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NOTETAKER CHECKLIST FORM

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Speaker's Name: Ileana Streinu

Talk Title: Metamaterial designs with expansive and auxetic deformations

Date: 10 /01 /2018 **Time:** 9 : 30 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences:
Geometric language is used to express expansive deformations of auxetic materials. In dimension two, expansive periodic structures can be understood in terms of pseudo-triangulations. This characterization allows an effective procedure to generate infinitely many planar periodic expansive designs. These techniques are illustrated with auxetic blueprints in dimension three.

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Speaker: Ileana Streinu

Title: Metamaterial designs with expansive and auxetic deformations

Note Taker: Małgorzata Marciniak

It is an adventure to venture outside your field into another one. A beautiful interplay can exists between geometry when you listen and get problems from other fields.

1. **Introduction. Auxetic metamaterials: a visual introduction**

Metamaterials are usually not found in nature and if they appear, they are sporadic. Auxetic material have negative Poisson's ratio: when you stretch it one direction it expands in all directions (other materials shrink in other directions). It is still a working definition, not sure how to define it precisely in mathematical terms. Usually we work within infinite lattices.

For the Hoberman's sphere one single force (one dimension) opens the entire structure. There is one degree of freedom for the bar-and-joint frameworks considered.

2. **Motivation.**

Deformations of flexible crystal framework and classifications which of them are auxetic. Try to apply mathematical techniques to see which crystal structures are auxetic? By comparison of pictures: with the same symmetry acting on both pictures, vertices are displaced and everything else is preserved.

Which principles can guide the design of new auxetic materials. Most frequently mentioned crystal is alpha-crystalobalite. Crystal deformations explained with an example of the Linus Pauling article from 1930 about sodalite, where the framework collapses. In fact, this metamaterial does not have one degree of freedom and the transformation actually goes through a singularity. Here mathematics helps understanding the behavior more deeply which may not be available by simply building a model.

Literature has a limited catalog of carefully analyzed designs usually based on Grimas rotating squares or Lake's "re-entrant" honeycomb. There are variations of these motifs, including 3D. The goal is to be able to design them systematically.

3. **Definitions: Geometric Auxetics**

Geometric auxetics are purely mechanic (kinematic) so far, without properties of atoms, forces between them or any physical aspects. When are they flexible and how do they deform? What makes structures expansive and auxetic?

Begin with an infinite periodic graph G with defined orbits and group Γ that act on it in a periodic way. In addition, we need placement p, and realization of the framework π (configuration space).

The geometric auxetic is defined as (G, Γ, p, π) so it lives in the realm of algebraic geometry and everything can be expressed in terms of a (finite) set of algebraic equations. Allow only deformations that preserve the periodicity under the group Γ. Specifying Γ is important since different groups acting on the same infinite graph create different periodic graphs.

Geometric Auxetic is a periodic structure of rigid bars connected by joints, constrained to move in a way that preserves the periodicity group. One degree-of-freedom deformation curve is seen as a curve in configuration space.

In one degree-of-freedom expansive deformation no two points get closer. In reverse direction the deformation is contractive.

This definition offers a lot of room for research and investigations in 3dimensions.

Definition: A one-parameter deformation of periodic framework is an auxetic path when for any $t_1 < t_2$, the linear operator taking the period lattice Λ_{t_2} to Λ_{t_1} is a contraction i.e., has operator norm at most 1.

Theorem: A one-parametric deformation of a periodic framework is an auxetic path when the curve given by the Gram matrices of a basis of periods has all velocity vectors (tangents) in the positive semidefinite cone.

The theorem above is an analogy to "casual lines" in special relativity i.e., curves with all its tangents are in the "light cone." In fact, in dimension 2 the statements are equivalent.

An auxetic material is defined to have an auxetic path stating from a "native" state.

4. Algorithms. Predicting expansiveness and auxeticity

Predicting whether a framework has non-trivial infinitesimal expansive deformation can be done with linear algebra. Deciding if a 2d generic framework has non-trivial infinitesimal expansive deformations can be done with combinatorial algorithms.

F has non-trivial infinitesimal auxetic deformations if and only if its spectrahedral cone is non-trivial. Thus, it can be decided using semi-definite programming (for fixed dimension in polynomial time) with the algorithm by Khachian and Porkolab (1991).

5. Large deformations. Recognizing the expansive and auxetic intervals

This theory is new and is still in process of implementation. Demonstration for 2-dimensional situation, where it is shown that only if the vertices lay on an ellipse the framework is auxetic.

6. Properties

Characterization of expansive planar periodic mechanisms with examples from the book by Grunbaum and Shephard "Tilings and Patterns"

Theorem: Periodic 1dof expansive mechanisms are kinematically equivalent to periodic pointed preudo-triangulations.

Proofs are inspired by results from two geometry papers (from 1864 and 1870) of James Clerk Maxwell:

Theorem (Maxwell-Laman type characterization of periodic minimally rigid bar-and-joined graphs):

A multi graph with n vertices and $dn + \binom{d}{2}$ edges is a quotient graph of some infinitesimally minimally rigid periodic bar-and-joint framework in $\mathbb{R}^d$ iff it contains a subgraph with $dn - d$ edges and sparsity type.

Maxwell's Theorem (1870): A planar geometric graph supports a nontrivial stress on its edges iffit has a dual reciprocal diagram iff it has a nontrivial lifting to 3D as a polyhedral terrain.

7. **Designs. Infinitely many designs of expansive and auxetic frameworks in dim 2 and higher.**

Recipe:

a) Start with an arbitrary unit cell and set of points
b) Add arbitrary edges, as long as they do not violate non-crossing and pointedness
c) When this is no longer possible: the result is a pointed pseudo -triangulation, which is expansive and 1dof

There is still no complete characterization.

KINARI, software implementation