2.3

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Mathematical Sciences Research Institute

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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: ORI KATZ Email/Phone: ORIKATZ.OK@gmail.com

Speaker's Name: Maciej Capinski

Talk Title: A topological mechanism for diffusion, with applications to the elliptic restricted 3-body problem

Date: 10/09/18 Time: 11:30 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: Capinski presents a topological mechanism of diffusion in a priori chaotic systems. The method leads to a proof of diffusion for an explicit range of perturbation parameters. Assumptions can be verified using interval arithmetic numerics leading to computer-assisted proofs. Example of application - proof of diffusion in the Neptune - Triton planar elliptic restricted 3-body problem.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☒ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☒ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - → <u>Computer Presentations</u>: Obtain a copy of their presentation
  - <u>Overhead</u>: Obtain a copy or use the originals and scan them
  - <u>Blackboard</u>: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - <u>Handouts</u>: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack <u>with this completed sheet on top</u> and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to <u>notes@msri.org</u> with the workshop name and your name in the subject line.

A topological mechanism for diffusion, with application to the elliptic restricted three body problem

Maciej Capiński
AGH University of Science and Technology, Kraków

Joint work with

Marian Gidea
Yeshiva University, New York

Plan of the presentation

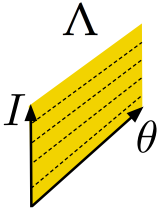
- Motivation
- Example
- Main result
- Covering relations
- Cone conditions
- Elliptic restricted three body problem - computer assisted proof

Motivation

$$x' = J\nabla(H(x) + \varepsilon G(t, x))$$

for $\varepsilon = 0$ we have:

- Λ NHIM, parameterised by I, θ
- Stable and unstable manifold intersect transversally
- Dynamics restricted to an energy level



Question: Can we diffuse in I over $[0, 1]$ for $\varepsilon > 0$?

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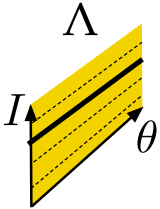

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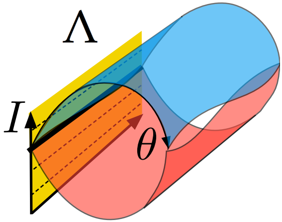

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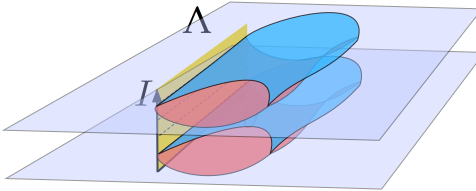

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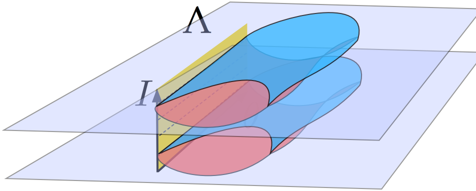

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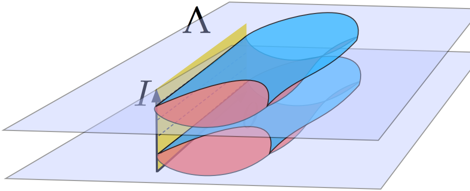
- Explicit range of $\varepsilon \in (0, \epsilon]$
- Estimates on diffusion time
- Symbolic dynamics
- Hausdorff dimension of the set of diffusing orbits
- Method suitable for computer assisted proofs



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Theorem

In the Neptune-Triton PER3BP

$$\mu = 0.000208923,$$

for any $\varepsilon \in (0, 0.000016]$ we have diffusion over energies of size $\frac{1}{4}10^{-8}$.

Motivating example

$$f : \mathbb{R}^2 \to \mathbb{R}^2$$

$$g : \mathbb{R}^2 \times \mathbb{S}^1 \to \mathbb{R}^2 \times \mathbb{S}^1$$

$$g(x, y, \theta) = (f(x, y), \theta + \omega)$$

$$F_\varepsilon : \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{R} \to \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{R}$$

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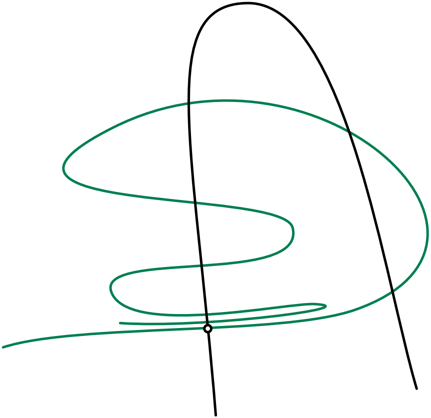

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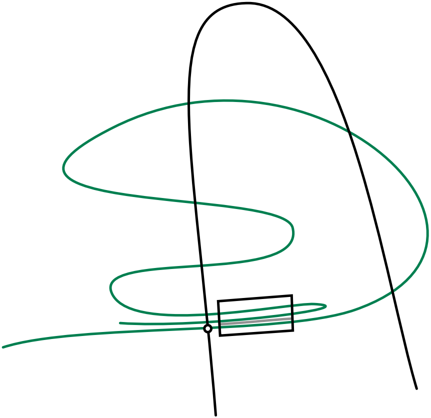

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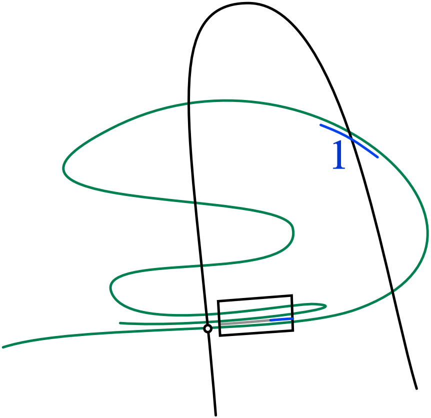

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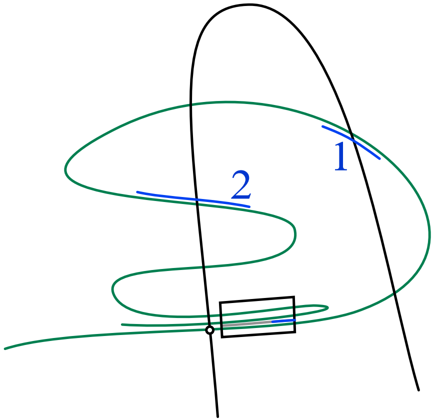

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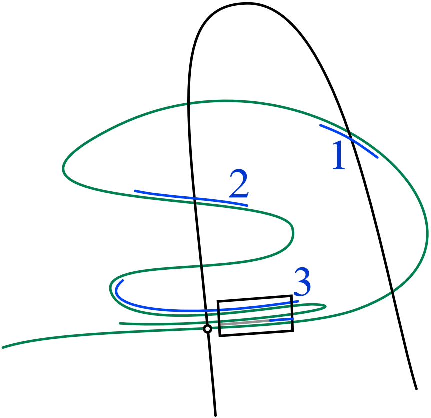

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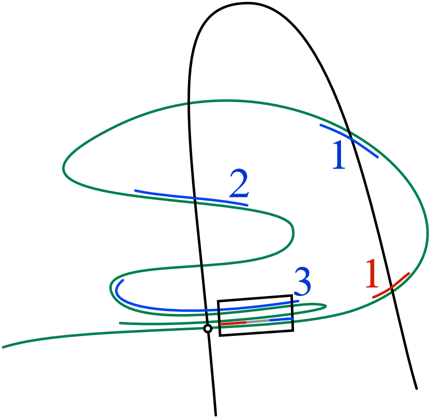

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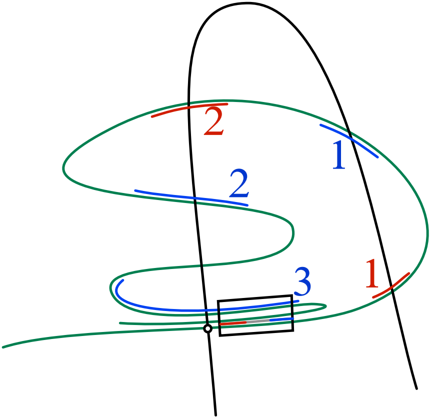

