll 1$$

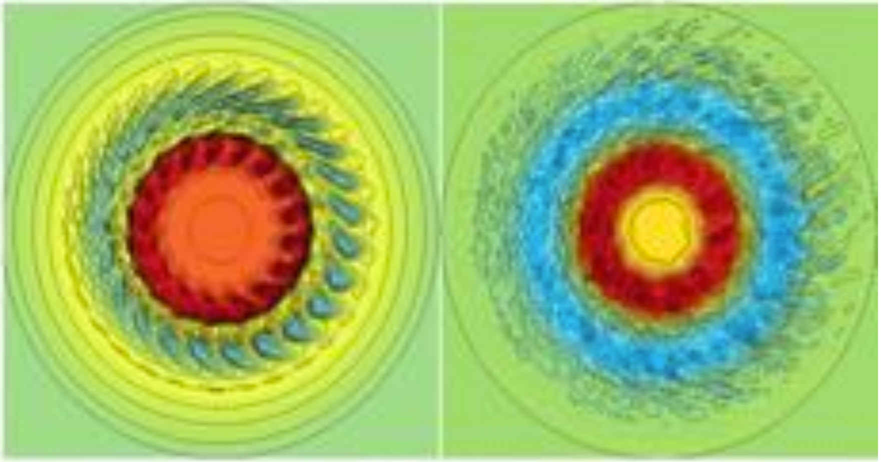
The center of instantaneous rotation (guiding-center) slowly drifts from the magnetic field: due to

- Magnetic field curvature
- Fluctuations of electromagnetic fields (when considered)

- Parallel ion velocity $\frac{v_{i\parallel}}{v_{i\perp}} \approx 100$
- Parallel electron velocity $\frac{v_{e\parallel}}{v_{e\perp}} \approx 10^4$

Natalia Tronko nataliat@ipp.mpg.de

Small parameters : 2) Anisotropy of turbulence



Lagrangian simulations with ORB5 code by L. Villard, SCP Lausanne

- Fluctuations of electrostatic potential: early (left) and late (right) stage of turbulence.
- Development of short wavelength perturbations with respect to the size of the tokamak
- 3D vue: elongation of perturbations along the magnetic field lines

$$k_\perp \rho_i \sim 1$$

Turbulent structures are of the Larmor radius size: small scales need to be solved

$$\epsilon_\delta = k_\perp \rho_i \frac{e\delta\phi}{T_i}$$

Typical parameter to characterize turbulent fluctuations

Natalia Tronko nataliat@ipp.mpg.de

GK Orderings

- **Guiding- center:** background quantities: $\epsilon_B = \rho_0 \, |\nabla B/B|$
- **Gyrocenter:** fluctuating fields:

$$\epsilon_\delta = (k_\perp \rho_i)\frac{e\delta\phi}{T_i}$$

$$\epsilon_\omega = \frac{\omega}{\Omega_{\rm ci}}$$

$$\epsilon_\parallel = k_\parallel / k_\perp \ll 1$$

$$\left.\right\} \epsilon_{||} \sim \epsilon_\omega \sim \epsilon_\delta$$

- Anisotropy of turbulence

- **Ordering defines physics: There is NO unique gyrokinetic model**

  - Gyrokinetics

    $$k_\perp \rho_i \sim 1$$

  - Drift-kinetics

    $$k_\perp \rho_i \ll 1$$

  - Maximal ordering

    $$\epsilon_B \sim \epsilon_\delta$$

  - Code ordering

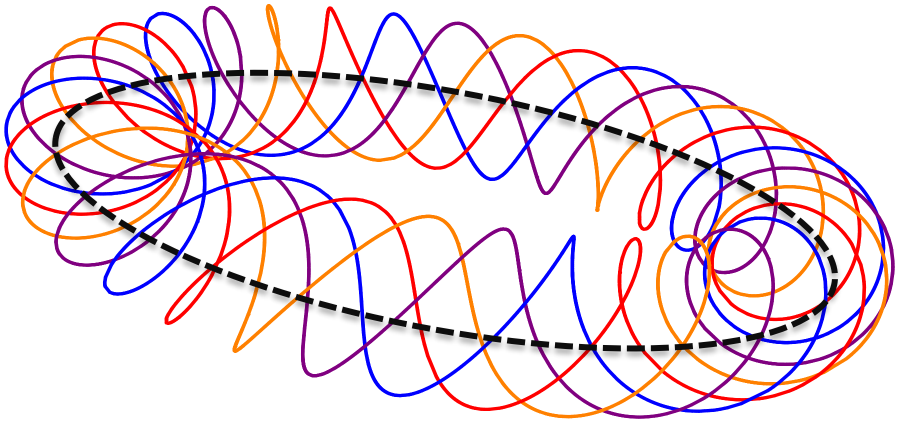
    $$\epsilon_B \ll \epsilon_\delta$$

***[Tronko, Chandre,** J.Plasma Phys., 2018]* $\epsilon_B = \epsilon_\delta^2$

***[Brizard, Hahm** Rev.Mod.Phys., 2007]* $\epsilon_B = \epsilon_\delta^{3/2}$

Natalia Tronko nataliat@ipp.mpg.de

Phase space Lagrangian formalism



- **Charged particle in external magnetic field** $\mathbf{B} = \nabla \times \mathbf{A}$

$$\gamma = L_p \; \mathrm{d}t$$

- Phase-space one form

$$L_p = \left(m\mathbf{v} + \frac{e}{c}\mathbf{A}(\mathbf{x}) \right) \cdot \dot{\mathbf{x}} - H(\mathbf{x}, \mathbf{v})$$

$$H = \frac{1}{2m} |\mathbf{v}|^2$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L_p}{\partial \dot{\mathbf{v}}} = \frac{\partial L_p}{\partial \mathbf{v}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L_p}{\partial \dot{\mathbf{x}}} = \frac{\partial L_p}{\partial \mathbf{x}}$$

$$m\dot{\mathbf{v}} = \frac{e}{c}\mathbf{v} \times \mathbf{B}$$

- Lorenz force

$$\mathbf{v} = \dot{\mathbf{x}}$$

- Lagrangian constraint

- **Starting point of gyrokientic reduction**

Natalia Tronko nataliat@ipp.mpg.de

Gyrocenter Lagrangian: GENE & ORB5

No θ dependency:

$$L_p = \left(\frac{e}{c}\mathbf{A} + \left(\frac{e}{c}\epsilon_\delta A_{1\|} + mv_\|\right)\widehat{\mathbf{b}}\right)\cdot\dot{\mathbf{X}} + \frac{mc}{e}\mu\dot{\theta} - H$$

- **Parallel Symplectic representation: GENE**

$$p_\| = mv_\| \qquad \mathbf{Z} = \left(\mathbf{X}, p_\|, \mu, \theta\right)$$

- Symplectic form: time dependent

$$\mathbf{B}^{**} = \boldsymbol{\nabla}\times\left(\mathbf{A} + \left[\epsilon_\delta A_{1\|} + \frac{c}{e}p_\|\right]\widehat{\mathbf{b}}\right)$$

- Characteristics with $\dfrac{\partial A_{1\|}}{\partial t}$

Dynamical reduction procedure
[Littlejohn 1983, Brizard 1989, Tronko&Chandre 2018]

- **Hamiltonian representation: ORB5**

$$p_z = mv_\| + \frac{e}{c}\epsilon_\delta A_{1\|} \qquad \mathbf{Z} = \left(\mathbf{X}, p_z, \mu, \theta\right)$$

- Symplectic form: time independent

$$\mathbf{B}^{*} = \boldsymbol{\nabla}\times\left(\mathbf{A} + \frac{c}{e}p_z\widehat{\mathbf{b}}\right)$$

[APS invited: Tronko, Bottino, Görler, Sonnendrücker, Told, Villard, PoP 2017]

Natalia Tronko nataliat@ipp.mpg.de

Hamiltonian hierarchy: Theory & ORB5

- **Hamiltonian model defines polarization and magnetization in the field equations**
- **Any approximated model can be used: Padé, adiabatic electrons**

$$H = H_0 + \epsilon_\delta H_1 + \epsilon_\delta^2 H_2$$

$$H_0^{Orb5} = \frac{p_z^2}{2m} + \mu B$$

$$H_1^{Orb5} = -e\mathcal{J}_0^{\rm gc}\left(\phi_1 - \frac{p_z}{m}A_{1\|}\right)$$

