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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Malgorzata Marciniak **Email/Phone:** mmarciniak@lagcc.cuny.edu 5734620411

Speaker's Name: Andy M Kraynik

Talk Title: The shape of random soap froth

Date: 10 /05 /2018 **Time:** 9 30 **[✓ am] / pm (circle one)**

Please summarize the lecture in 5 or fewer sentences:
The geometrical problems in the soap froth is explored through simulations with the Surface Evolver. Foam structures are ranging in complexity from perfectly ordered foams based on the Kelvin cell to random polydisperse foams with 12^3 cells. The individual cells have a wide distribution of shapes and sizes. The connection between elastic-plastic rheology and foam structure involves intermittent cascades of topological transitions; this cell-neighbor switching is a fundamental mechanism of foam flow.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - **Computer Presentations**: Obtain a copy of their presentation
    - **Overhead**: Obtain a copy or use the originals and scan them
    - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
    - **Handouts**: Obtain copies of and scan all handouts
- ☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Sandia National Laboratories

The shape (feel and aging) of random soap froth

MANCHESTER 1824

FAU FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

UNIVERSITÉ PARIS SUD

Caltech

TEXAS The University of Texas at Austin

Durham University

Andy Kraynik
Sandia National Labs (retired)

Doug Reinelt (SMU)

Frank van Swol (Sandia - retired)

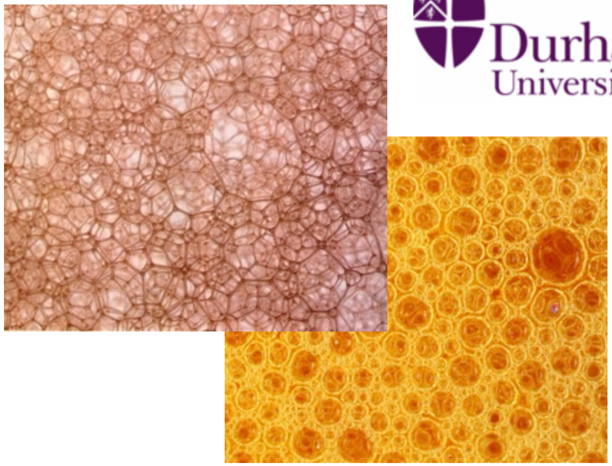
Sascha Hilgenfeldt (Illinois)

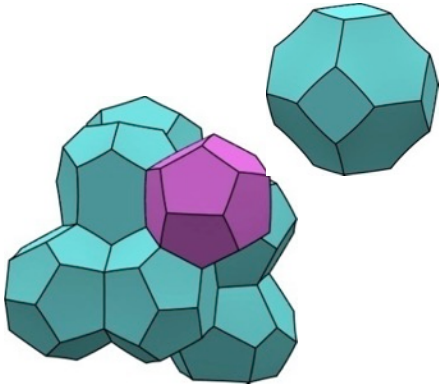
Myfanwy Evans (TU Berlin)

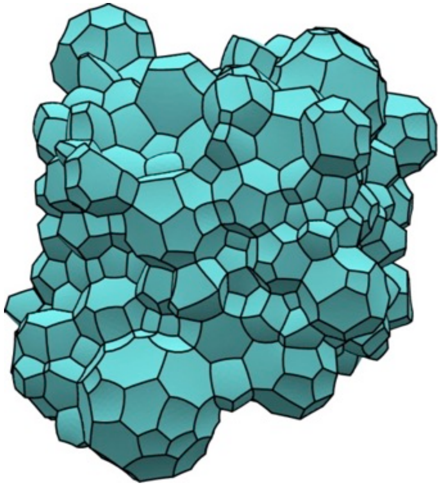
Gerd Schroeder-Turk (Murdoch)

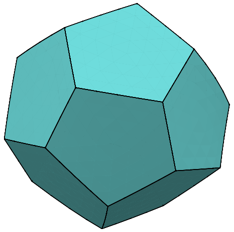
Klaus Mecke (Erlangen)

