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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Jeffrey Heninger **Email/Phone:** jeffrey.heninger@yahoo.com

Speaker's Name: Kwang Je Kim

Talk Title: From Particle Noise to Coherent X-Rays: Beam Dynamics of X-Ray Free-Electron Lasers

Date: 11 / 29 / 2018 **Time:** 11 : 00 **am** / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: A bunch of electrons from an accelerator enters a magnetic field that makes them oscillate and radiate. The radiation grows exponentially can becomes coherent. Model this using Vlasov-Maxwell with a smooth background and a small perturbation corresponding to the discreteness of the electrons. Solve using van Kampen's normal mode expansion - after a long time, one mode will dominate. The dispersion relation can be solved using a variational method, which makes the design of free electron lasers simpler.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

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- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
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- ☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
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MSRI, BERKELEY, NOVEMBER 26-30, 2018

Argonne NATIONAL LABORATORY

Hamiltonian systems, from topology to applications through analysis II

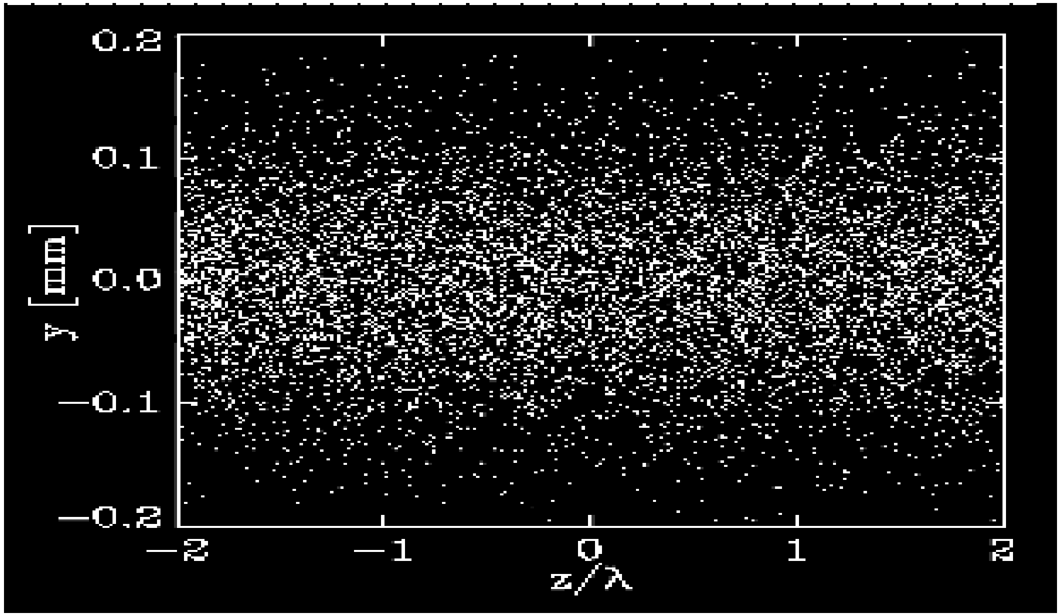
FROM PARTICLE NOISE TO COHERENT X-RAYS: BEAM DYNAMICS OF X-RAY FREE-ELECTRON LASERS

KWANG-JE KIM

University of Chicago & ANL

November 29, 2018
Berkeley, CA

COLLECTIVE MOTION IN A SYSTEM OF DISCRETE PARTICLES I

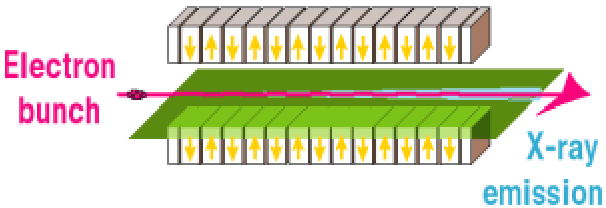


COLLECTIVE MOTION IN A SYSTEM OF DISCRETE PARTICLES II

- **A system of discrete particles interacting via EM field exhibits both collective and individual particle behavior, e.g., plasma oscillation & Debye screening [1]**
- **These phenomena can be discussed conveniently by representing particles by delta functions in phase space—Klimontovitch distribution function**
- **The initial value problem of the linearized, coupled equations for Klimontovich distribution and EM field can be solved in 1D by Laplace transform and inversion via Landau contour [2]**
- **Here we discuss another example of practical importance, a high-gain FEL, in which the initial noise becomes organized into quasi-coherent collective motion, giving rise to intense X-ray pulses known as SASE [3]**
- **For 3D, we use the normal mode expansion by Van Kampen [4.5]**

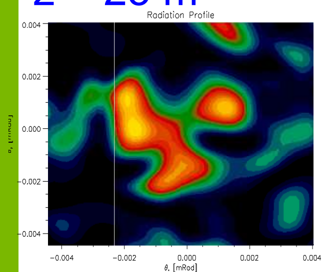
1. D. Pines & Bohm, PR 85, 338 (1952)
2. KJK & R. R. Lindberg, FEL 2011
3. KJK, PRL 51, 1871 (1986)
4.N. G. Van Kampen, Physica XXI, 949 (1955)
5. K. M. Case, Ann. Phys. 7, 349 (1959)