- **Theory:** Hamiltonian **correspondance to Hahm's 1988 electrostatic model**

$$H_2^{Theory} = \frac{e^2}{2mc^2}\mathcal{J}_0^{\rm gc}\left(A_{1\|}\left(\mathbf{X}+\boldsymbol{\rho}_0\right)^2\right) - \frac{e^2}{2B}\mathcal{J}_0^{\rm gc}\left(\frac{\partial}{\partial\mu}\phi_1(\mathbf{X}+\boldsymbol{\rho}_0) - \frac{p_z}{m}A_{1\|}(\mathbf{X}+\boldsymbol{\rho}_0)\right)$$

Electromagnetic coupling between GK Poisson and Ampère equations

- **ORB5 semi-electromagnetic**

$$H_2^{Orb5} = \frac{e^2}{2mc^2}A_{1\|}(\mathbf{X})^2 + \frac{\mu}{2B}\left|\boldsymbol{\nabla}_\perp A_{1\|}\right|^2 + \frac{1}{2}\frac{\mu}{B}A_{1\|}\boldsymbol{\nabla}_\perp^2 A_{1\|}(\mathbf{X}) - \frac{e^2}{2B}\mathcal{J}_0^{\rm gc}\left(\frac{\partial}{\partial\mu}\phi_1(\mathbf{X}+\boldsymbol{\rho}_0)^2\right)$$

Uncoupled GK Poisson and Ampère equations

Natalia Tronko nataliat@ipp.mpg.de

Gyrokinetic field theory: concept

Particles(GK) dynamical reduction
Goal: remove fastest scale of motion

[Littlejohn 1983] — Gauge transformation & Canonical Lie-Transform

Guiding-center

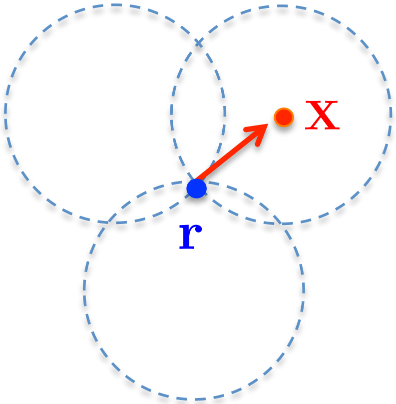
[Brizard 1989] — Canonical Lie-Transform

Gyrocenter

Variational principle:

Coupling fields & gyrocenters

Reduced GK Vlasov+ GK Maxwell equations



Bonus: Noether theorem for energy conservation diagnostics

- **Polarization effects: fields and particles are not evaluated at the same position anymore: polarization shifts in GK Ampére and Poisson equations**

Field theory guarantees consistency

Natalia Tronko nataliat@ipp.mpg.de

Gyrokinetic field theory: GENE & ORB5

- **Common framework for code models derivation:** *[**Sugama** Phys. Pl. 2000, **Brizard** PRL 2000]*

$$\mathcal{L} = \sum_{s} \int d\Omega \; f(\mathbf{Z}_0, t_0) L_p \left(\mathbf{Z}[\mathbf{Z}_0, t_0], \dot{\mathbf{Z}}[\mathbf{Z}_0, t_0]; t \right) + \int dV \frac{|\mathbf{E}_1|^2 - |\mathbf{B}_1|^2}{8\pi}$$

- Phase-space volume $d\Omega = dV\,dW$
- **Field terms: option to couple with fluid model**

- Time-dependent: **GENE**

$$\mathbf{Z} = \left(\mathbf{X}, p_{\|}, \mu, \theta\right); dW = \frac{2\pi}{m^2} B_{\|}^{**} dp_{\|} d\mu$$

- Time-independent: **ORB5**

$$\mathbf{Z} = \left(\mathbf{X}, p_z, \mu, \theta\right); dW = \frac{2\pi}{m^2} B_{\|}^{*} dp_z d\mu$$

Goal: Coupling reduced particle dynamics with fields within the common mathematical structure

↓

Getting consistently reduced set of Maxwell-Vlasov equations

$f(\mathbf{Z}_0, t_0)$ Distribution function of species "sp" at arbitrary initial time t_0

L_p Gyrocenter Lagrangian: reduced motion of a single particle

Natalia Tronko nataliat@ipp.mpg.de

Lagrangian formulation of GK for ORB5

*[**Tronko et al.** Phys. Pl. 2016]*

- The expression for action principle corresponding to Orb5 code model

$$\mathcal{L} = \sum_{\mathrm{s}} \int d\Omega \left(e\mathbf{A}^* \cdot \dot{\mathbf{X}} + \frac{e}{c}\mu\dot{\theta} - (H_0 + \epsilon_\delta H_1) \right) f - \epsilon_\delta^2 \sum_{\mathrm{s}\neq e} \int d\Omega \; H_2 f_0 - \epsilon_\delta^2 \int dV \; \frac{\left|\boldsymbol{\nabla}_\perp A_{1\|}\right|^2}{8\pi}$$

$$\mathbf{Z} = (\mathbf{X}, p_z, \mu, \theta)\,; dW = \frac{2\pi}{m^2} B_\|^* dp_z d\mu$$

Further approximations:

- Quasineutrality- eliminating perturbed electric field
- Low magnetic pressure $\mathbf{B} = \widehat{\mathbf{b}} \times \boldsymbol{\nabla} A_{1\|}$
- δf-approximation $f = f_0 + \epsilon_\delta \delta f$

Natalia Tronko nataliat@ipp.mpg.de

Linearized Uncoupled GK Poisson and Ampère equations: ORB5

- **Polarization equation**

$$\frac{\delta L}{\delta \phi_1} \circ \phi_1 = 0 \quad \Rightarrow$$

$$\sum_{\mathrm{s}} \int \mathrm{d}\Omega \; f \; q_s \; \mathcal{J}_0^{\mathrm{gc}}(\phi_1) = \epsilon_\delta \sum_s \int \mathrm{d}\Omega \; f_C \; \frac{q_s^2}{B m_s} \frac{\partial}{\partial \mu} \left(\mathcal{J}_0^{\mathrm{gc}}(\phi_1^2) - \left[\mathcal{J}_0^{\mathrm{gc}}(\phi_1)\right]^2 \right)$$

- **Ampère's equation**

$$\frac{\delta L}{\delta A_{1\|}} \circ A_{1\|} = 0 \quad \Rightarrow$$

$$\epsilon_\delta \int \frac{dV}{4\pi} \left| \nabla_\perp A_{1\|} \right|^2 = \sum_{\mathrm{s}} \int \mathrm{d}\Omega \; f \; \frac{p_z}{m_s} \mathcal{J}_0^{\mathrm{gc}}(A_{1\|})$$

$$- \sum_{\mathrm{s} \neq e} \epsilon_\delta \int \mathrm{d}\Omega \; f_C \left(\frac{q_s^2}{m_s} A_{1\|}^2 + \frac{m_s \mu}{B} \left[A_{1\|} \nabla_\perp^2 A_{1\|} + A_{1\|} \nabla_\perp^2 A_{1\|} \right] \right)$$

Natalia Tronko nataliat@ipp.mpg.de

GK Vlasov equation: ORB5

- **Vlasov equation is reconstructed from the characteristics**

$$\frac{\delta L}{\delta \mathbf{Z}} = 0$$

$$\mathbf{Z} = (\mathbf{X}, p_z, \mu)$$

$$\dot{\mathbf{X}} = \frac{\partial (H_0 + \epsilon_\delta H_1)}{\partial p_z} \frac{\mathbf{B}^*}{B_\parallel^*} - \frac{c}{eB_\parallel^*} \widehat{\mathbf{b}} \times \boldsymbol{\nabla}(H_0 + \epsilon_\delta H_1)$$

$$\dot{p}_z = -\frac{\mathbf{B}^*}{B_\parallel^*} \cdot \boldsymbol{\nabla} (H_0 + \epsilon_\delta H_1)$$

$$\frac{d}{dt} f\left(\mathbf{Z}[\mathbf{Z}_0, t_0, t]; t\right) = \frac{\partial}{\partial t} f\left(\mathbf{Z}, t\right) + \frac{d\mathbf{Z}}{dt} \cdot \frac{\partial}{\partial \mathbf{Z}} f(\mathbf{Z}, t)$$

- **δf model requiers first order characteristics : only H_0 and H_1**
- **Full-f (nonlinear) model requires H_2 contributions in the characteristics**