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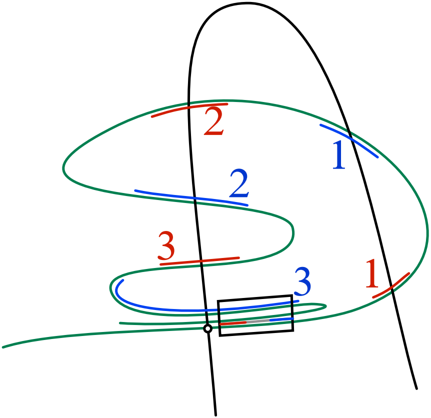

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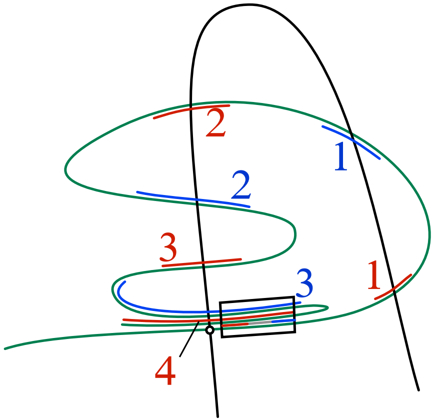

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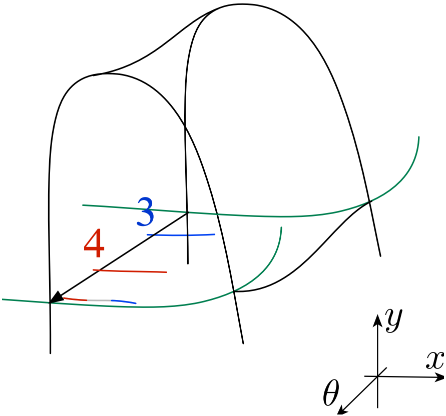

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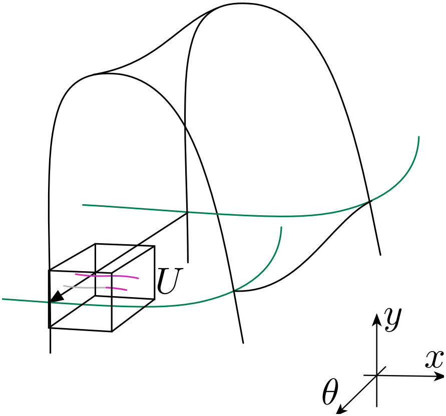

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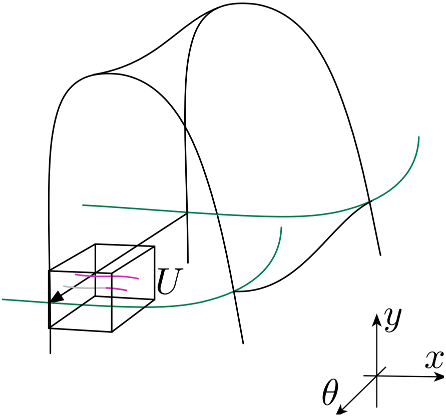


Main result

$$F_\varepsilon : \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{R} \to \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{R}$$

$$F_{\varepsilon=0}(x, y, \theta, I) = (f(x, y), \theta + \omega, I)$$

Theorem

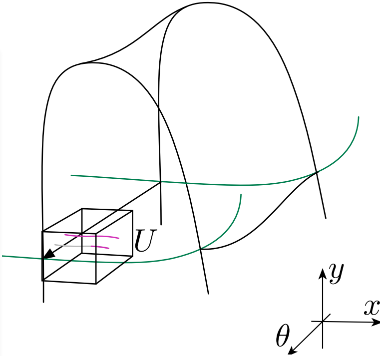
If any horizontal disc 'returns' to U and for points x that return

$$\pi_I F_\varepsilon^n(x) > \pi_I x + c\varepsilon$$

then for any $\varepsilon > 0$ $\exists \tilde{x}$ and $\exists N$ for which

$$\pi_I \tilde{x} = 0,$$

$$\pi_I F_\varepsilon^N(\tilde{x}) > 1.$$



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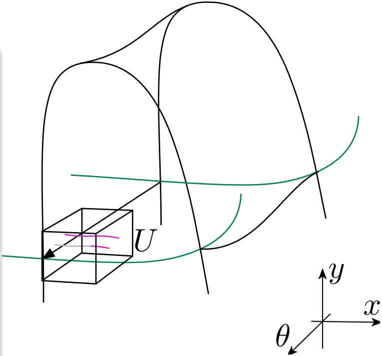

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Theorem

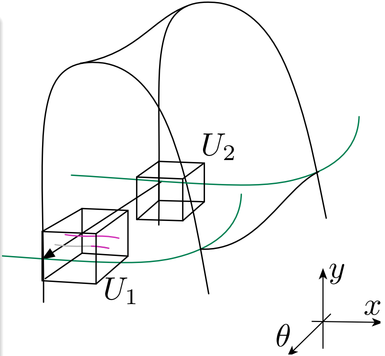
Horizontal discs can 'travel' between U_i and U_j for $i, j \in \{1, 2\}$.
For points that travel from U_1 to U_1

$$\pi_I F_\varepsilon^n(x) > \pi_I x + c\varepsilon.$$

For points that travel from U_2 to U_2

$$\pi_I F_\varepsilon^n(x) < \pi_I x - c\varepsilon.$$

Then we have symbolic dynamics in I.



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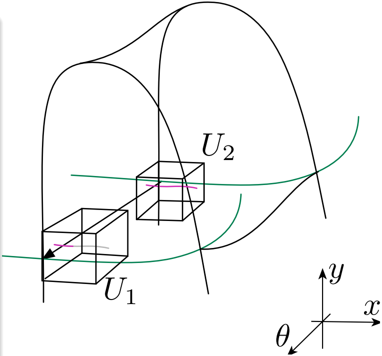

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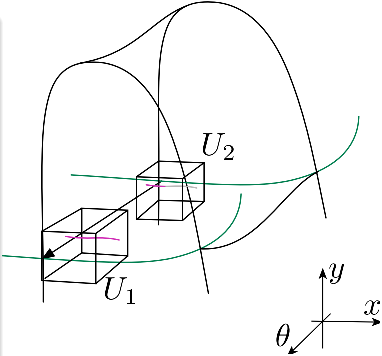

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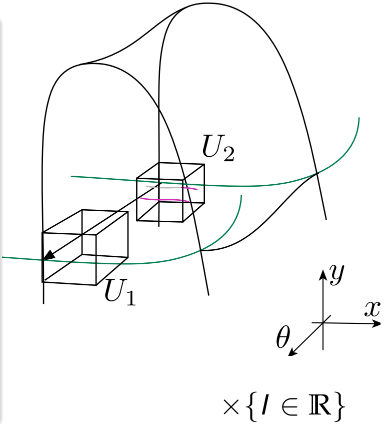
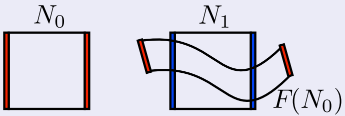


Tools

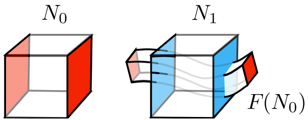
Verification of assumptions

Definition

Covering relation, $N_0 \overset{F}{\Longrightarrow} N_1$



One dimensional unstable coordinate:



Theorem ([1])

$N_0 \overset{F}{\Longrightarrow} N_1 \overset{F}{\Longrightarrow} \cdots \overset{F}{\Longrightarrow} N_k$

Then there exists a trajectory that passes through the sets.