SELF AMPLIFIED SPONTANEOUS EMISSION(SASE)

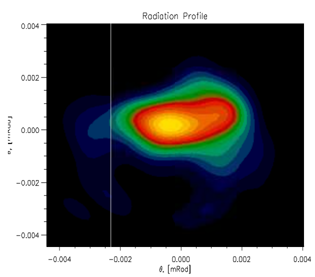


Transverse mode

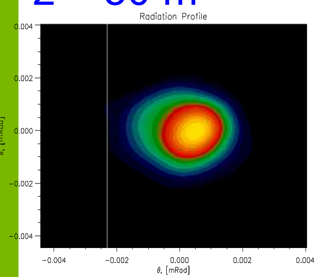
z = 25 m



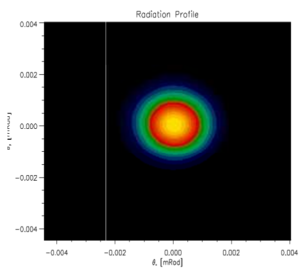
z = 37.5 m

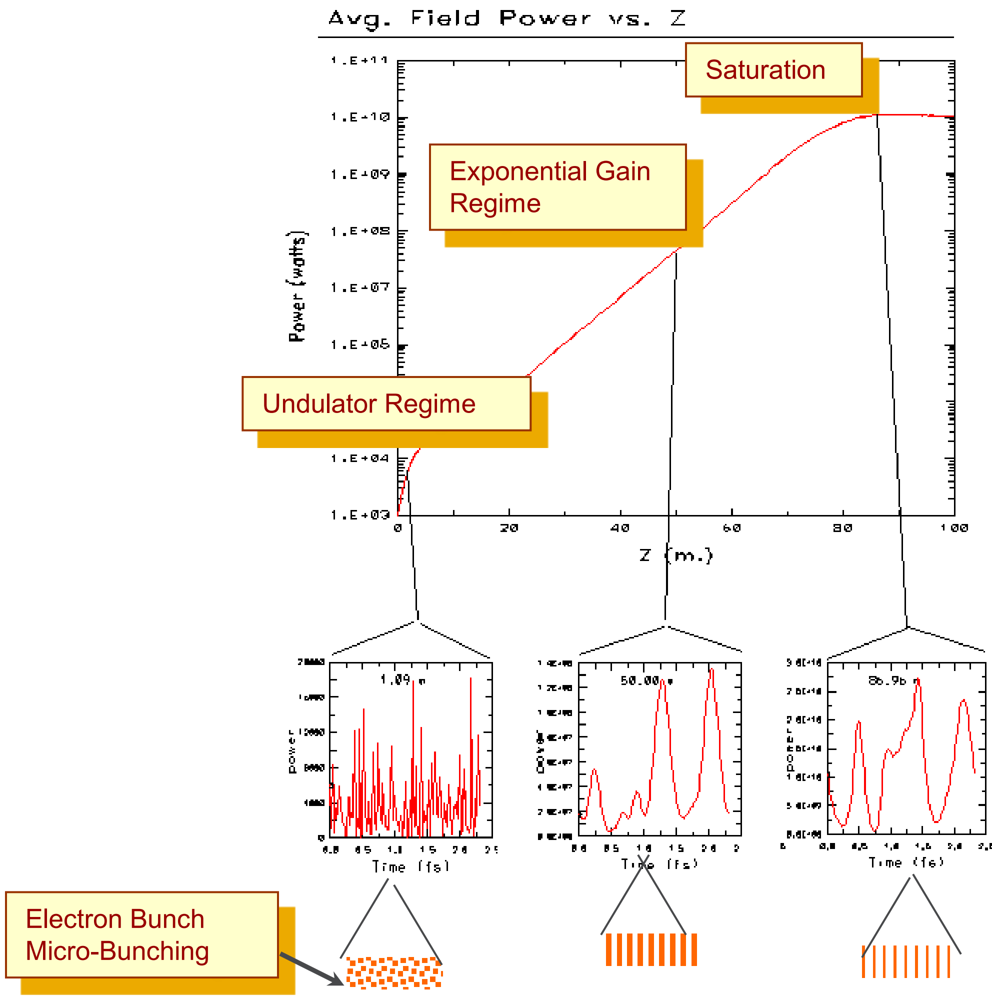


z = 50 m

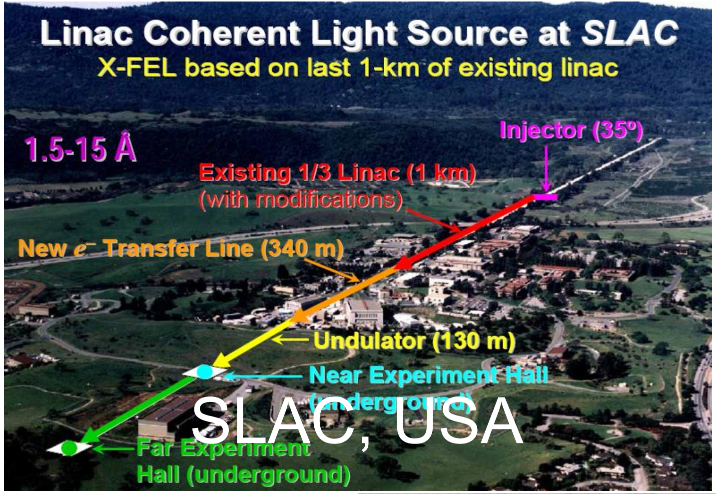


z = 90 m






DESY, Germany


Linac Coherent Light Source at SLAC
X-FEL based on last 1-km of existing linac
1.5-15 Å
Injector (35º)
Existing 1/3 Linac (1 km)
(with modifications)
New e^- Transfer Line (340 m)
Undulator (130 m)
Near Experiment Hall
(underground)
Far Experiment
Hall (underground)
SLAC, USA