Natalia Tronko nataliat@ipp.mpg.de

Noether's method

Time invariance ⟺ Energy conservation

$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}t} = \frac{\delta\mathcal{L}}{\delta\mathbf{X}}\cdot\dot{\mathbf{X}} + \frac{\delta\mathcal{L}}{\delta p_z}\dot{p_z} + \frac{\delta\mathcal{L}}{\delta\theta}\dot{\theta} + \frac{\delta\mathcal{L}}{\delta\mu}\dot{\mu} + \underbrace{\frac{\delta\mathcal{L}}{\delta\phi_1}\cdot\frac{\partial\phi_1}{\partial t} + \frac{\delta\mathcal{L}}{\delta A_{1\|}}\cdot\frac{\partial A_{1\|}}{\partial t}}_{\text{GK field equations}}$$

Gyrocenter characteristics $\mathbf{Z} = (\mathbf{X}, p_z, \mu, \theta)$

$$\frac{\delta\mathcal{L}}{\delta\mathbf{Z}}\cdot\widehat{\mathbf{Z}} = \int dV\ dW\left(\frac{\partial L}{\partial\mathbf{Z}}\cdot\widehat{\mathbf{Z}} - \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\mathbf{Z}}}\cdot\widehat{\mathbf{Z}} + \frac{d}{dt}\left[\frac{\partial\mathcal{L}}{\partial\dot{\mathbf{Z}}}\cdot\widehat{\mathbf{Z}}\right]\right)f = \int dV\ dW\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\mathbf{Z}}}\cdot\widehat{\mathbf{Z}}\right)$$

⬇

$$\frac{d}{dt}\left(\mathcal{L} - \int dV\ \ dW\left[\frac{\partial\mathcal{L}}{\partial\dot{\mathbf{X}}}\cdot\widehat{\mathbf{X}} - \frac{\partial\mathcal{L}}{\partial\dot{\theta}}\cdot\widehat{\theta}\right]f\right) = \frac{d}{dt}\mathcal{E}^{\mathrm{EM}} = 0,$$

Natalia Tronko nataliat@ipp.mpg.de

Universal energy diagnostics

$$\mathcal{E}^{\mathrm{EM}} = \sum_{\mathrm{s}} \int dV\ dW\ \left(H_0 + \epsilon_\delta H_1\right)\ f + \epsilon_\delta^2 \sum_{\mathrm{s}} \int dV\ dW\ H_2\ f_C - \epsilon_\delta^2 \sum_{\mathrm{s}\neq e} \int\ dt\ dV dW\ H_2 f_C$$

$$-\epsilon_\delta^2 \int\ dt\ dV\ \frac{\left|\boldsymbol{\nabla}_\perp A_{1\|}\right|^2}{8\pi}$$

- Simplifying with making use of the Euler-Lagrange (GK Poisson and GK Ampere) equations

$$\mathcal{E}^{\mathrm{EM}} = \frac{1}{2} \sum_{\mathrm{s}} q_s \int dV\ dW\ \left(\mathcal{J}_0^{\mathrm{gc}}\left(\phi_1\right) - \frac{1}{c}\frac{p_z}{m_s}\mathcal{J}_0^{\mathrm{gc}}\left(A_{1\|}\right)\right) f + \sum_{\mathrm{s}} \epsilon_\delta \int dV\ dW\ f\left(\frac{p_z^2}{2m_s} + \mu B\right)$$

Universal for all nonlinear electromagnetic models

Natalia Tronko nataliat@ipp.mpg.de

Numerical advantages of a consistent theory derivation

ORB5

- Clarifying connections to fundamental GK derivation from the variational principle

- **New** understanding of electromagnetic microinstabilities; differencies with electrostatic case

Noether theorem

Energy conservation

↔

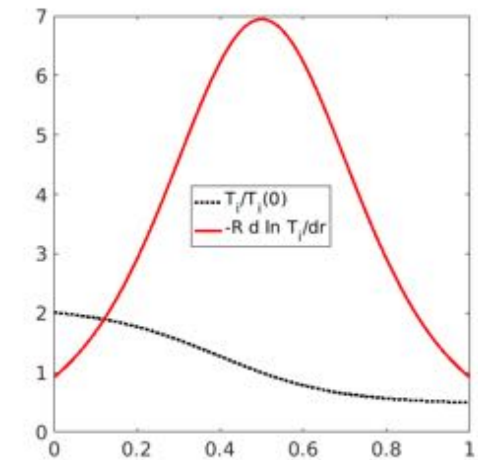
Controlling the quality of simulations

Accessing instabilities mechanisms

Natalia Tronko nataliat@ipp.mpg.de

Cyclone Base Case

- Common framework for benchmark: *[**Dimits**, Phys. Pl. 2000]*



electrostatic simulations, adiabatic electrons

- The original discharge DIII-D:

H- mode shot #81499 at t=4000 ms;

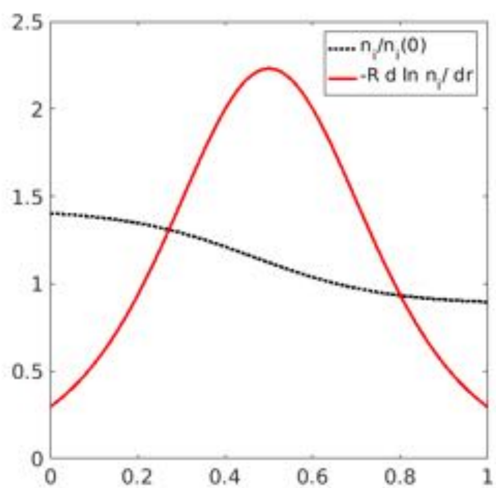
flux tube label r=0.5a

$$q(r) = 0.86 - 0.16(r/a) + 2.52(r/a)^2$$

$$A(r) = A(r_0)\exp\left[-\kappa_A a \Delta A \tanh\left(\frac{r-r_0}{\Delta A a}\right)\right]$$

$$\Delta T_i = \Delta n = 0.3$$

$$\kappa_{T_i} = 6.96 \qquad \kappa_n = 2.23 \qquad T_e/T_i = 1$$



Natalia Tronko nataliat@ipp.mpg.de

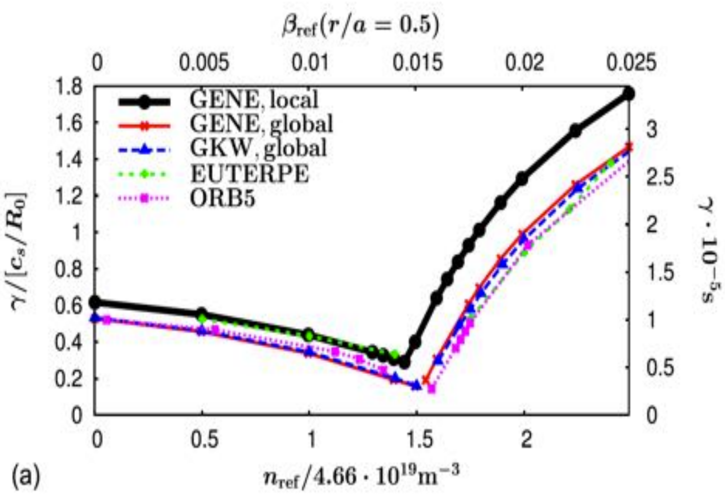
Linear electromagnetic β-scan: 5 codes

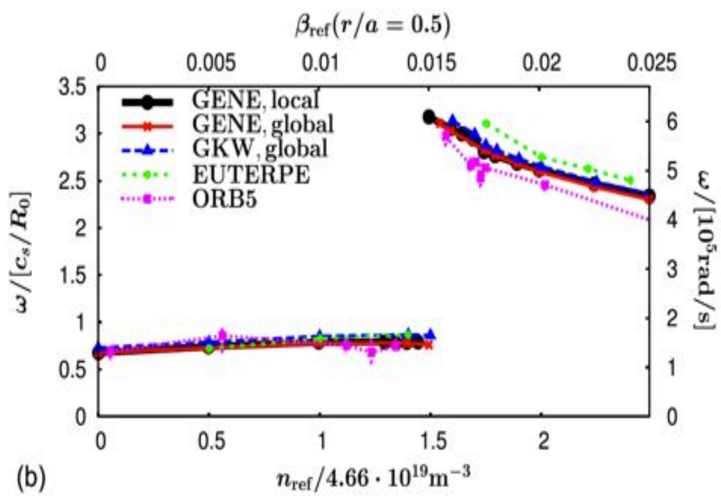
[Goerler, Tronko, Hornsby et al, PoP 2016]