[1] M. Gidea, P. Zgliczyński, "Covering relations for multidimensional dynamical systems" JDE

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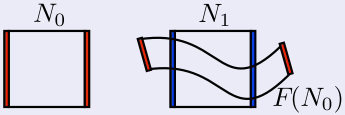

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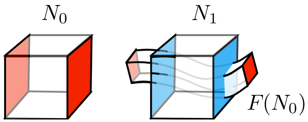

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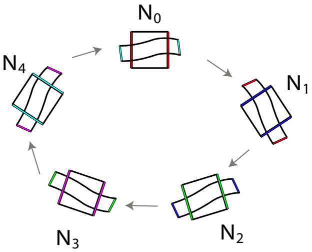

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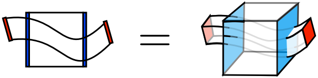
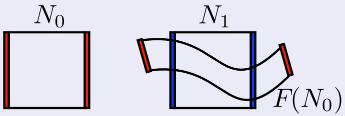
Tools

Verification of assumptions

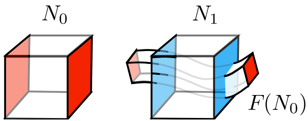


Definition

Covering relation, $N_0 \overset{F}{\Longrightarrow} N_1$



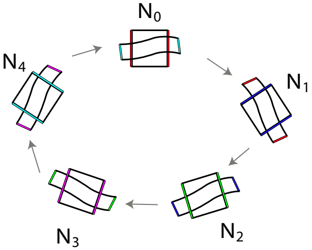
One dimensional unstable coordinate:



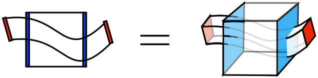
Theorem ([1])

$$N_0 \overset{F}{\Longrightarrow} N_1 \overset{F}{\Longrightarrow} \cdots \overset{F}{\Longrightarrow} N_k$$

Then there exists a trajectory that passes through the sets.

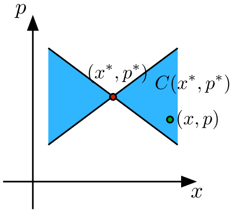


[1] M. Gidea, P. Zgliczyński, "Covering relations for multidimensional dynamical systems" JDE



$\alpha > 0$. Coordinates $(x, p) \in \mathbb{R} \times \mathbb{R}^3$.
Cone:

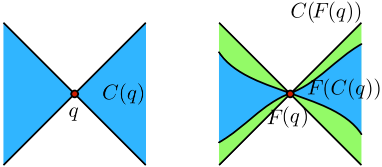
$$C(x^*, p^*) = \{(x, p) : |x^* - x| \geq \alpha \|p^* - p\|\}$$



Definition

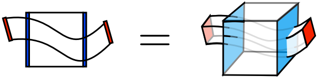
F satisfies cone condition iff

$$F(C(q)) \subset C(F(q))$$



Verification of assumptions

Cone conditions



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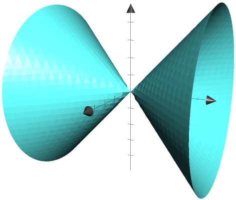

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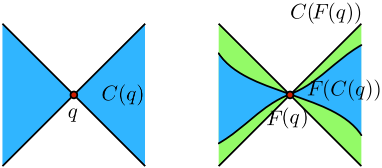

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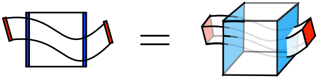

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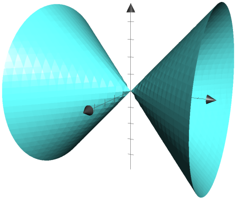

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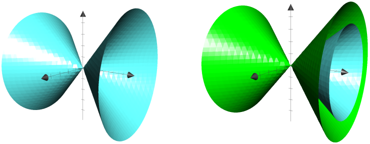

Verification of assumptions

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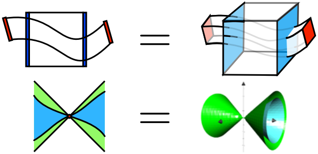

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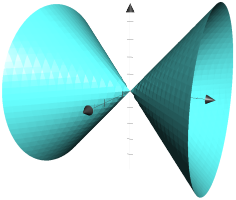

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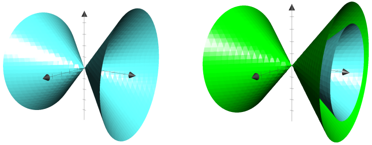

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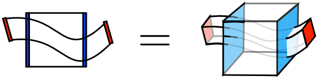
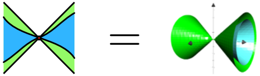
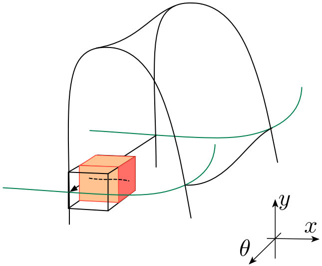
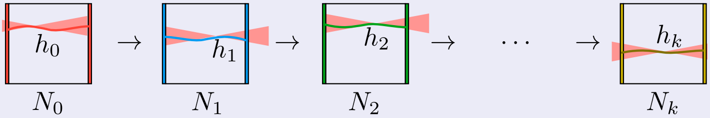






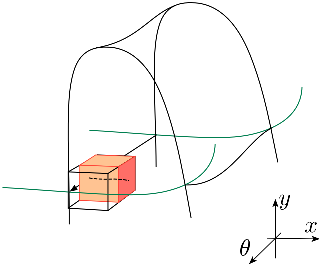
Theorem ([2])

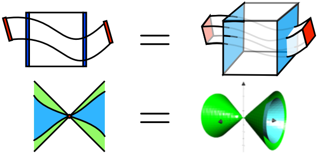
Cone conditions and $N_0 \overset{F}{\Longrightarrow} N_1 \overset{F}{\Longrightarrow} \cdots \overset{F}{\Longrightarrow} N_k$ *imply*

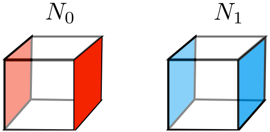


[2] P. Zgliczyński, "Covering relations, cone conditions and the stable manifold theorem", JDE

Verification of assumptions







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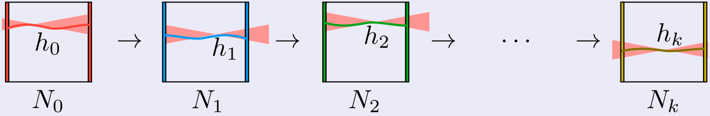

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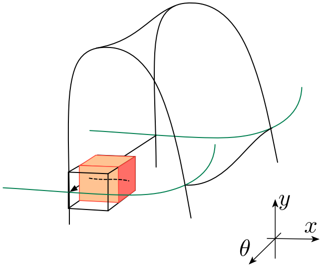

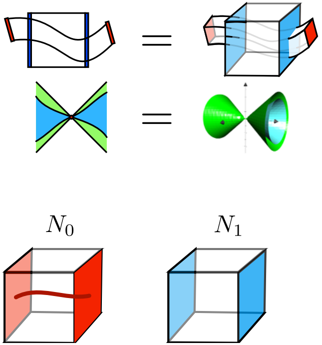

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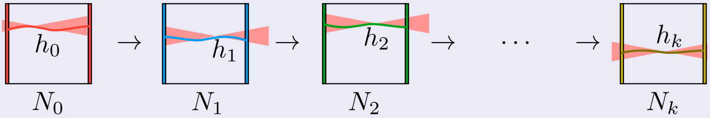

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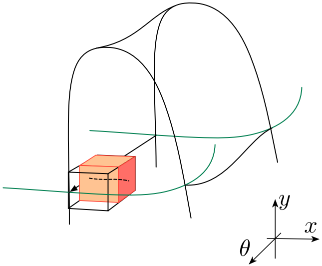