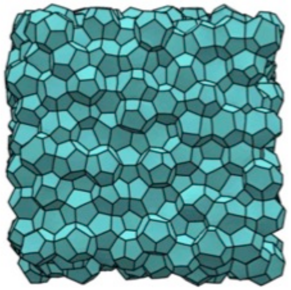
Claudia Redenbach
TU Kaiserslautern

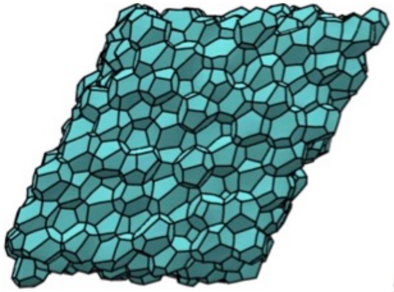


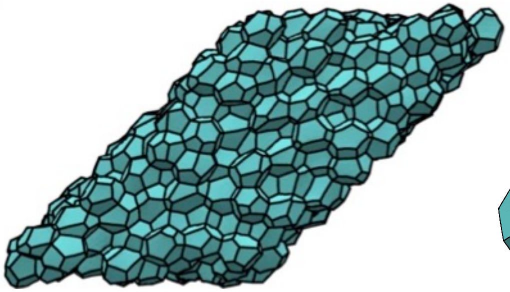


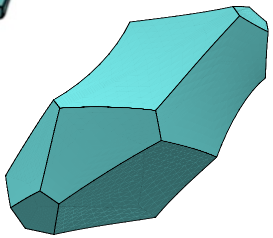








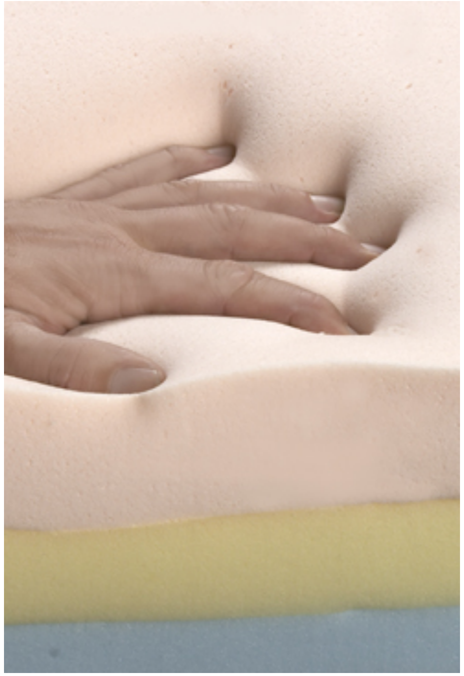




Applications

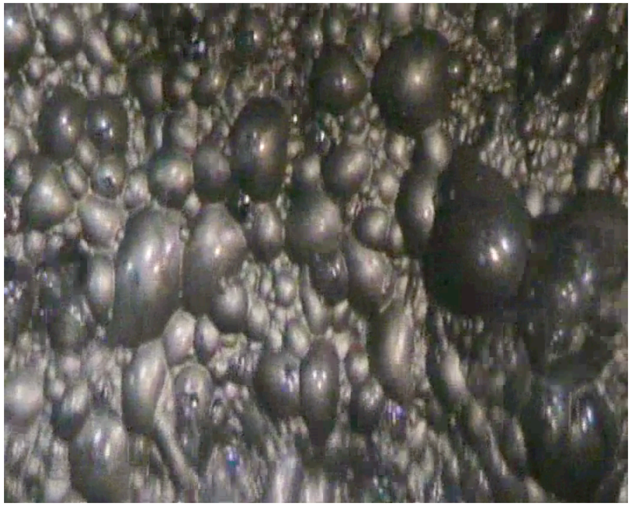
Personal Care

Food and Beverage



Solid Foams

Drilling Fluid



Froth Flotation

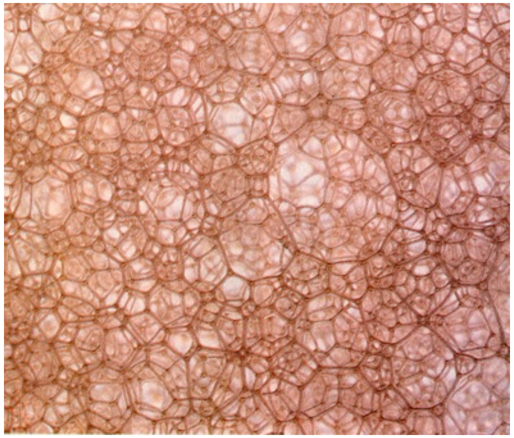
Fire Fighting

Familiar Foams

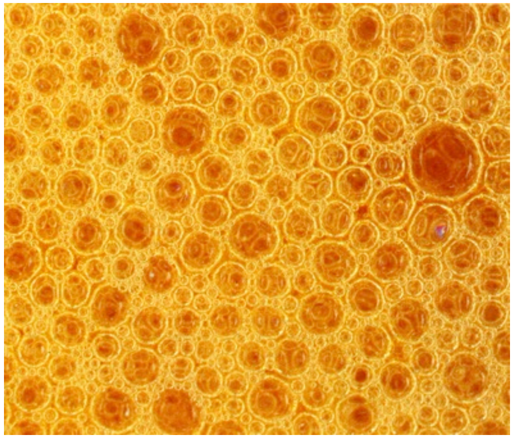
Low-Density Foams

Dense Foams

Liquid Foams

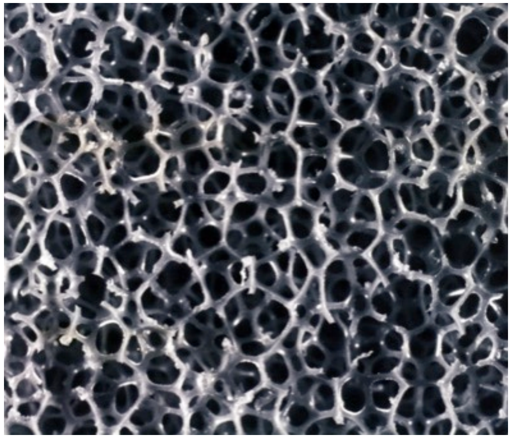


Soap Froth - “Dry” Foam
Drained Beer Foam