- Looking at one of the most unstable modes **n=19**
  - Successful comparison of 4 different codes (2xPIC and 2xEulerian)
  - All codes agree at the ITG/KBM transition
  - Threshold shifted comparing to flux-tube growth rate

Important for experiment

- ***Growth Rate and Frequency scan***





- **ORB5 code**

$$\mathrm{nptot}_{\mathrm{De}} = 8 \times 10^6$$

$$\mathrm{nptot}_{\mathrm{e}} = 16 \times 10^6$$

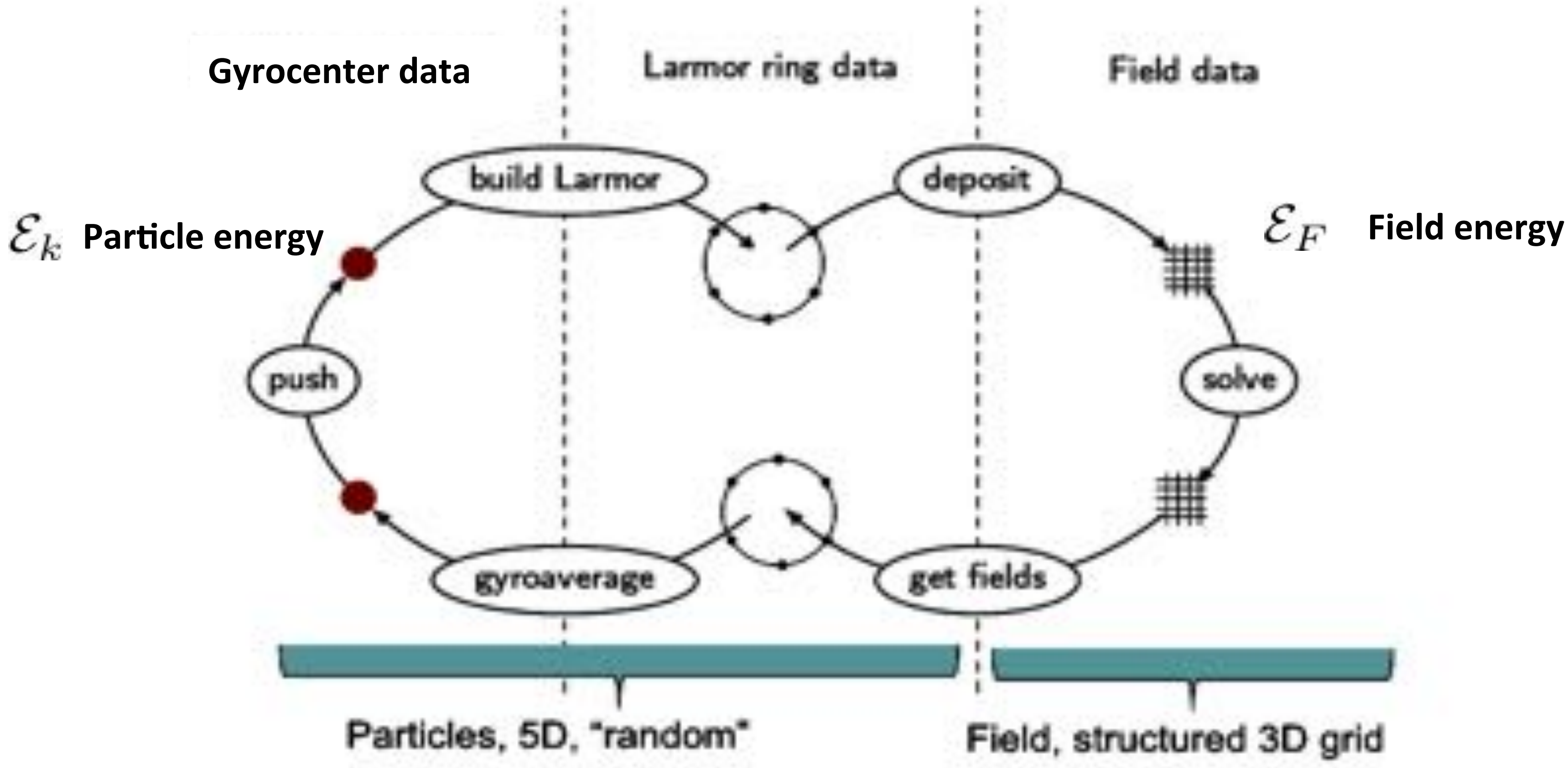
4 global codes 1 local (GENE)

Size of the system

$$\rho_* = \frac{\rho_L}{a} = \frac{1}{180}$$

Natalia Tronko nataliat@ipp.mpg.de

Particle-In-Cell code concept: ORB5



Use mathematical tools to verify quality of the simulations: field-particle energy balance

$$\frac{d\mathcal{E}}{dt} = 0 \Rightarrow \frac{d\mathcal{E}_k}{dt} = -\frac{d\mathcal{E}_F}{dt}$$

Natalia Tronko nataliat@ipp.mpg.de

Electromagnetic Powerbalance ORB5: for quality control

- **Noether theorem**

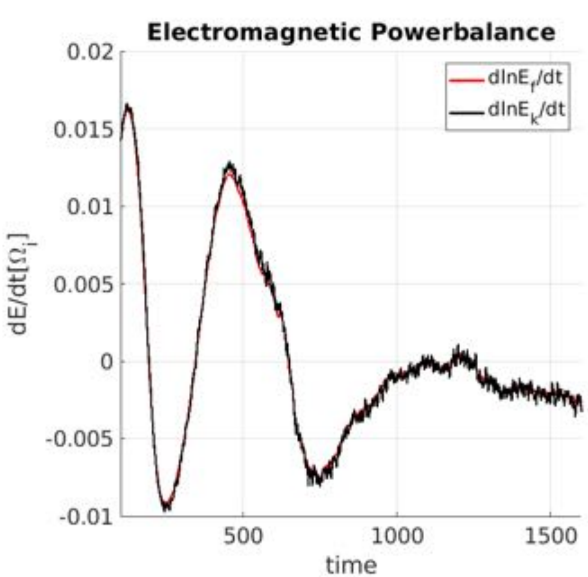
$$\mathcal{E}_k = \sum_s \int \mathrm{d}\Omega\ f \left(\frac{p_z^2}{2m_s} + \mu B \right)$$ Particles energy

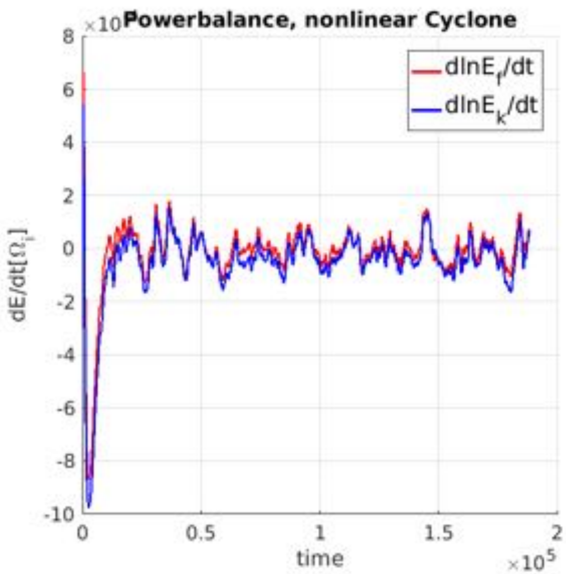
$$\mathcal{E} = \mathcal{E}_k + \mathcal{E}_F$$

$$\mathcal{E}_F = \frac{1}{2} \sum_s \int \mathrm{d}\Omega\ f \left(\phi_1 - \frac{e p_z}{mc} A_{1\|} \right)$$ Fields energy

- **Verification of energy conservation in the simulations**

$$\frac{d \ln \mathcal{E}_k}{dt} = - \frac{d \ln \mathcal{E}_F}{dt}$$





Natalia Tronko nataliat@ipp.mpg.de

E_F diagnostics: How much “electromagnetic” is the instability ?

$$\mathcal{E}_F = \frac{1}{2}\sum_{\mathrm{s}}\int \mathrm{d}\Omega\; f\left(\phi_1 - \frac{ep_z}{mc}A_{1\|}\right) \qquad \equiv E_{\mathrm{F;ES}} - E_{\mathrm{F;EM}}$$