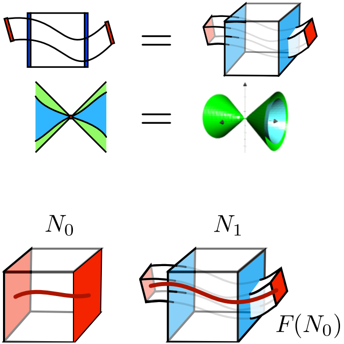

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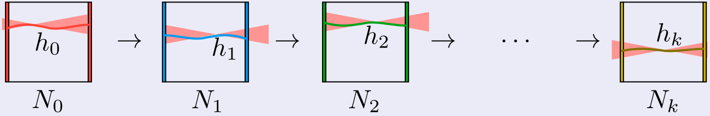

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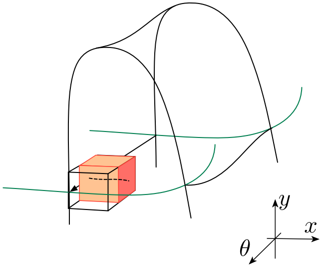

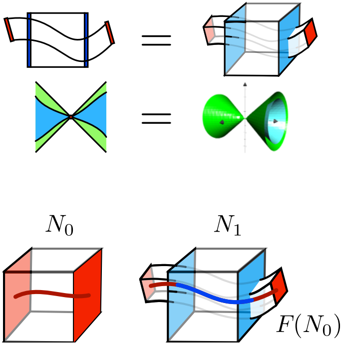

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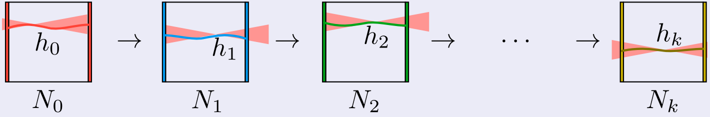

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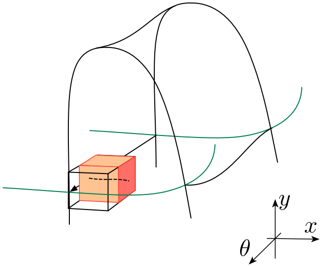

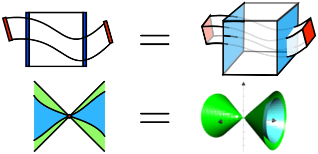

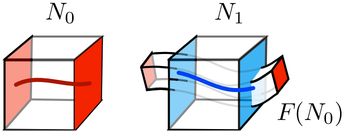

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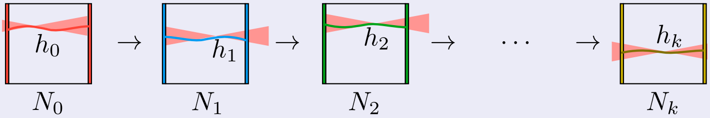

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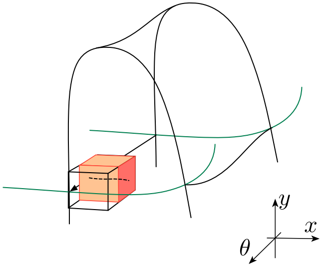

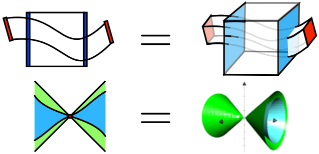

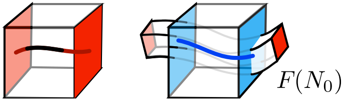

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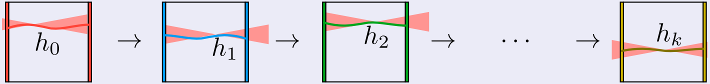

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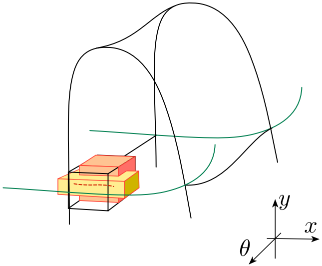

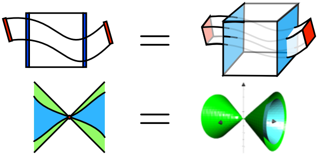

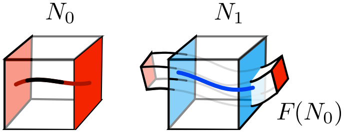

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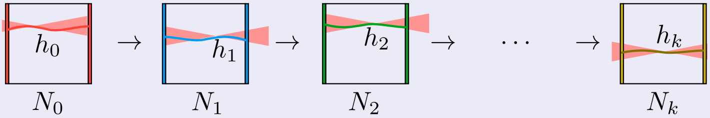

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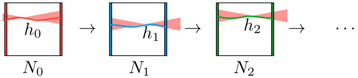
Connecting Sequence



$\mathbf{S} = [-1, 1] \times [-1, 1] \times \mathbb{R} \times [s_1, s_2]$
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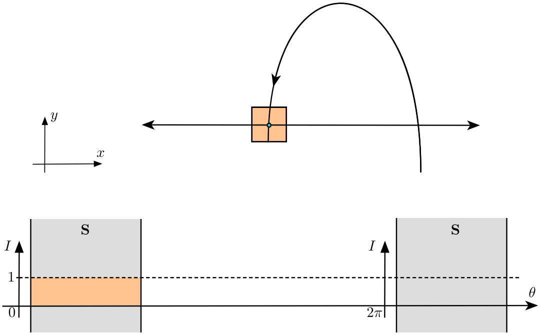
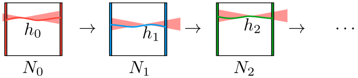


- Assume that we can enclose any horizontal disc in D by a set N_0
- Assume that we have a sequence of coverings that lands back to **S**

Connecting Sequence



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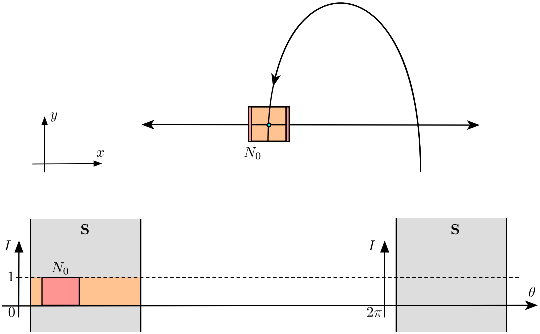

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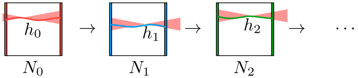

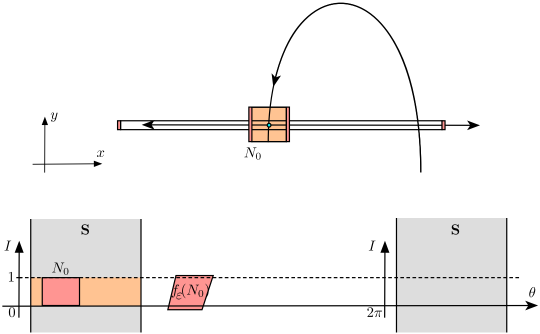

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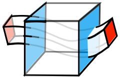

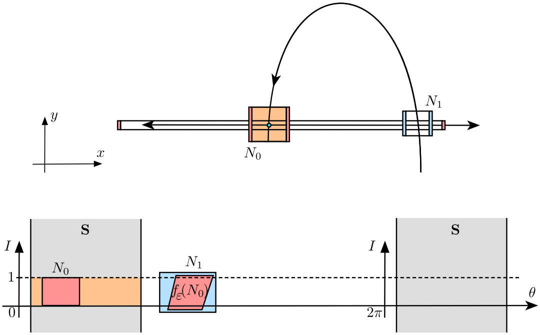

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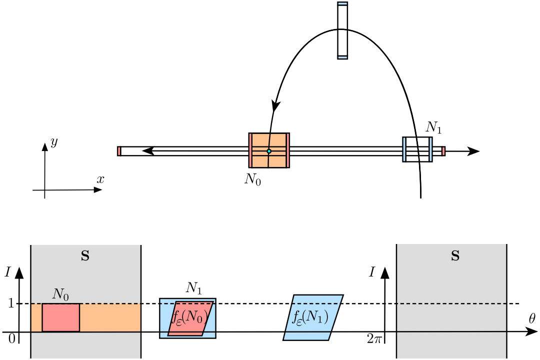

