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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Chanel Lee Email/Phone: chanelclee@gmail.com

Speaker's Name: Maria Beatrice Pozzetti

Talk Title: Orbit Growth Rate for Maximal Representations

Date: 08/15/2019 Time: 9:30 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: The lecture covers maximal reprensations, character varieties, and orbital growth rate.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - **Computer Presentations**: Obtain a copy of their presentation
    - **Overhead**: Obtain a copy or use the originals and scan them
    - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
    - **Handouts**: Obtain copies of and scan all handouts
- ☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
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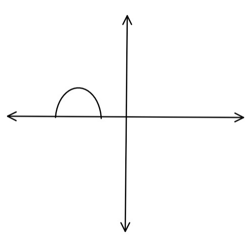
Orbit Growth Rate for Maximal Representations

Maria Beatrice Pozzetti

August 15, 2019

I. Max Reps and Character Varieties

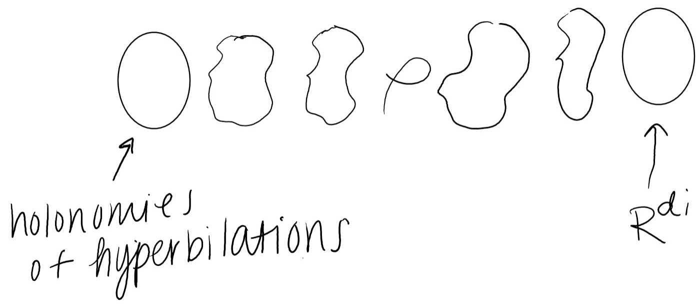
$\Gamma_g = \pi_g(S_g)$
G - semi-simple Lie group of the non compact type
$G = \mathrm{Iso}^\circ(\chi)$
χ - symmetric space
G/k - non-positively curved homogenous Riemann manifold
e.g. $G = \mathrm{PSL}_2\mathbb{R}$
$\chi = \mathbb{H}^2$ - hyperbolic plane



Character variety

$$\begin{aligned} R(\Gamma_g, G) &= \mathrm{Hom}(\Gamma_g, G)/G \text{ (conjugation)} \\ &= \text{"actions of } \Gamma_g \text{ on } \chi\text{"} \end{aligned}$$

$R^{di}(\Gamma_g, G) \subset R(\Gamma_g, G)$
$R^{di}(\Gamma_g, G)$ injective homorphisms with discrete image
e.g. $G = \mathrm{PSL}_2\mathbb{R}$
Goldman $R(\Gamma_g, G)$ has $4g - 3$ connected components.



$$R^{di}(\Gamma_g, G) = \text{Teichmuller space of } S_g$$
$$\overset{unif.}{=} \text{marked hyperbolic structures on } S_g$$

Q. Are there other Lie groups G for which R^{di} contains a union of components?
Higher Rank Teichmuller.
e.g. $R(T_g, \mathrm{PSL}_2\mathbb{C})$ connected
components of $R(\Gamma_g, C_2)$ contains in $R^{di}(\Gamma_g, C_2)$

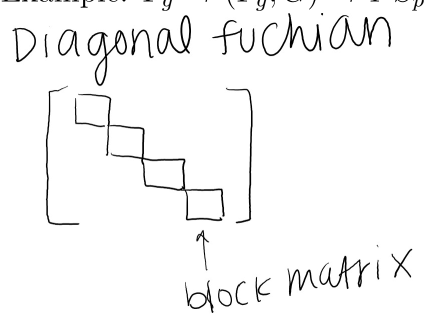
<u>Maximal Representations</u>

χ is herimitian $\rightarrow \chi$ is Kahler, $\omega \in \Omega^2(\chi)^G$

$\rho \in R(\Gamma_g, G) \rightarrow T(\rho) = \int_{S_g} \rho^x \omega$

$|T(\rho)| \leq 2\pi\chi(S_g) rk(G)$

Def. If the above holds, the ρ is maximal.
Thm. [Burgec, Iazzi, Labourie, Weinhard]
$M(\Gamma_g, G) \subset R^{di}(\Gamma_g, G)$ maximal representation.
Example. $\Gamma_g \xrightarrow{i} (\Gamma_g, G) \xrightarrow{\Delta} PS_p(z_n, \mathbb{R})$ (diagonal Fuchsian) is maximal.



<u>Tool</u>: Boundary Maps
<u>Key</u>:

$\rho : \Gamma_g \rightarrow S_p(z_n, \mathbb{R})$ is maximal iff it admits a continuous <u>monotone</u> equivariant map
$\zeta : \partial T \rightarrow \mathcal{L}(\mathbb{R}^{2n}) = G/Q$ where $T = S'$ and $\mathcal{L}(\mathbb{R}^{2n}) \underline{\subseteq} \partial_n \chi$

Rmk: $\zeta(\partial T)$ is locally the graph of the Lipschitz function.

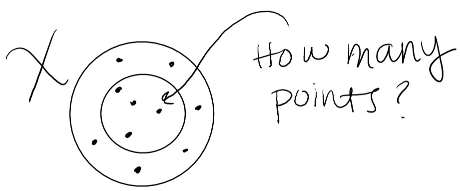
Hitchin Components

G split Lie group connected components of $\Gamma_g \xrightarrow{i} \mathrm{PSL}_2\mathbb{R} \xrightarrow{\text{irreducible}} \mathrm{PSL}_n\mathbb{R}$

Positive Representation (Guichard-Wienhard)

$\rho : \Gamma_g \to S_0(p,q)$

II. Orbit Growth Rate



$$h_\rho^\chi = \lim_{R\to\infty} \frac{1}{R} log\#\{\gamma \in \Gamma_g | d(0, \gamma\circ) < R\}$$

Thm:

(PSW) $\rho : \Gamma \to$ G is maximal, then $h_p\chi \leq h_{p_0}\chi$ for ρ_0 Diagonal Fuchian rep.

On the proof:

χ is symmetric space G^+ weye chamber $= \chi^2/G$.

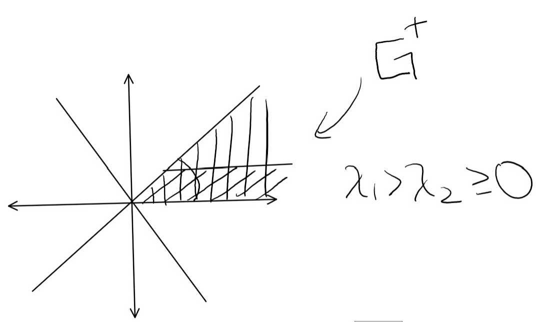
Cartan projection list of logs of singular values of $\rho(\gamma)$.

$d^{G^+}(0, \rho(\gamma)\circ)$

$\rho : \Gamma_g \to S_\rho(\mathbb{C}, \mathbb{R})$

Given $\rho : G^+ \to \mathbb{R}$

$$h_\rho^\rho = \lim_{R\to\infty} \frac{1}{R} log\#\{\gamma \in \Gamma_g | \rho(d(0, \rho(\gamma)\circ) < R\}$$



Thm B

$\forall \rho$ maximal $h^{z\partial n}(\rho) = 1$