- Electrostatic component of field energy:independent of β

$$E_{\mathrm{F;ES}} = \frac{1}{2}\sum_{\mathrm{s}}\int \mathrm{d}\Omega\; f\;\phi_1$$

- Electromagnetic component of field energy: **depending on β**

$$E_{\mathrm{F;EM}} = \frac{1}{2}\sum_{\mathrm{s}}\int \mathrm{d}\Omega\; f\;\frac{ep_z}{mc}A_{1\|}$$

$$\Delta E_{\mathrm{F}} = \frac{E_{\mathrm{F;ES}} - E_{\mathrm{F;EM}}}{E_{\mathrm{F;ES}}}$$

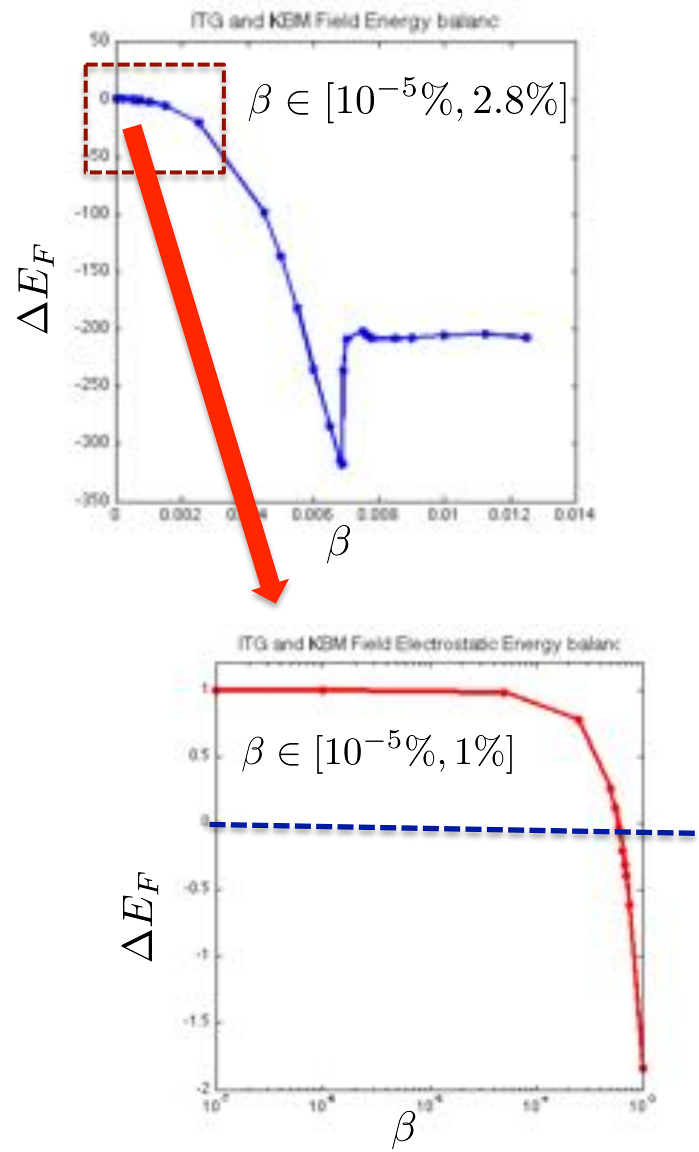
Natalia Tronko nataliat@ipp.mpg.de

Analysis of $\Delta E_F(\beta)$ function: min; zeros?

$$\Delta E_{\rm F} = \frac{E_{\rm F;ES} - E_{\rm F;EM}}{E_{\rm F;ES}}$$

Magnetic beta

$$\beta = \frac{P_{\rm kin}}{P_{\rm magn}}$$



1) $\min \Delta E_F(\beta) \Rightarrow \beta = 1.378\%$

Bifurcation ITG to KBM

2) Zoom on zero of

$$\Delta E_F(\beta) = 0 \Rightarrow \beta = 0.1\%$$

shows transition to the electromagnetic regime

$$\beta = 0.1\%$$

$$E_{\rm F;ES} = E_{\rm F;EM}$$

Natalia Tronko nataliat@ipp.mpg.de

Powerbalance as growth rate measure

- **Can also be used for Growth rate measure when E_F different from 0**

$$\mathcal{E}_F = \mathcal{E}_F e^{-\gamma t} \Rightarrow \gamma = \frac{1}{2\mathcal{E}_F}\frac{d\mathcal{E}_F}{dt} = -\frac{1}{2\mathcal{E}_F}\frac{d\mathcal{E}_k}{dt}$$

- Anytime we can calculate dynamical contributions directly from the characteristics

$$\frac{d\mathcal{E}_F}{dt} = \sum_s \int d\Omega\ f\ \nabla \mathcal{J}_0^{\mathrm{gc}}\left(\phi_1 - \frac{e p_z}{mc} A_{1\|}\right) \cdot \left(\mathbf{v}_\| + \mathbf{v}_{\nabla B}\right) - \sum_s \int \mathcal{J}_0^{\mathrm{gc}}\left(A_{1\|}\right)\left(\frac{\mu B}{m}\nabla \cdot \widehat{\mathbf{b}}\right)$$

Parallel velocity contains electromagnetic component

$$v_\| = \frac{p_z}{m} = \frac{1}{m}\left(m v_\| - \frac{e}{c} A_{1\|}\right)$$

Curvature contribution

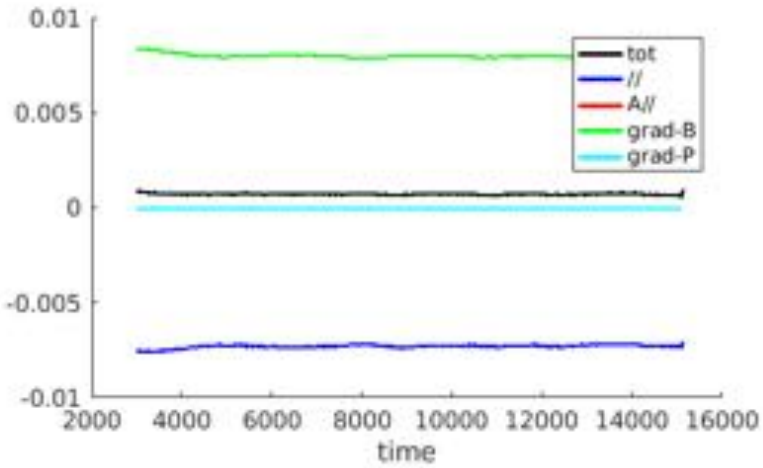
$$\mathbf{v}_{\nabla B} = \left(\frac{\mu}{m} + \left(\frac{p_z}{m}\right)^2\right)\frac{m}{e B_\|^*}\widehat{\mathbf{b}} \times \frac{\nabla B}{B}$$

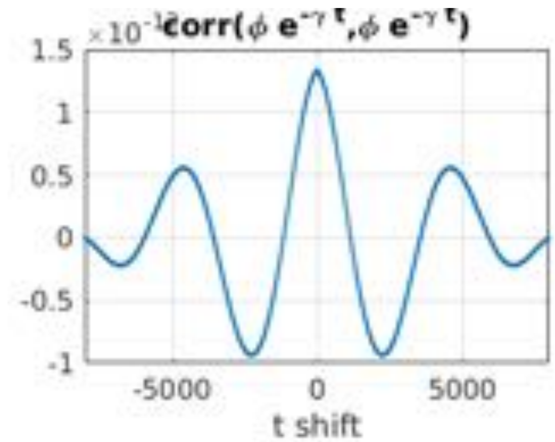
- **New result:** Electromagnetic ITG and KBM have the same main destabilizing mechanism: they are mainly destabilize by kinetic effects rather than by curvature of magnetic fields