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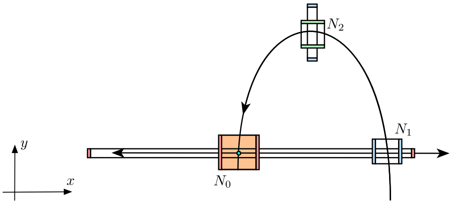

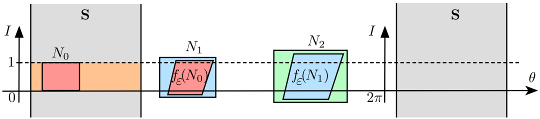

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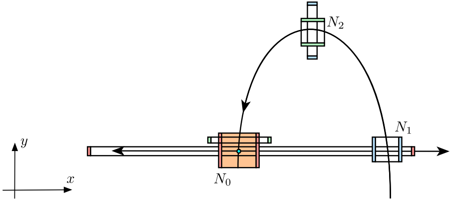

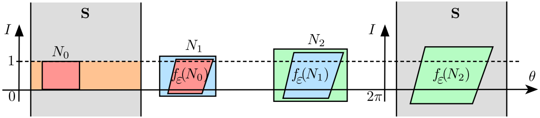

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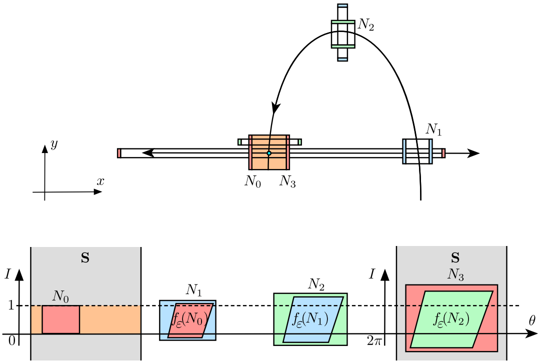

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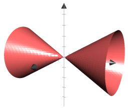

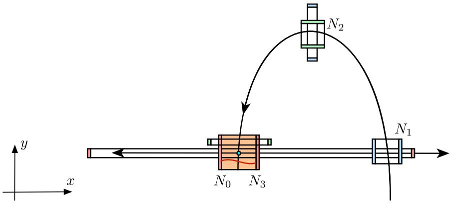

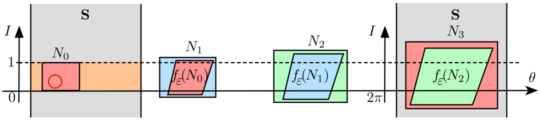

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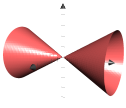

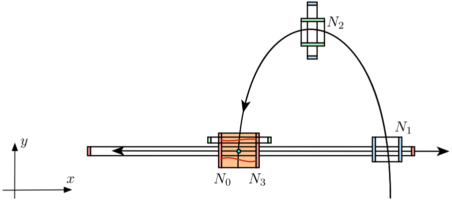

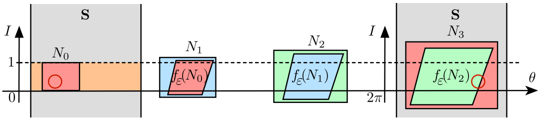

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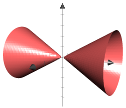

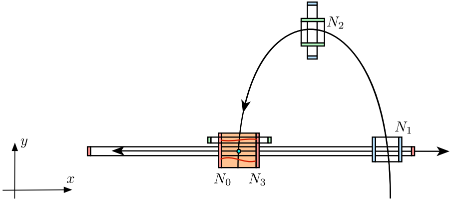

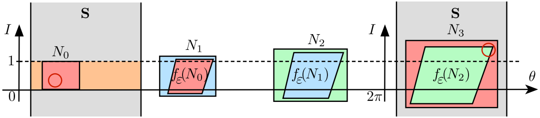

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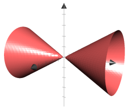

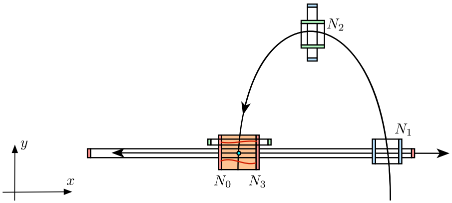

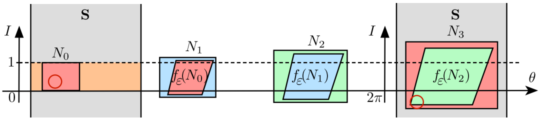

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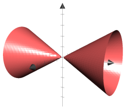

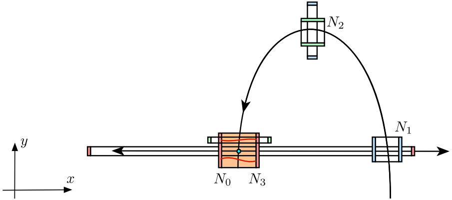

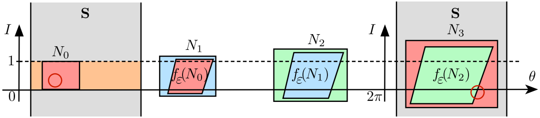

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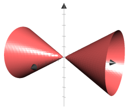

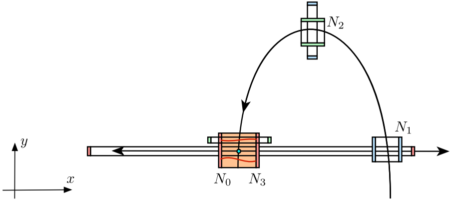

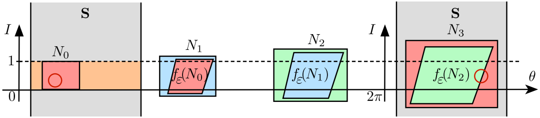

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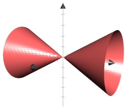

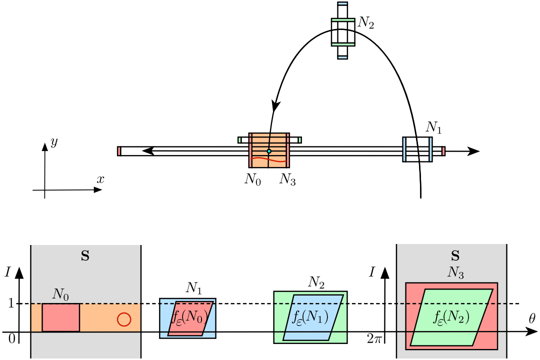

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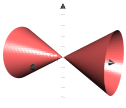

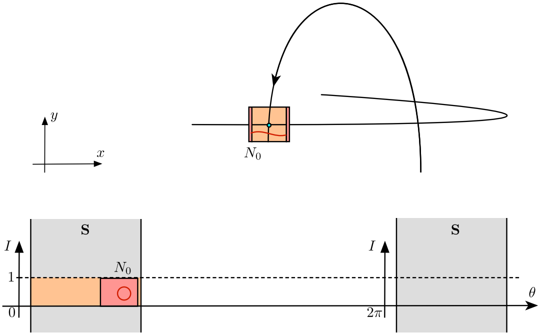

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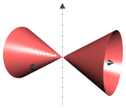