SACLA
Spring-8, Japan


PAL-XFEL, Korea

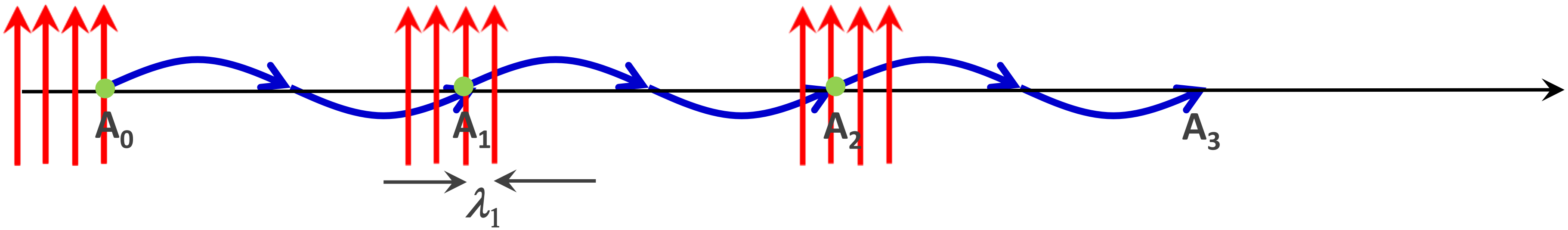

SwissFEL


FERMI, Italy

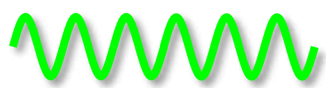

SINAP, China

EM wave amplification by e-beam in undulator

- When the EM wavelength satisfies the undulator condition, an electron sees the same EM field in the successive period→ sustained energy exchange



- An e^- arriving at A_0 loses energy to the field ($e\boldsymbol{v}\bullet\boldsymbol{E}<0$). Similarly the e^- at distance $n\lambda_1$, n=1,2,… also loses energy. However, those at $\lambda_1(1/2+n)$ away gain energy.
- The electron beam develops energy modulation (period length λ_1).



- Higher energy electrons are faster → density modulation develops

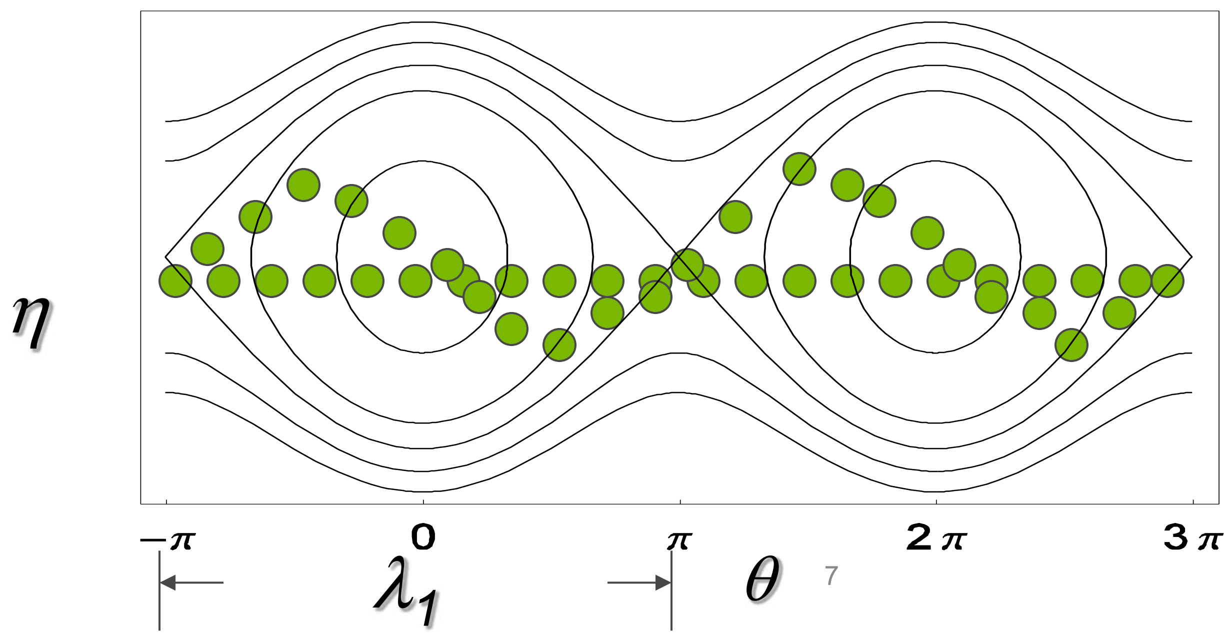
- Coherent EM of wavelength λ_1 is generated→ "Free electron laser"