Natalia Tronko nataliat@ipp.mpg.de

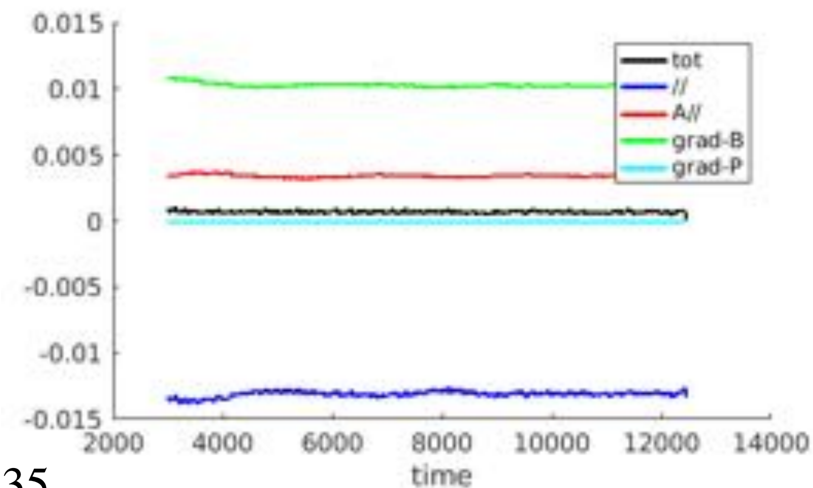
Instabilities driving mechanisms

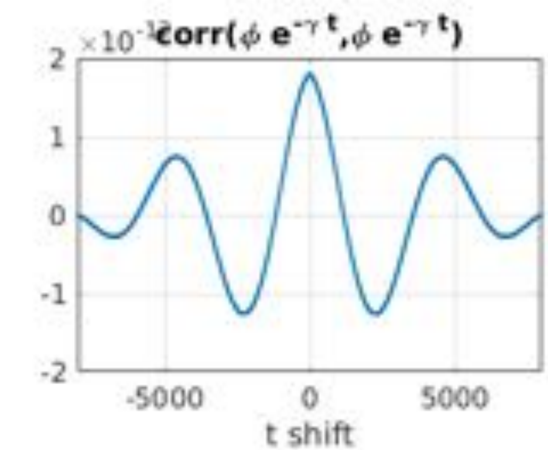
β=0.00001%: Elecrostatic, classic ITG mode





β=0.05%: increasing EM contribution

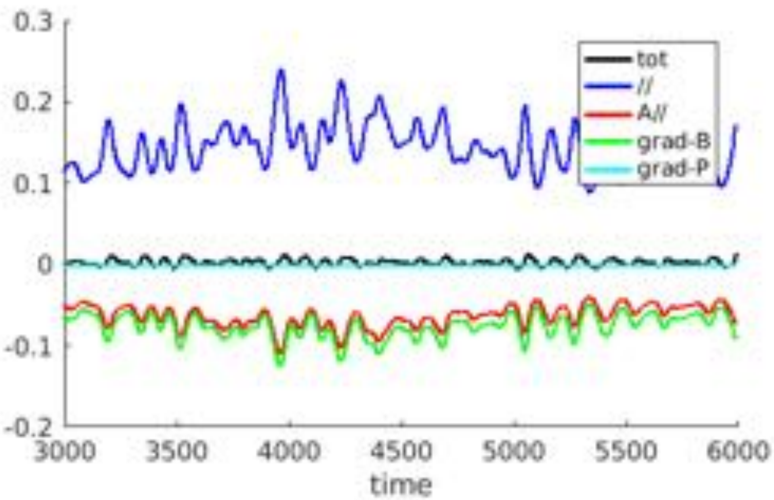


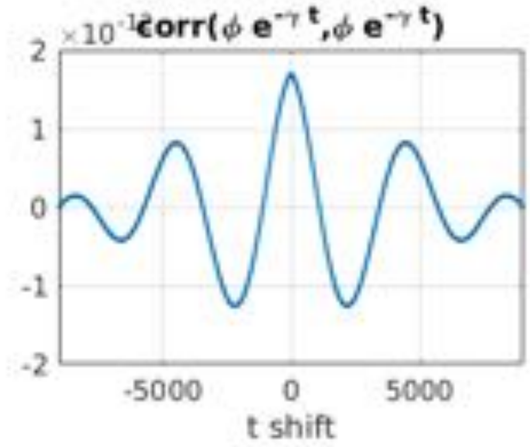


Natalia Tronko nataliat@ipp.mpg.de

Instabilities driving mechanisms

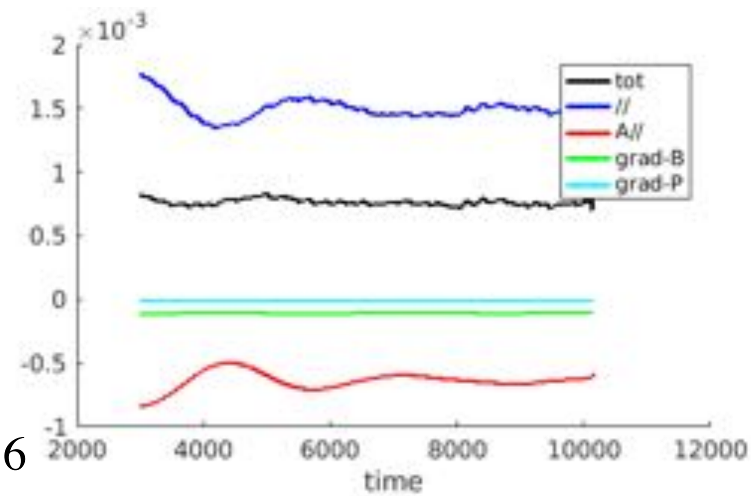
β=0.135%: ΔE_F=0.0034

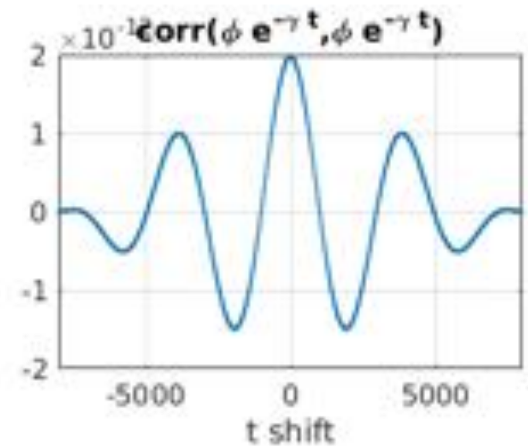




β=1%: exchange of destabilizing mechanisms: Electromagnetic ITG

No mode change: low frequency

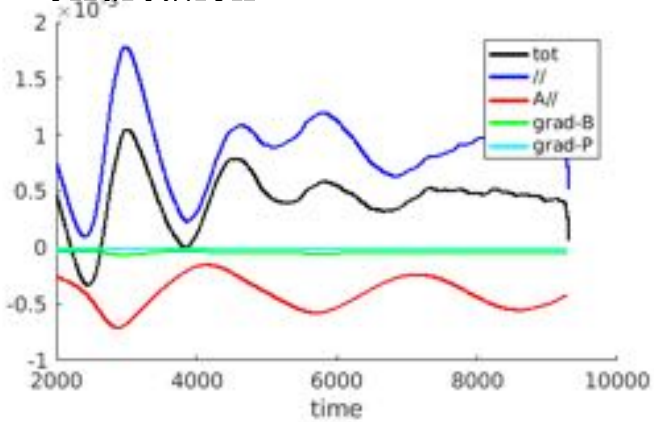


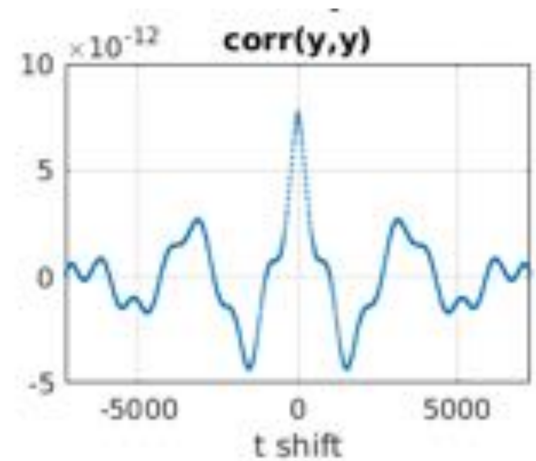


Natalia Tronko nataliat@ipp.mpg.de

Instabilities driving mechanisms

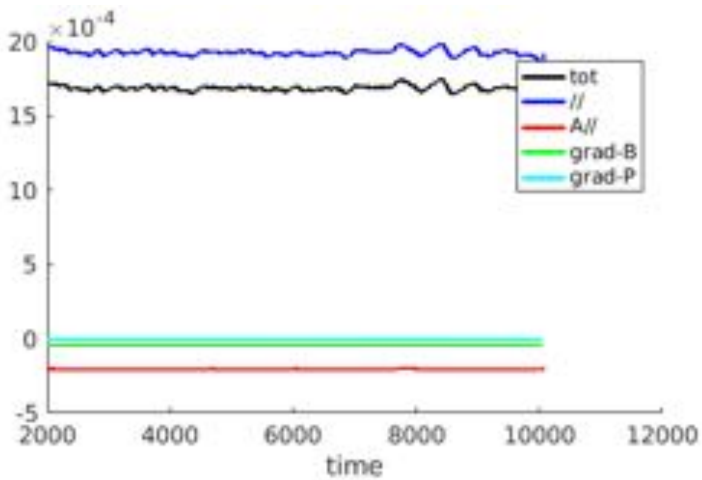
β=0,69 %; minimum ΔE_F=-269,33; mode bifurcation