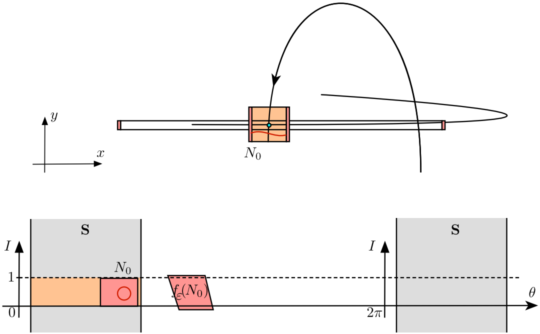

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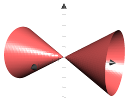

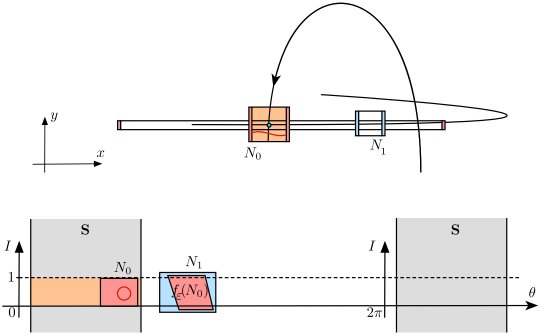

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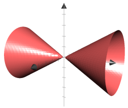

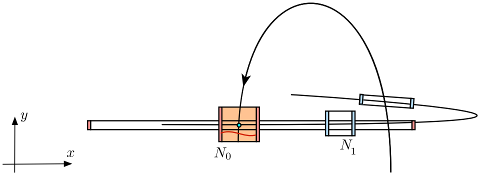

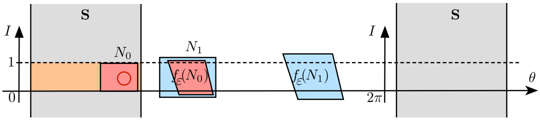

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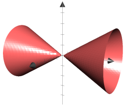

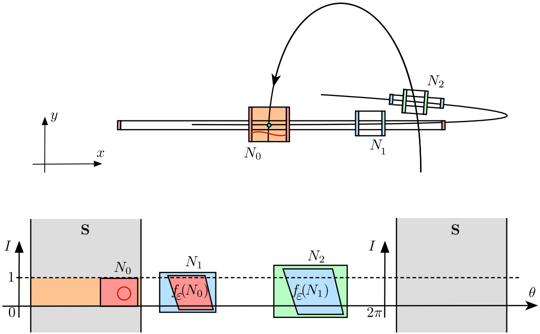

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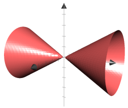

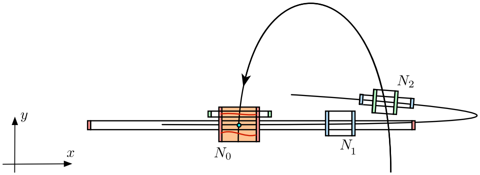

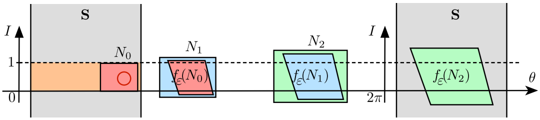

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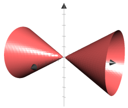

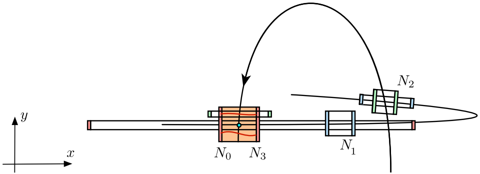

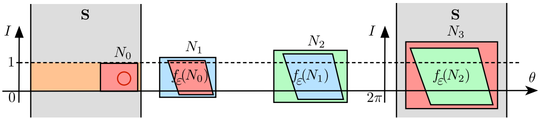

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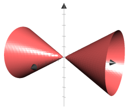

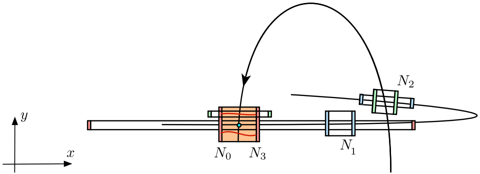

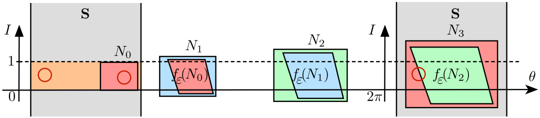

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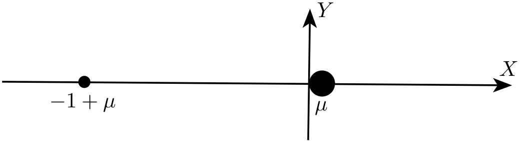
Restricted three body problem

Equations - elliptic case



ε eccentricity

θ true anomaly (if $\varepsilon = 0$, $\theta = t$)

$$H_\varepsilon(X, Y, P_X, P_Y, \theta) = \frac{(P_X + Y)^2 + (P_Y - X)^2}{2} - \frac{\Omega(X, Y)}{1 + \varepsilon\cos(\theta)}.$$

$$\frac{dX}{d\theta} = \frac{\partial H_\varepsilon}{\partial P_X}, \qquad \frac{dP_X}{d\theta} = -\frac{\partial H_\varepsilon}{\partial X},$$

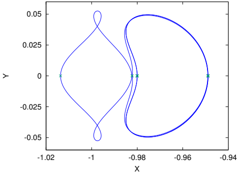
$$\frac{dY}{d\theta} = \frac{\partial H_\varepsilon}{\partial P_Y}, \qquad \frac{dP_Y}{d\theta} = -\frac{\partial H_\varepsilon}{\partial Y}.$$

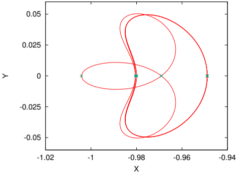
[3] V. G. Szebehely, "Theory of Orbits, The R3BP", Academic Press 1967

Restricted three body problem

Neptune-Triton system: $\mu = 0.000208923$,

For $\varepsilon = 0$ we have





F_ε is the map from $\{Y = 0\}$ to $\{Y = 0\}$. Sets $N_0, \ldots, N_k$ on $\{Y = 0\}$.

$N_0 \Rightarrow N_1 \Rightarrow \ldots \Rightarrow N_{13} \Rightarrow N_0$ $\quad$ $N_0 \Rightarrow N_1 \Rightarrow \ldots \Rightarrow N_{15} \Rightarrow N_0$

- (X, Y, P_X, P_Y, θ)
- (X, P_X, P_Y, θ)
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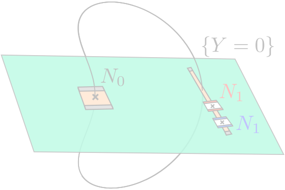

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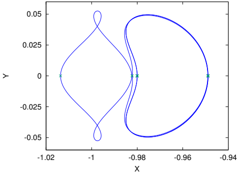

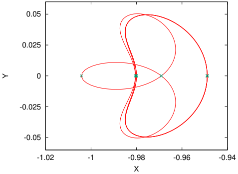

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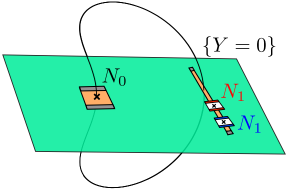

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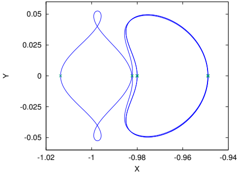

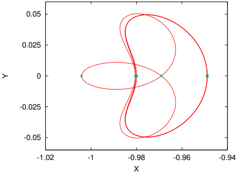

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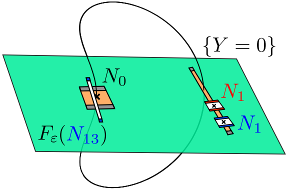