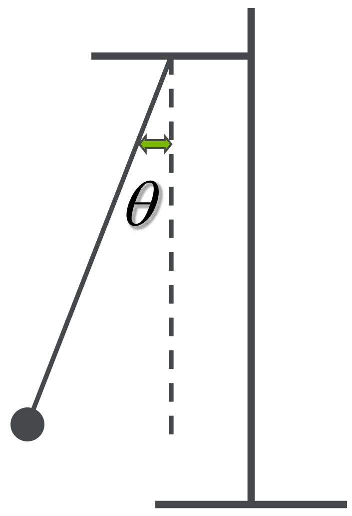
1D FEL pendulum equation for electron motion in combined undulator & radiation field

$$\boxed{\frac{d\theta_j}{dz} = 2\eta_j k_u, \qquad \frac{d\eta_j}{dz} = \chi_1\left(\tilde{E}e^{i\theta_j} + \tilde{E}^* e^{-i\theta_j}\right)}$$

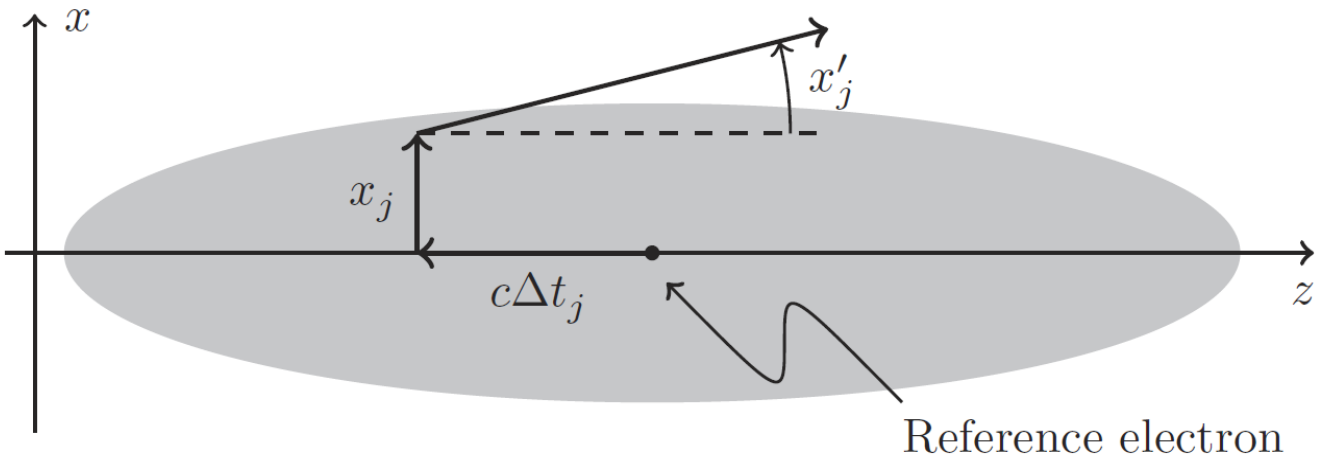
$$\boxed{\chi_1 = \frac{eK[\mathrm{JJ}]}{(2\gamma_0^2 mc^2)}}$$

$$\boxed{[\mathrm{JJ}] = J_0\left(\frac{K^2}{4+2K^2}\right) - J_1\left(\frac{K^2}{4+2K^2}\right)}$$





3D VARIABLES



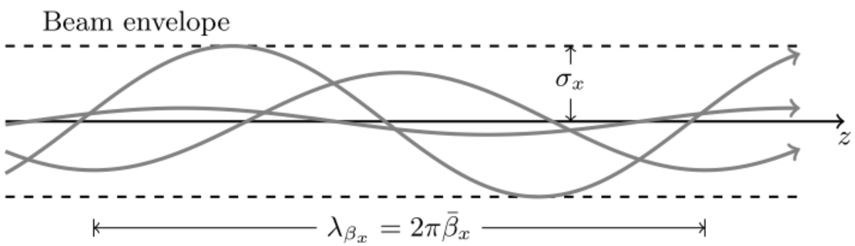
- Variables:
  - "Time" : z
  - Long. position: $\theta_j = 2\pi c \Delta t_j / \lambda$
  - Long. momentum: $\eta_j = \Delta\gamma_j / \gamma_0$
  - Trans. position: x_j
  - Trans. momentum: $p_j = x'_j = dx_j / dz$

3D pendulum equation including the transverse betatron motion

$$\frac{d\theta}{dz} = 2k_u\eta - \frac{k_1}{2}(p^2 + k_\beta^2 x^2),$$

$$\frac{d\eta}{dz} = \chi_1 \int d\nu \, e^{i\nu\theta} E_\nu(x; z) + c.c.,$$

$$\frac{dx}{dz} = p, \qquad \frac{dp}{dz} = -k_\beta^2 x,$$



- In the transverse plane, the electrons perform betatron oscillations, which can be approximated by harmonic motion.