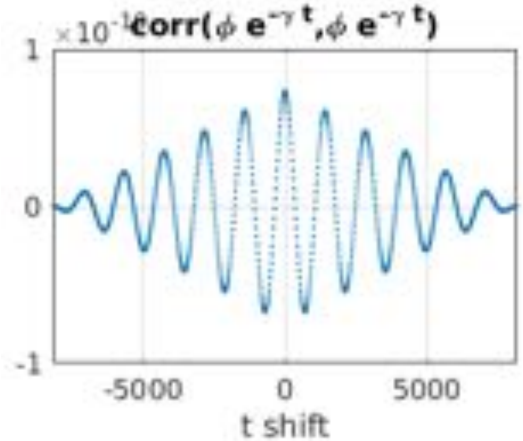




β=0,9 %; classic KBM mode: kinetically destabilized

Mode change: high frequency





Natalia Tronko nataliat@ipp.mpg.de

Open questions and further developments

- **GK codes: significant development: electromagnetic implementations**
  - **Electrostatic gyrokinetic implementations :** theory & simulations: **well established for core of Tokamak**
  - **Electromagnetic gyrokinetic implementations:**
    - Next level of complexity: Alfvén physics, **different instabilities mechanisms**
    - A lot of freedom for approximations (Poisson and Ampère equations)
    - **Noether method: primary tool for quality control and instabilities investigation**
  - **Need to question existing orderings**
    - **Comparing with experiments**
    - **From the core to the edge of devices: very different physical properties**
    - **Exploring model validity in new regimes**
    - **New magnetic geometries (Stellamak: under construction)**

Natalia Tronko nataliat@ipp.mpg.de

Guiding-center generating function

$$H = \frac{1}{2} m W_{\|}^2 + \frac{1}{2} m W_{\perp}^2 + m(W_{\|}\mathcal{W}_{\|} + W_{\perp}\mathcal{W}_{\perp}) + \mathcal{O}(\epsilon_B^2)$$

$$\underbrace{\frac{1}{2} m W_{\|}^2 + \frac{1}{2} m W_{\perp}^2}_{H_0} \qquad \underbrace{m(W_{\|}\mathcal{W}_{\|} + W_{\perp}\mathcal{W}_{\perp})}_{H_1 \sim \mathcal{O}(\epsilon_B)}$$

- **Formal scales separation in the Poisson bracket**

$$\{F, G\}_{\mathrm{gc}} = \{F, G\}_{-1} + \{F, G\}_0 + \{F, G\}_1$$

- **Averaged and fluctuating parts of Hamiltonian**

$$H_1 = \widetilde{H_1} + \langle H_1 \rangle \qquad \langle H_1 \rangle = (2\pi)^{-1} \int_0^{2\pi} \mathrm{d}\Theta \; H_1$$

- **Define a generating function by cancelling lower order fluctuating terms**

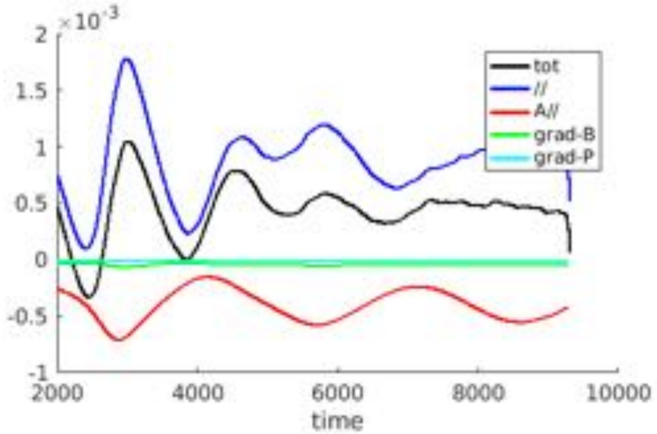
$$\bar{H} = H_0 + \epsilon_B \left(\langle H_1 \rangle + \boxed{\widetilde{H}_1 - \{S_1, H_0\}_{-1}} + \{S_1, H_0\}_0 - \{S_1, H_0\}_1 \right) + \mathcal{O}(\epsilon_B^2)$$

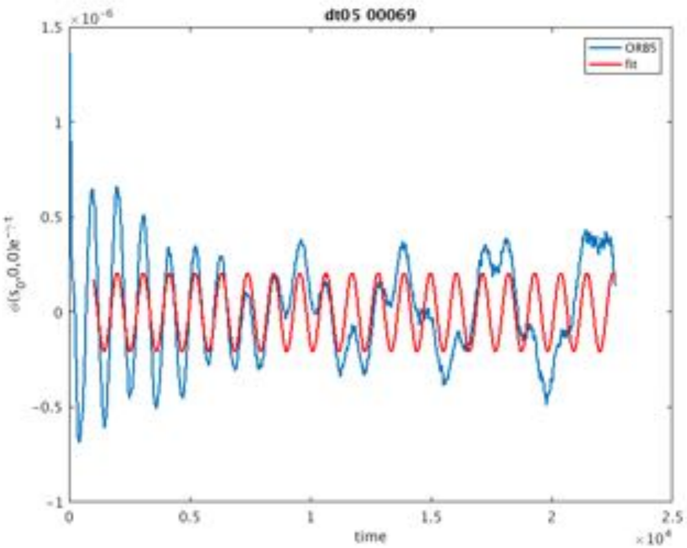
$\{S_1, H_0\}_0 \sim \mathcal{O}(\epsilon_B)$ $\qquad$ $\{S_1, H_0\}_1 \sim \mathcal{O}(\epsilon_B)$

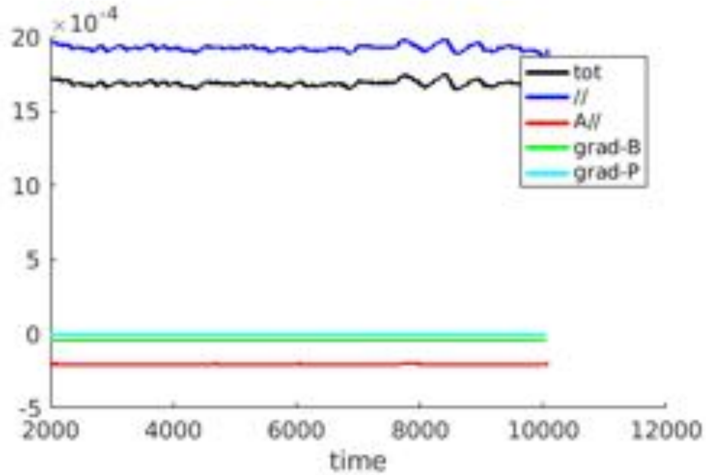
$$\boxed{\frac{\partial S_1}{\partial \Theta} + \frac{m^2 c}{eB} (W_{\|}\mathcal{W}_{\|} + W_{\perp}\mathcal{W}_{\perp}) = 0}$$

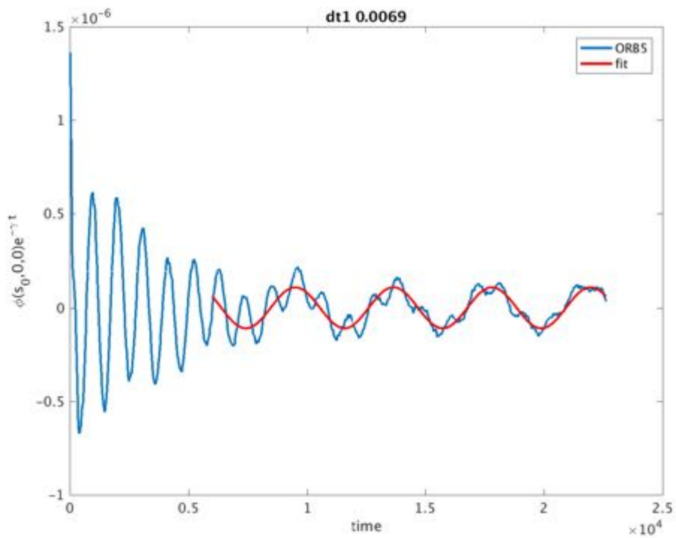
Natalia Tronko nataliat@ipp.mpg.de

ITG to KBM transition









Natalia Tronko nataliat@ipp.mpg.de

Guiding-center dynamical reduction

Splitting difficulties: first solving problem for particle motion in external non-uniform magnetic field