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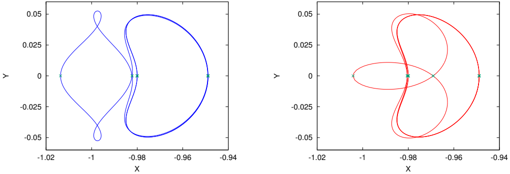

F_ε is the map from $\{Y = 0\}$ to $\{Y = 0\}$. Sets $N_0, \ldots, N_k$ on $\{Y = 0\}$.

$$N_0 \Rightarrow N_1 \Rightarrow \ldots \Rightarrow N_{13} \Rightarrow N_0 \qquad N_0 \Rightarrow N_1 \Rightarrow \ldots \Rightarrow N_{15} \Rightarrow N_0$$

- (X, Y, P_X, P_Y, θ)
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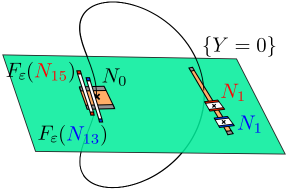
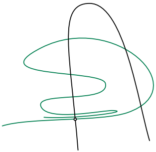
- $(x, y, I, \theta) \in \mathbb{R} \times \mathbb{R} \times [0, I^*] \times \mathbb{S}^1$



Restricted three body problem



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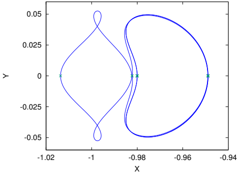

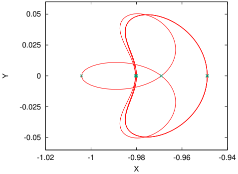

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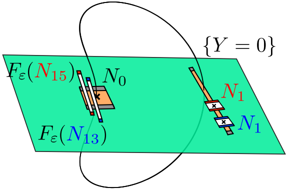

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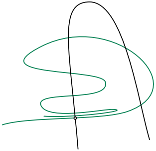

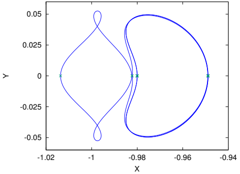

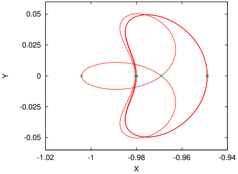

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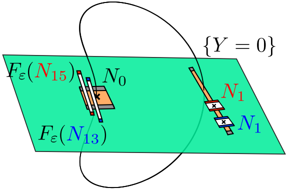
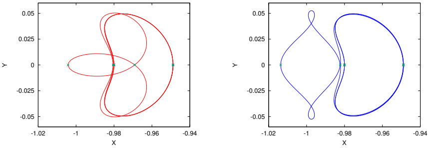


Restricted three body problem

Neptune-Triton: choice of θ



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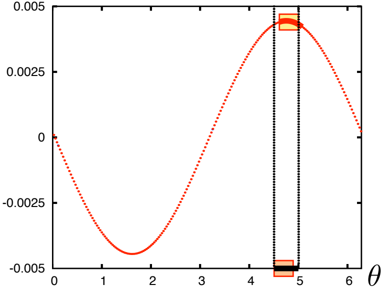
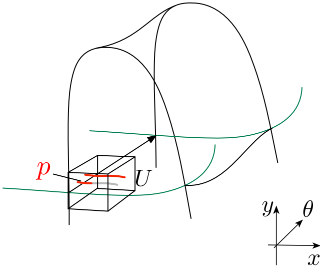


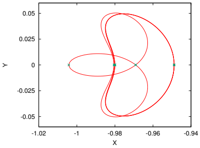


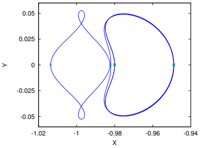
$$\theta \to \left(\pi_\theta (F_\varepsilon)^n (p, \theta), \frac{\partial}{\partial \varepsilon} \pi_I (F_\varepsilon)^n (p, \theta) \right)$$

$n = 16$, $n = 14$

Restricted three body problem

Neptune-Triton: choice of θ





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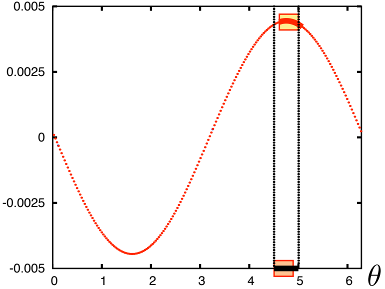

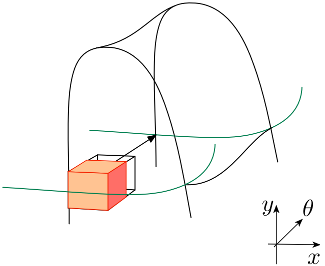

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Restricted three body problem

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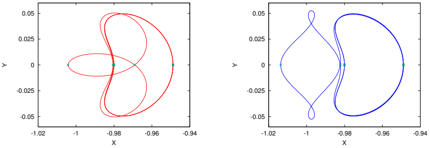

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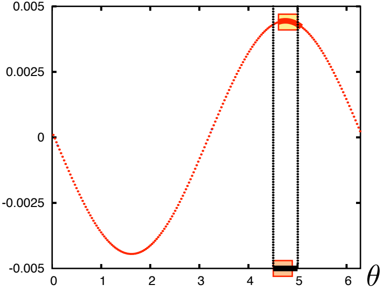

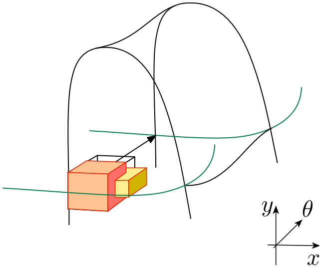

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Restricted three body problem

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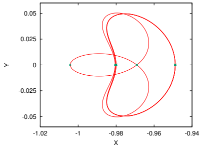

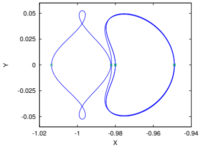

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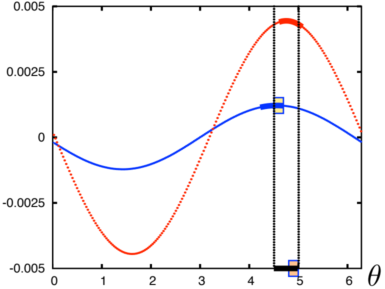

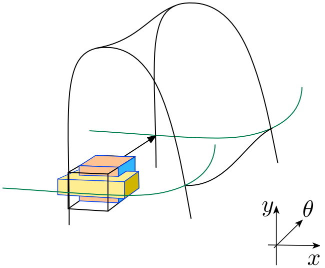

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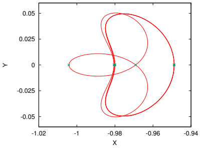

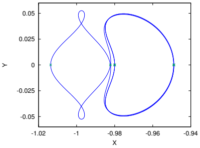

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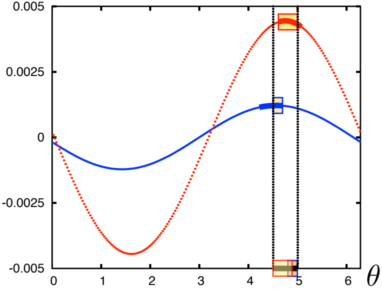

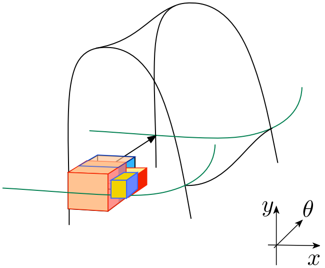

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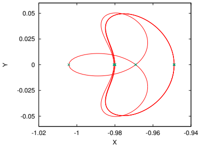
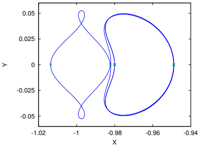
Restricted three body problem

Neptune-Triton: the result



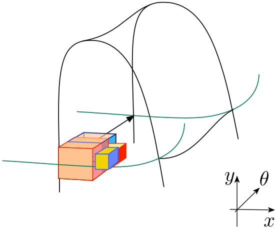


Theorem

Neptune-Triton: $\mu = 0.000208923$

For any $\varepsilon \in (0, 0.000016]$, *there exists an orbit with the change of energy:*

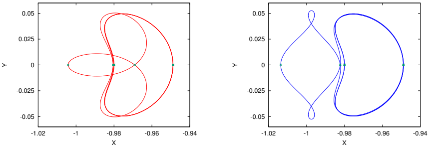
$$H(\varphi^{\varepsilon}_{t(\varepsilon)}(q)) = H(q) + \frac{1}{4}10^{-8}$$



- good news: explicit range of ε
- not so good: $\frac{1}{4}10^{-8}$ is small...

