Joint work with

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$\mathcal{L}M = C^\infty(S; M)$ (finite dim manifold M) $\qquad (C^\infty(S))^{\dim M}$

$\mathcal{L}_s M = H^s(S; M) \qquad s > \frac{1}{2}$

want to think it as a manifold

$C^\infty(M)$

$\downarrow$ pull back

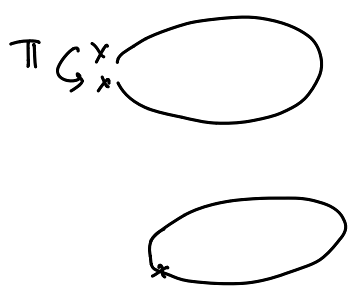
(trivial) $\quad \mathcal{L}M \leftsquigarrow C^\infty(M)$

$\mathrm{Diff}^+(S) \times \mathcal{L}M \to \mathcal{L}M.$

Example:

Transgression

$\mathcal{L}M \to \mathbb{T} \cong U(1)$ (holonomy) $\quad \longleftarrow \quad L \to M$ (L complex, Hermition line bundle)



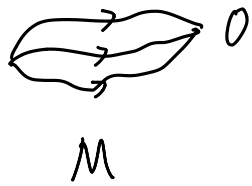
Invariant under reparameterize

$PM = C^\infty([0,1]; M)$

$\downarrow$ endpoints

M^2

Fusion



$P_s^{[3]}M \rightrightarrows \mathcal{L}_s M$

$\frac{1}{2} < s < \frac{3}{2}$

Fusive: Fusion + Diff^+ equivariance

$\widehat{LG}$ $\quad$ $\widehat{LE}$

$\uparrow$ $\quad$ $\uparrow$

$\mathcal{L}G$ — $\mathcal{L}E$ $\qquad$ G — E $\qquad$ V vector bundle

Pull back $\qquad$ $\downarrow$ $\quad$ ⇠ $\quad$ $\downarrow$

$\mathcal{L}M$ $\qquad$ M $\qquad$ M "Dirac operators"

Principal bundle

$\downarrow$

$\mathbb{T} \to \widehat{\mathcal{L}G} \to \mathcal{L}G$

? Cortas extension

$G = \mathbb{T}$

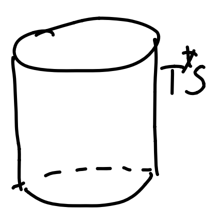
"$G = \mathbb{T}$" $\qquad$ $\mathcal{L}G \ni f : C^\infty(S) \to C^\infty(S)$

$\times f$

$HC^\infty(S)$

$\| $

$\mathrm{span}\{e^{ik\phi}, k \geq 0\}$



$H^{-} \oplus = L^2(S)$

$\ell \overset{\sigma}{\leftsquigarrow} \pi \ell \pi : H \to H$ $\quad C^{\infty}(S \times S)$

$$\pi \ell_1 \pi \pi \ell_2 \pi = \pi \ell_1 \ell_2 \pi + \pi \alpha \pi$$

$\ell_0 \pi \subset \mathcal{L}\pi$

$$\mathcal{G}_0 = \{ \pi \ell \pi + \pi \beta \pi : \beta \in \Psi^{-\infty}(S) \}$$

$\downarrow$ winding # = 0

$\det_H : G^{-\infty}(H) \to \mathbb{C}^*$

$G^{-\infty}(H) \to \mathcal{G}_j \xrightarrow{\sigma} I\pi$

$G^{-\infty}(H) = (Id + \pi \Psi^{-\infty}(S) \pi)^{-1}$

$G^{-\infty}_{\det = 1}(H)$ is ___?___ in $\mathcal{G}_j$

$$\hat{\mathcal{L}}_0 \pi = \mathcal{G}_0 / G^{-\infty}_{\det}(H).$$

Fock Space:

GT of "H-liti" (?) space $C^{\infty}(S)$

$G^{-\infty}(S) / G_H(S)$ $\qquad$ $G_H^{-\infty} \ni g \in G_T^{-\infty}(S)$, $gH = H$

$\ell : Gr \to Gr$

$$\begin{pmatrix} 0 & \pi \ell \pi + \beta \\ \pi \ell \pi + \beta & 0 \end{pmatrix}$$

$\mathcal{H} = gH$

$g \in G^{-\infty}(\mathcal{S})$

$\ell \cdot \mathcal{H} = \ell_g \hat{\ell}$

$\uparrow$

Gr

Line bundle

$$\mathcal{D} = G^{-\infty}(\mathcal{S}) / G_{H, \det_\pi = 1}(\mathcal{S})$$

$\downarrow$

Gr

$\hat{\mathcal{L}}_\pi$ act on $\mathcal{F}$

?

$\mathcal{F}$ = holomorphic ______ $A \; \mathcal{D}'$

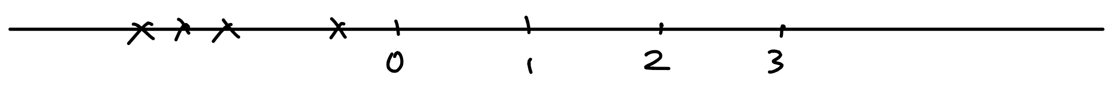
Fock space

$\downarrow$

Gr

$g \in G^{-\infty} \Rightarrow \sigma_g \in \mathcal{F}$

$G^{-\infty}(S) \ni h \longmapsto \det_H({}^t g h)$



S_-

$S \subset \mathbb{N}' \setminus S_+$

$|S_-| = |S_+|$

$H_y \in Gr \qquad e^{ih\theta}, \theta \in \mathcal{S}$

λ_S^{-1}	$Id \setminus \Pi_{S_+}$
$Id - \Pi_{S_-}$	λ_S

$\sigma_S \in \mathcal{L}$

Hilbert

$\sum_S C_S \sigma_S \qquad C_S \in \ell^2(\cdot)$ "L^2"

$$\ell(\mathcal{S}) = \sum_{h \in S_+} k - \lambda_S(h)$$

$$\mathcal{L}^t = \{ \sum C_S \sigma_S ; \ C_S \in \ell(S)^{t/2} \in \ell^2(\downarrow) \}$$

$\mathcal{L}^\infty \qquad \widehat{D\#}^+(S)$ act $\mathcal{L}^t$

$SO(n) \leftarrow Spin(n) \leftarrow Str(n) \leftarrow$

∂

$\mathcal{L}M$