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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Jeffrey Heninger **Email/Phone:** jeffrey.heninger@yahoo.com

Speaker's Name: Marian Gidea

Talk Title: Hamiltonian Instability via Geometric Method

Date: 11 / 30 / 2018 **Time:** 11 : 00 **am** / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: Extends Arnold diffusion to any system with a normally hyperbolic invariant manifold (NHIM). Uses outer dynamics along the hyperbolic orbits to define a scattering map on the NHIM. Builds a pseudo-orbit by iterating the inner dynamics and the scattering map alternatively. There are several scattering lemma that ensure that, if the inner dynamics are iterated sufficiently many times between iterations of the scattering map, a real orbit exists close to the pseudo-orbit. This argument only requires Poincare recurrence to show diffusion and if there isn't Poincare recurrence, diffusion is immediate.

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HAMILTONIAN INSTABILITY VIA GEOMETRIC METHOD

MARIAN GIDEA   Notes by Jeffrey Heninger

Arnold Diffusion Problem

rotator + pendulum + small perturbation

for small enough ε, there are trajectories for which the action changes by an amount ($\mathcal{O}(1)$)
which is independent of the perturbation

add extra variable to make it time-independent

identify tori, manifolds in unperturbed system

small positive ε – doesn't destroy the tori (not generic)

stable & unstable manifolds split & cross with neighboring manifolds

creates a transition chain – can move between ~~varied~~ values of action by
following a chain of unstable & stable manifolds

a general perturbation would destroy some of the tori – KAM

gaps ~ $\sqrt{\varepsilon}$, transition link ~ ε $\Rightarrow$ chain can't directly cross a gap.

can use transition chains, but not just of tori – also more complicated objects

integrable Hamiltonian system, $n > 2$

"generic" perturbation $\Rightarrow$ $\exists$ trajectories whose action changes by $\mathcal{O}(1)$

classification

a priori stable – expressed only in terms of action, completely foliated by tori

a priori unstable – unperturbed system has hyperbolicity – Arnold's case

a priori chaotic – hyperbolic basic set

Questions:

ask ~~⇒~~ about a specific (not generic) Hamiltonian – existence of diffusing orbits

interesting for applications – 3 body problem, accelerators, plasma confinement, flows, ...

unperturbed Hamiltonian may not be convex/superlinear $\rightarrow$ no variational method

can you control the system using the perturbation

quantitative estimates on diffusing orbits – speed, Hausdorff dim of ICs, ...

Geometric Method:

existence of normally hyperbolic invariant manifold (NHIM)

two dynamics

inner – in NHIM, outer – along heteroclinic orbits

this work mostly looks at outer dynamics – only require Poincaré recurrance on inner dynamics

does not ~~see~~ require invariant tori, Aubrey-Mather sets, etc. on inner dynamics

in explicit applications, you can use these structures to improve on Poincaré recurrance time

Q2

NHIM – invariant manifold Λ with associated stable & unstable manifolds

stable & unstable manifolds foliated by fibers – equivariant under dynamics

Scattering Map

assume stable & unstable manifolds intersect transversally

follow fibers to footpoints on NHIM x_-, x_+

the map between footpoints ($x_- \mapsto x_+$) is the scattering map

~~pseudo~~ forward iterates of ~~[illegible]~~ intersection converge to forward iterates of x_+

pseudo-orbit – iterate inner dynamics, then scattering map, then inner dynamics, ...

looks ~~[illegible]~~ like an orbit that goes through the intersection

Perturbed System Scattering Map

expand scattering map of perturbed system in terms of scattering map of unperturbed system

Melnikov integrals

orbits of unperturbed system & form of perturbation $\rightarrow$ take one integral

~~[illegible]~~ Set-up for Main Results

$(\mathcal{M}, \omega)$ – symplectic manifold

family of symplectic maps – depend on ε

~~[illegible]~~ NHIM for each ε – smooth parametrization in terms of unperturbed NHIM

family of scattering maps – expanded in ε.

take unperturbed scattering map = identity

interated function system (IFS) – inner dynamics & scattering maps

Objectives

find pseudo-orbits of IFS that diffuse

show that there are ~~[illegible]~~ real orbits that follow these ~~pseudo-orb~~ pseudo-orbits

Ex

a bunch of coupled pendula – fails convexity

Shadowing lemma 1

take a pseudoorbit $y_{i+1} = \underset{\uparrow \text{inner}}{f_\varepsilon^{m_i}} \circ \underset{\uparrow \text{scattering}}{\sigma_\varepsilon^{I(i)}} \circ \underset{\uparrow \text{inner}}{f_\varepsilon^{n_i}}(y_i)$ look for true orbit δ-close

have to choose m_i, n_i (times for inner dynamics) carefully

choice of m sufficiently big – in terms of all previous history

Remarks

shadowing lemma for hyperbolic systems don't apply

no assumption on inner dynamics

can't choose n, m uniformly for all time

[Gelfreich & Turaev] – similar result

Shadowing lemma 2

consider an orbit of IFS created by applying a scattering map (or multiple) map times

Why would there be multiple? Multiple heteroclinic connections.

assume inner dynamics are recurrant

then there is a true orbit ~~whoos~~ whose orbit is close to this orbit of IFS

use alternating Shadowing lemma 1 & Poincaré Recurrence to prove

to use Poincaré Recurrance, we might need larger & larger times for inner dynamics

Shadowing lemma 3

scattering maps are given in terms of expansions

~~see~~ orbit of IFS is close to a real trajectory of the original Hamiltonian dynamics

Thm

regularity / continuity
must look like a pendulum
nondegeneracy – in term of Melnikov integrals

$\Rightarrow$ exist diffusing orbits

No assumptions of convexity, anything about rotator

Problem in higher dim

to have Poincaré recurrance, inner dynamics must be confined

Dichotomy:

if Poincaré recurrance, above argument

if no Poincaré recurrance, then inner dynamics by itself has diffusion

Connection with Control theory

Hörmander Condition – at each point, Lie algebra of Hamiltonian vector field spans whole space

Idea of Proof – call 2 theorems

(1) Any 2 points can be connected by piecewise smooth curves [Chow, Rashevsky] either in positive or negative time

(2) Each pseudoorbit can be approx. by a pseudoorbit only in positive time

You have a pseudoorbit, so you can shadow it.

Controllability

you can construct a true orbit that visits a sequence of prescribed targets along the path, even though you can't exactly follow the path.

needs to have at least two scattering maps