- The longitudinal position θ is delayed due to the transverse action

Vlasov-Maxwell formalism I

- The interaction between the electron beam and the FEL radiation can be described in the framework of the Vlasov-Maxwell equations.

- The e-beam is described in terms of a distribution function $F = F(\theta, \eta, \boldsymbol{x}, \boldsymbol{p}; z)$ in 6D-phase space. In view of the importance of electron discreteness, we use the Klimontovich distribution:

$$F(\theta, \eta, \boldsymbol{x}, \boldsymbol{p}; z) = \frac{k_1}{n_e} \sum_{j=1}^{N_e} \delta[\theta - \theta_j(z)]\delta[\eta - \eta_j(z)] \\ \times \delta[\boldsymbol{x} - \boldsymbol{x}_j(z)]\delta[\boldsymbol{p} - \boldsymbol{p}_j(z)],$$

n_e: on-axis electron number density

- The distribution function is governed by the Vlasov equation

$$\frac{\partial F}{\partial z} + \frac{d\theta}{dz}\frac{\partial F}{\partial \theta} + \frac{d\eta}{dz}\frac{\partial F}{\partial \eta} + \frac{d\boldsymbol{x}}{dz} \cdot \frac{\partial F}{\partial \boldsymbol{x}} + \frac{d\boldsymbol{p}}{dz} \cdot \frac{\partial F}{\partial \boldsymbol{p}} = 0,$$

K.-J. Kim, PRL 57, 1871 (1986)
K.-J. Kim, Z. Huang, R. Lindberg, Synchrotron Radiation and FELs (Cambridge Press, 2017)

Vlasov-Maxwell formalism II

- In the small signal regime, the equation is linearized:

  - Decompose the distribution function into a background distribution function $\bar{F}$ and a small perturbation δF i.e. $F = \bar{F} + \delta F$. We work with the Fourier amplitude $F_\nu = (1/2\pi)\int d\theta(\delta F)e^{-i\nu\theta}$ and $\delta F = \int d\nu F_\nu e^{i\nu\theta}$.

  - Treat F_ν and E_ν for $\nu \simeq 1$ as small compared to $\bar{F}$.

- After some manipulation (which involves using the equations of motion), we obtain a linearized Vlasov equation:

$$\left\{\frac{\partial}{\partial z} + p\cdot\frac{\partial}{\partial x} - k_\beta^2 x\cdot\frac{\partial}{\partial p} + i\nu\left[2\eta k_u - \frac{k_1}{2}(p^2 + k_\beta^2 x^2)\right]\right\} F_\nu = -\chi_1 E_\nu \frac{\partial}{\partial \eta}\bar{F}$$

Vlasov-Maxwell formalism III

- We also use a paraxial wave equation for the radiation field:

$$\left(\frac{\partial}{\partial z} + i\Delta\nu k_u + \underbrace{\frac{\nabla_\perp^2}{2ik_1}}_{\text{3D term giving rise to diffraction}}\right) E_\nu(x;z) = -\chi_2 \frac{k_1}{2\pi} \sum_{j=1}^{N_e} e^{-i\nu\theta_j(z)} \delta[x - x_j(z)]$$

$$\chi_2 \equiv eK[\mathrm{JJ}]/2\varepsilon_0\gamma_r$$

- In terms of the distribution function, the paraxial becomes

$$\left(\frac{\partial}{\partial z} + i\Delta\nu k_u + \frac{\nabla_\perp^2}{2ik_1}\right) E_\nu = -\chi_2 n_e \underbrace{\int dp d\eta \, F_\nu}_{\text{current as a momentum integral of } F_\nu}$$

- These linearized Vlasov-Maxwell equations accurately describe the FEL operation up to the onset of nonlinear, saturation effects.

Scaled equations

- We introduce a set of convenient scaled quantities

$$\hat{z} = 2\rho k_u z \qquad \hat{\eta} = \frac{\eta}{\rho}, \qquad a_\nu = \frac{\chi_1}{2k_u\rho^2}E_\nu = \frac{eK\,[\mathrm{JJ}]}{4\gamma_r^2 mc^2 k_u\rho^2}E_\nu,$$

$$\hat{x} = x\sqrt{2k_1k_u\rho} \qquad \hat{p} = p\sqrt{\frac{k_1}{2k_u\rho}}. \qquad f_\nu = \frac{2k_u\rho^2}{k_1}F_\nu. \qquad \hat{k}_\beta = k_\beta/(2k_u\rho)$$

Pierce-or FEL-parameter

$$\rho = \left[\frac{n_e\chi_1\chi_2}{(2k_u)^2}\right]^{1/3} = \left(\frac{e^2K^2[\mathrm{JJ}]^2 n_e}{32\epsilon_0\gamma_r^3 mc^2 k_u^2}\right)^{1/3}$$

$$= \left[\frac{1}{8\pi}\frac{I}{I_A}\left(\frac{K[\mathrm{JJ}]}{1+K^2/2}\right)^2\frac{\gamma\lambda_1^2}{2\pi\sigma_x^2}\right]^{1/3}$$