Goal: removing gyroangle dynamical dependencies up to the first order in ϵ_B

$$\epsilon_B = \rho_0 \left| \frac{\nabla B}{B} \right| \sim \rho_i \rho_j \left| \partial^2 \mathbf{A} / \partial \bar{x}_i \partial \bar{x}_j \right|$$

- **Physical ordering: with respect to curvature of magnetic field**

- **Exact solution in SLAB geometry exists, i.e. for** $\epsilon_B = 0$

- **Step1: infinitesimal shift in velocity space:**
  - **Second order in ϵ_B shift in velocity:**

$$\mathbf{w} = \mathbf{v} + \frac{e}{mc} \left[\mathbf{A}\left(\bar{\mathbf{x}} + \boldsymbol{\rho}_0\right) - \mathbf{A}\left(\bar{\mathbf{x}}\right) - \left(\boldsymbol{\rho}_0 \cdot \nabla\right) \mathbf{A}(\bar{\mathbf{x}}) - \frac{1}{2} \left(\boldsymbol{\rho}_0 \boldsymbol{\rho}_0 : \nabla\nabla\right) \mathbf{A}(\bar{\mathbf{x}}) \right]$$

$$\sim \mathcal{O}(\epsilon_B^2)$$

$$\mathbf{w} = w_{\|} \hat{\mathbf{b}}(\mathbf{x}) + w_{\perp} \hat{\perp}(\theta, \mathbf{x})$$

  - **Particle position decomposition : instantaneous rotation center and Larmor radius** $\mathbf{x} = \bar{\mathbf{x}} + \boldsymbol{\rho}_0$

Natalia Tronko nataliat@ipp.mpg.de

Some resolutions for kinetic simulations

Typical turbulence frequencies are much lower:
idea of gyrokinetic reduction

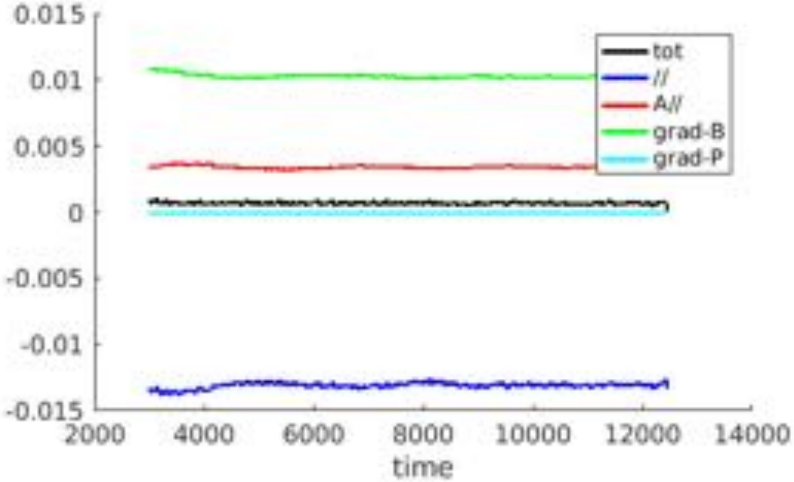
$$\epsilon_\omega = \frac{\omega}{\Omega_{ci}} \sim 10^{-3}$$

Multi-scaled dynamics :

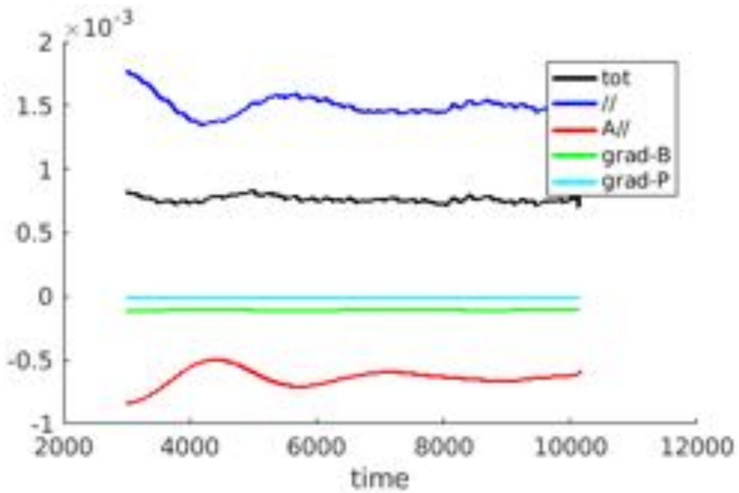
- Use adiabatic invariant: 4D instead of 6D phase space
- Eliminating fastest time scale: increasing time step by 1000 for ions
- Use adapted to magnetic geometry coordinates!
  Adapting spatial resolution
- Store only moments of the distribution function (integrated on the velocity space)

Sign changing ΔE_F

- Two cases of low-frequency mode (before the bifurcation) with different sign of ΔE_F



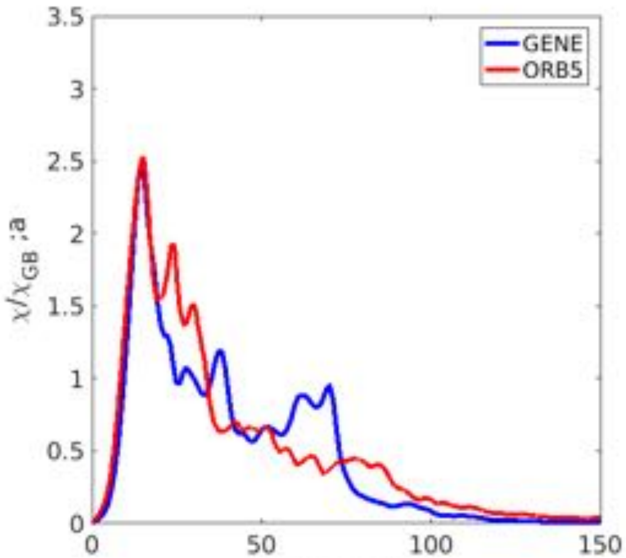
- β=0.05% ΔE_F>0
- Mainly destabilized by magnetic curvature
- Parallel velocity component is stabilizing

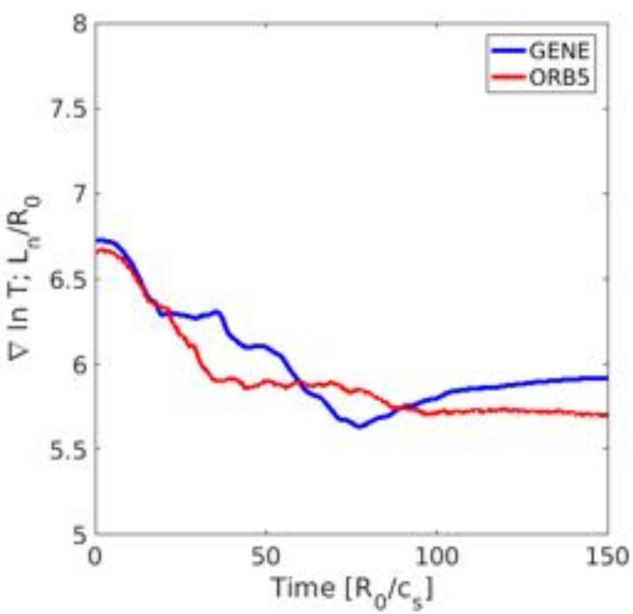


- β=1% ΔE_F<0
- Mainly destabilized by EM mechanisms contained in parallel velocity

Natalia Tronko nataliat@ipp.mpg.de

Nonlinear Benchmark: adiabatic electrons relaxation ORB5&GENE





- High sensibility to initial conditions (matching first peak)
- NO sources
- Reasonable cost : One run 19200 node-hours on SKL Marconi

[Lapillonne, McMillan, PoP 2010]

$$\chi_i = \langle Q_i \rangle \,/\, \langle |\nabla T_i| \rangle \qquad \chi_{\rm GB} = \rho_s^2 c_s / a$$

More codes are welcome to join!!!

Natalia Tronko nataliat@ipp.mpg.de