Restricted three body problem

Neptune-Triton: the result



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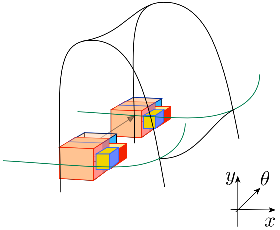

- We can prove symbolic dynamics
- Hausdorff dimension of diffusing orbits $>$ state space dimension -1.

Restricted three body problem

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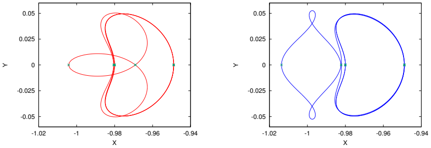

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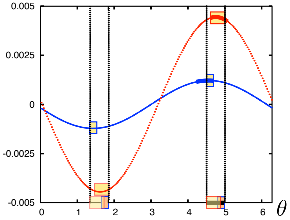
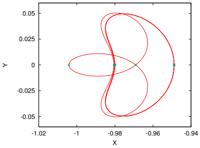
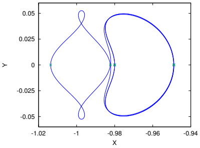


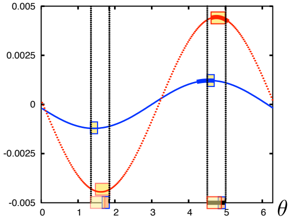
- We can prove symbolic dynamics
- Hausdorff dimension of diffusing orbits $>$ state space dimension -1.

Closing remarks





- Works in higher dimensions
- Explicit parameter range
- Explicit bound on diffusion time: C/ε
- Symbolic dynamics in energy
- Bound on Hausdorff dimension
- Suitable for computer assisted proofs



Thank you for your attention.

A topological mechanism for diffusion, with application to the elliptic restricted three body problem - Maciej Capinski

Lecture notes - Ori S. Katz

October 10, 2018

Abstract

We present a topological mechanism of diffusion in a priori chaotic systems. The method leads to a proof of diffusion for an explicit range of perturbation parameters. The assumptions of our theorem can be verified using interval arithmetic numerics, leading to computer assisted proofs. As an example of application we prove diffusion in the Neptune-Triton planar elliptic restricted three body problem. Joint work with Marian Gidea.

1 Lecture notes

Motivation:

Given a Hamiltonian system, with ϵ the perturbation parameter. $\epsilon = 0$ -> Hamiltonian is autonomous, energy is conserved.

NHIM - normally hyperbolic invariant manifold. Assume the NHIM is 1-dimensional, parametrized by I, θ.

Focusing on one action, the stable and unstable manifolds intersect transversally, and dynamics restricted to an energy level.

When we switch on a perturbation, can we have diffusion in the energy - in the action - for an arbitrarily small ϵ? (in chaotic systems)

Objectives (goals):

- We want to develop a method that work for all perturbations taken from a given interval $(0, \epsilon]$.
- We want to obtain estimates on the diffusion time, considering chaotic diffusion.
- What could we say about the Hausdorff dimension?
- We would like a method that is suitable for computer assisted proofs.

Teaser: Theorem about the planar elliptic restricter 3 body problem (PER3BP) - we will talk about it in more detail later on.

Start with a map of the phase plane with a hyperbolic fixed point, stable and unstable manifolds intersecting. Consider a 1-dimensional curve aligned with the unstable manifold. After a certain number of iterates we return to a previous position (arbitrarily close). This 1D curve is referred to as a horizontal disc.

Next stage - add rotation in an additional variable. Now, if the horizontal disc returns it will return at a different angle (in the new variable). Iterating enough time, we can find a given set U that the trajectory returns to.

Extending by one more direction trivially, we add a constant in another direction. This is not completely trivial.

The full system is defined as the unperturbed system.

Now, consider a family of maps parameterized by ϵ. For $\epsilon = 0$, we return to the previous case.

For the theorem (page 7/17), the following assumptions are made:

1. The horizontal disc returns to U.
2. Consider points x that return, with the number of returns increasing by $c\epsilon$.

How do we validate these assumptions? The key element is propagating and controlling the evolution of the horizontal discs.

Tools: The covering relation relates to exit sets, describing the expanding and contracting directions of the evolution.

What is the covering relation useful for?

Theorem ([1]) (page 8/17) - given a loop of covering relations, there exists a trajectory that passes through the sets.

Another tool: Cone conditions. It means that the image of a cone given the dynamics lies inside the cone. How is this verified? It is easy to validate given bounds on the derivatives of the functions.

How does this look like in higher dimensions? The picture is similar.

Theorem ([2]) (page 10/17) requires assumptions of cone condition and covering.

Connecting sequence (an attempt to draw 4 dimensions) - this cannot be done indefinitely, eventually the enclosing volume will blow up. However, since we look only at the horizontal disc we know it will not grow. Through this covering sequence, we know the energy is increasing. If the increase is above 1, we have achieved diffusion. If not, we iterate again.

Example: The restricted 3-body problem.

Equations - elliptic case. If $\epsilon = 0$ the dynamics are on circles. What is the motion of the small mass?

If $\epsilon \neq 0$, what are the dynamics? Obtain pulsating. There is a change of coordinates to pulsating coordinates in which the two primaries are motionless, as in the circulate ($\epsilon = 0$) case. What is the difference? The problem becomes non-autonomous time-periodic, as the systems we used to prove the theorems in the previous sections.

Circular problem - Neptune-Triton system. Choose just two homoclinic orbits that derive from a periodic solution. Define a map F_ϵ on which we will construct the iterated sequence. Iterating the map enough times, we return to the initial volume.

Comment about dimensions: We're talking about the circular problem here - 4 dimensions. But the circular problem is autonomous, energy is conserved, so we can drop one dimension. Further restricting to Poincare, we have 2 dimensions. What happens when we turn on the perturbation? No conservation of energy, more coordinates. Taking the section $Y = 0$ we drop one dimension, instead of using P_Y we can use the energy, and then we can align the remaining vectors to obtain the same picture we used previously.

Finally we obtain the theorem that shows an increase in energy (page 15/17). Although the increase is small, this process can be reiterated again and again to gain an arbitrary growth of energy.

We can prove symbolic dynamics by showing that given two regions the trajectory gains energy in one and loses energy in the other.

Also, we have gained "for free" a lower bound on the Hausdorff dimension of diffusing orbits.

Closing remarks:

This is the simplest, lowest dimensional case of a more general mechanism. This is a different approach than the usual KAM approach.

We have an explicit parameter range and an explicit bound on the diffusion time C/ϵ. Symbolic dynamics in energy, bound on Hausdorff dimension, suitable for computer assisted proofs.

2 Questions:

- From symbolic dynamics, can we prove an infinite number of periodic trajectories?

There is an infinite number of periodic trajectories.

- How sensitive is the method to the transversality of the intersections of the invariant manifolds, since you claimed you don't need to check transversality?

Transversality is useful when you want to position your sets - they are positioned on the transversal intersections. They could be positioned in other methods but this would be harder.