- The linearized FEL equations become

$$\left(\frac{\partial}{\partial\hat{z}} + i\frac{\Delta\nu}{2\rho} + \frac{\hat{\nabla}_\perp^2}{2i}\right)a_\nu(\hat{x};\hat{z}) = -\int d\hat{\eta}d\hat{p}\; f_\nu(\hat{\eta},\hat{x},\hat{p};\hat{z})$$

$$\left(\frac{\partial}{\partial\hat{z}} + i\dot{\theta} + \hat{p}\cdot\frac{\partial}{\partial\hat{x}} - \hat{k}_\beta^2\hat{x}\frac{\partial}{\partial\hat{p}}\right)f_\nu = -a_\nu\frac{\partial\bar{f}_0}{\partial\hat{\eta}},$$

phase derivative

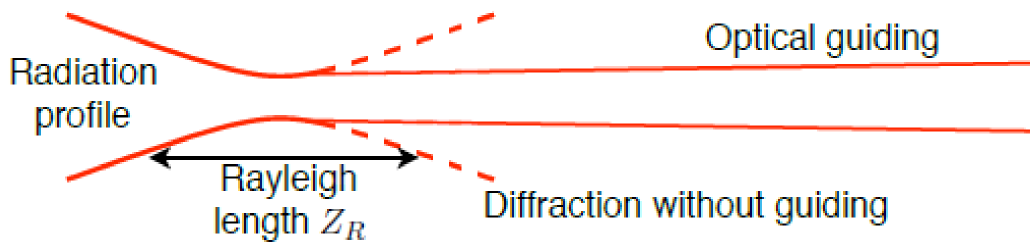
$$\dot{\theta} = \frac{d\theta}{d\hat{z}} = \hat{\eta} - \frac{\hat{p}^2 + \hat{k}_\beta^2\hat{x}^2}{2}$$

Van Kampen's normal mode expansion I

- We seek the eigenmodes of the FEL equations in the form :

$$\Psi = \begin{bmatrix} a_\nu(\hat{\boldsymbol{x}};\hat{z}) \\ f_\nu(\hat{\eta},\hat{\boldsymbol{x}},\hat{\boldsymbol{p}},\hat{z}) \end{bmatrix} = e^{-i\mu_\ell \hat{z}} \begin{bmatrix} \mathcal{A}_\ell(\hat{\boldsymbol{x}}) \\ \mathcal{F}_\ell(\hat{\boldsymbol{x}},\hat{\boldsymbol{p}},\hat{\eta}) \end{bmatrix}$$

- Each mode is characterized by a constant growth rate μ_l and a z-independent radiation/density mode profile A_l/F_l
- **If one mode dominates→ Optical guiding**



- Substituting into the Vlasov-Maxwell (FEL) equations, we obtain two coupled relations for the growth rate and the mode amplitudes:

$$\begin{bmatrix} \mu_\ell \mathcal{A}_\ell + \left(-\frac{\Delta\nu}{2\rho} + \frac{1}{2}\hat{\boldsymbol{\nabla}}_\perp^2\right)\mathcal{A}_\ell + i\int d\hat{\boldsymbol{p}}d\hat{\eta}\;\mathcal{F}_\ell \\ \mu_\ell \mathcal{F}_\ell + i\mathcal{A}_\ell \frac{\partial \bar{f}_0}{\partial \hat{\eta}} + \left\{-\nu\dot{\theta} + i\left(\hat{\boldsymbol{p}}\cdot\frac{\partial}{\partial \hat{\boldsymbol{x}}} - \hat{k}_\beta^2 \hat{\boldsymbol{x}}\cdot\frac{\partial}{\partial \hat{\boldsymbol{p}}}\right)\right\}\mathcal{F}_\ell \end{bmatrix} = 0.$$

Van Kampen's normal mode expansion II

- The second equation can be solved by the method of characteristics:

$$\mathcal{F}_\ell = -\frac{\partial \bar{f}_0}{\partial \hat{\eta}} \int_{-\infty}^{0} d\tau \, \mathcal{A}_\ell(\hat{\boldsymbol{x}}_+) e^{i(\nu\dot{\theta} - \mu_\ell)\tau}$$

- Inserting this into the first, we obtain the "dispersion equation" for the growth rate (eigenvalue) and the eigenmode:

$$\left(\mu_\ell - \frac{\Delta\nu}{2\rho} + \frac{1}{2}\hat{\boldsymbol{\nabla}}_\perp^2\right)\mathcal{A}_\ell(\hat{x})$$
$$- i\int d\hat{p}\,d\hat{\eta}\int_{-\infty}^{0} d\tau\; e^{i(\nu\dot{\theta}-\mu_\ell)\tau}\frac{d\bar{f}_0}{d\hat{\eta}}\,\mathcal{A}_\ell(\hat{x}_+) = 0.$$

$$\hat{\boldsymbol{x}}_+(\tau) \equiv \hat{\boldsymbol{x}}\cos(\hat{k}_\beta\tau) + (\hat{\boldsymbol{p}}/\hat{k}_\beta)\sin(\hat{k}_\beta\tau)$$

- This equation can be solved numerically or by variational method (later)

Van Kampen's normal mode expansion III

- Eigenmode equation can be written as $(\mu_\ell + \mathbf{M})\Psi_\ell = 0$
- The matrix operator ***M*** is not Hermitian & eigenvalue μ_ℓ can be complex
- Van Kampen: introduce the scalar product and adjoint operator $\mathbf{M}^\dagger$

$$(\Psi_1, \Psi_2) \equiv \int d\hat{\mathbf{x}}\, a_{1\nu} a_{2\nu} + \int d\hat{\mathbf{x}} d\hat{\mathbf{p}} d\eta\, f_{1\nu} f_{2\nu} \qquad \left(\mathbf{M}^\dagger \Psi_\ell^\dagger, \Psi\right) = \left(\Psi_\ell^\dagger, \mathbf{M}\Psi\right)$$

- The adjoint eigenvalue equation

$$\left(\mu_\ell^\dagger + \mathbf{M}^\dagger\right) \Psi_\ell^\dagger = 0.$$

- We find that the adjoint dispersion equation is the same as the original equation when the electrons' momentum distribution is symmetric under

$$\hat{\boldsymbol{p}} \to -\hat{\boldsymbol{p}} \rightarrow \mu_\ell^\dagger = \mu_\ell$$

- Thus the Van Kampen orthogonality follows:

$$(\mu_\ell - \mu_k)\left(\Psi_k^\dagger, \Psi_\ell\right) = \left(\Psi_k^\dagger, \mathbf{M}\Psi_\ell\right) - \left(\mathbf{M}^\dagger \Psi_k^\dagger, \Psi_\ell\right) = 0 \rightarrow \left(\Psi_k^\dagger, \Psi_l\right) = \delta_{k,\ell}\left(\Psi_l^\dagger, \Psi_\ell\right)$$

- Thus the initial value problem is solved as

$$\Psi(\hat{z}) = \sum_\ell C_\ell \Psi_\ell e^{-i\mu_\ell \hat{z}} = \sum_\ell \frac{\left(\Psi_\ell^\dagger, \Psi(0)\right)}{\left(\Psi_\ell^\dagger, \Psi_\ell\right)} \Psi_\ell e^{-i\mu_\ell \hat{z}}$$

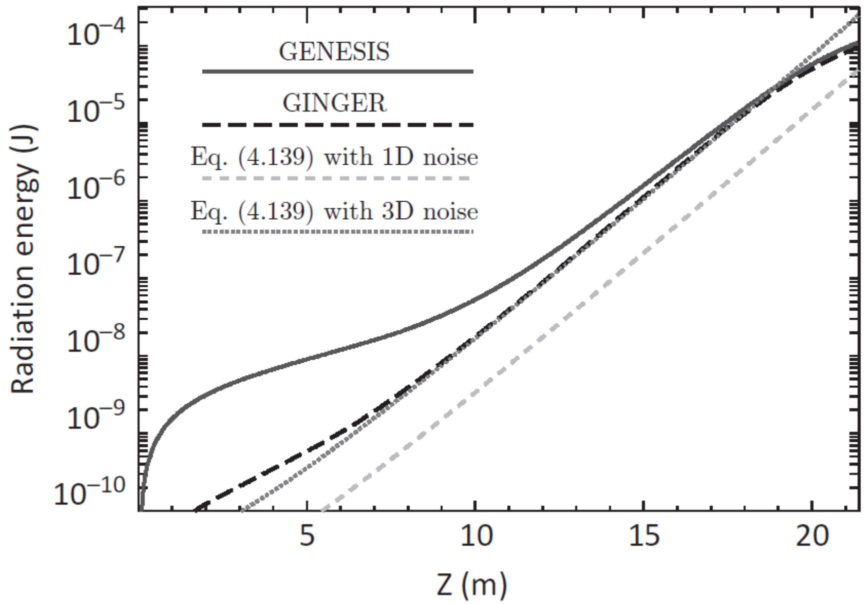
3D solution

- Using a specific $\overline{f}_0$, we obtain an explicit dispersion relation:

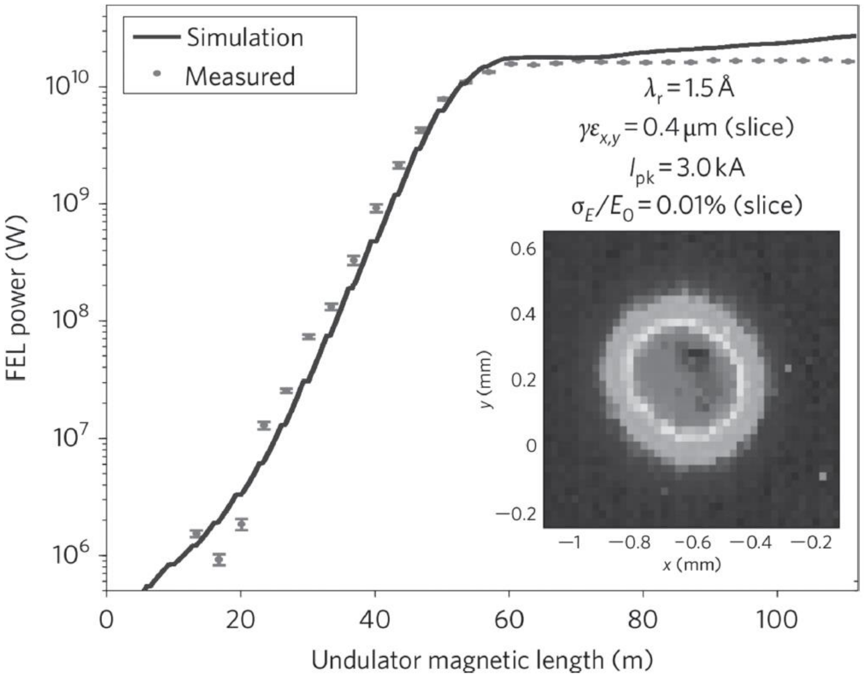
$$\left(\mu - \frac{\Delta\nu}{2\rho} + \frac{1}{2}\hat{\nabla}_\perp^2\right)\mathcal{A}(\hat{x}) - \frac{1}{2\pi\hat{k}_\beta^2\hat{\sigma}_x^2}\int_{-\infty}^{0} d\tau\, \tau e^{-\hat{\sigma}_\eta^2\tau^2/2 - i\mu\tau}$$
$$\times \int d\hat{p}\, \mathcal{A}[\hat{x}_+(\hat{x},\hat{p},\tau)] \exp\left[-\frac{1 + i\tau\hat{k}_\beta^2\hat{\sigma}_x^2}{2\hat{k}_\beta^2\hat{\sigma}_x^2}\left(\hat{p}^2 + \hat{k}_\beta^2\hat{x}^2\right)\right] = 0.$$

- There are four dimensionless parameters that affect the growth rate:
  - ➢ $\hat{\sigma}_x$ is a quantitative measure of the diffraction effect
  - ➢ $\hat{\sigma}_x\hat{k}_\beta$ is a measure of the emittance effect
  - ➢ $\hat{\sigma}_\eta$ represents the energy spread effect
  - ➢ $\Delta\nu/(2\rho)$ is scaled frequency detuning
- The DR can be solved numerically—elaborate but faster than simulation
- Ming Xie (PAC1995, page 183) used a variational technique to obtain a fitting formula that captures all these effects→ FEL design became a simple exercise on spread sheet !

Theory, simulation, and experiment



Power growth in LEUTL FEL



Power growth in LCLS

Concluding remarks

- X-ray FELs have so far have been mainly based on self-amplified spontaneous emission, in which the initial noise due to particle discreteness evolves into gain guided transverse mode
- The system can be succintly discussed by the coupled Klimontovich-Maxwell equations
- The solution of the initial value problem of SASE including electrons' betatron oscillation and 3 D Maxwell equation in terms of Van Kampen mode expansion is formally elegant and provides practical approach for numerical analysis
- The construction of an X-ray FEL theory, the application of the theory to the design and interpretation of actual experiments have been one of the most exciting and successful beam dynamics activities during the last several decades

M1

FROM PARTICLE NOISE TO COHERENT X-RAYS
BEAM DYNAMICS OF X-RAY FREE-ELECTRON LASERS

KWANG-JE KIM   Notes by Jeffrey Heninger

Collective Motion in a System of Discrete Particles

- interacting via EM fields – plasma oscillation & Debye ~~shielding~~ screening
- Klimontovich distribution function – δ functions in phase space
- initial value problem in 1D – Laplace transform, Landau contour for inverse Laplace transform
- for 3D – van Kampen – normal mode expansion

Self Amplified Spontaneous Emission

- electron bunch from accelerator
- magnetic field to make them oscillate
- → radiation
- organizes as it travels. chaotic → Gaussian, clear bands. intensity ↑

EM Wave Amplification by Electron Beam Undulator

- electron undulating, light traveling past
- if there is resonance between the 2 motions, significant energy transfer
- energy transfer different for different electrons → energy modulation
- higher energy electrons faster → density modulation
- exponential growth of EM wave

pendulum equation for motion of electrons

coordinates arranged around electron bunch

in transverse plane, focus → betatron oscillation – approximately harmonic motion

Vlasov – Maxwell formalism

- discreteness of electrons important – use Klimontovich distribution
- Vlasov equation for dynamics of distribution function
- perturbation expansion – smooth background + small perturbation
  - discreteness & small fluctuations
  - → linearized Vlasov equation
- Paraxial Wave Equation
  - ~~light~~ light emitted by electrons mostly travels in forward direction
- linearized – accurate up until saturation (compare to numerics)
- scale variables → nondimensional FEL parameter

van Kampen's Normal Mode Expansion

- single frequency along beam path ($\hat{z}$, i.e. time)
- initial value problem as a linear ~~supp~~ superposition of normal modes.
- after a long time, a single mode will dominate – called optical guiding here

M2

What are the normal modes?

2^{nd} equation by method of characteristics – turns 1^{st} equation into "dispersion relation"

can solve dispersion equation numerically or by a variational method

↳ made FEL design simple

use normal modes to solve initial value problem

general solution is sum over normal modes – determine coefficients using orthogonality of modes

eigenmode equation is not Hermitian – define adjoint using inner product → left & right eigenvectors

if the electron's momentum distribution function is symmetric under $\hat{p} \to -\hat{p}$,

then eigenvalues of M and M^{+} are the same → van Kampen orthogonality

Comparison between theory, simulation, and experiment