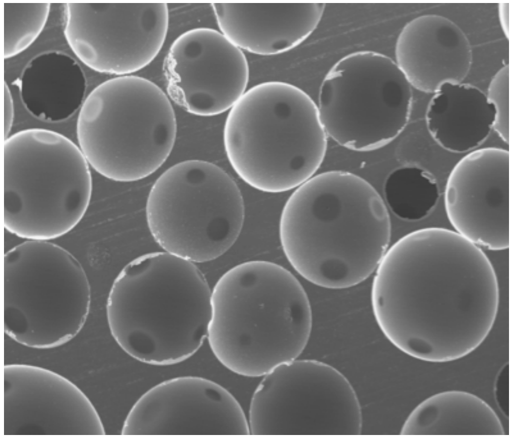


“Wet” Foam
Fresh Beer Foam

Solid Foams



Low-Density Open-Cell Foam
Flexible Polyurethane Foam



Dense Closed-Cell Foam
Rigid Polyurethane Foam

“Foam Micromechanics”

foam energy E , stress $\boldsymbol{\Sigma}_{ij}$ *, shear modulus G* ~ σ/R , $\sigma/V^{1/3}$

Plateau’s Laws

Simulating random soap froth with the Surface Evolver

Foam and cell morphology
- topological and geometric statistics
  - Matzke (1946)

Rheology – quasi-static simple shearing flow
- shear modulus
  - Princen & Kiss (1986)
- yield stress

Diffusive coarsening – aging mechanism
- von Neumann’s law in 3D

Mechanics of solid foams with open cells

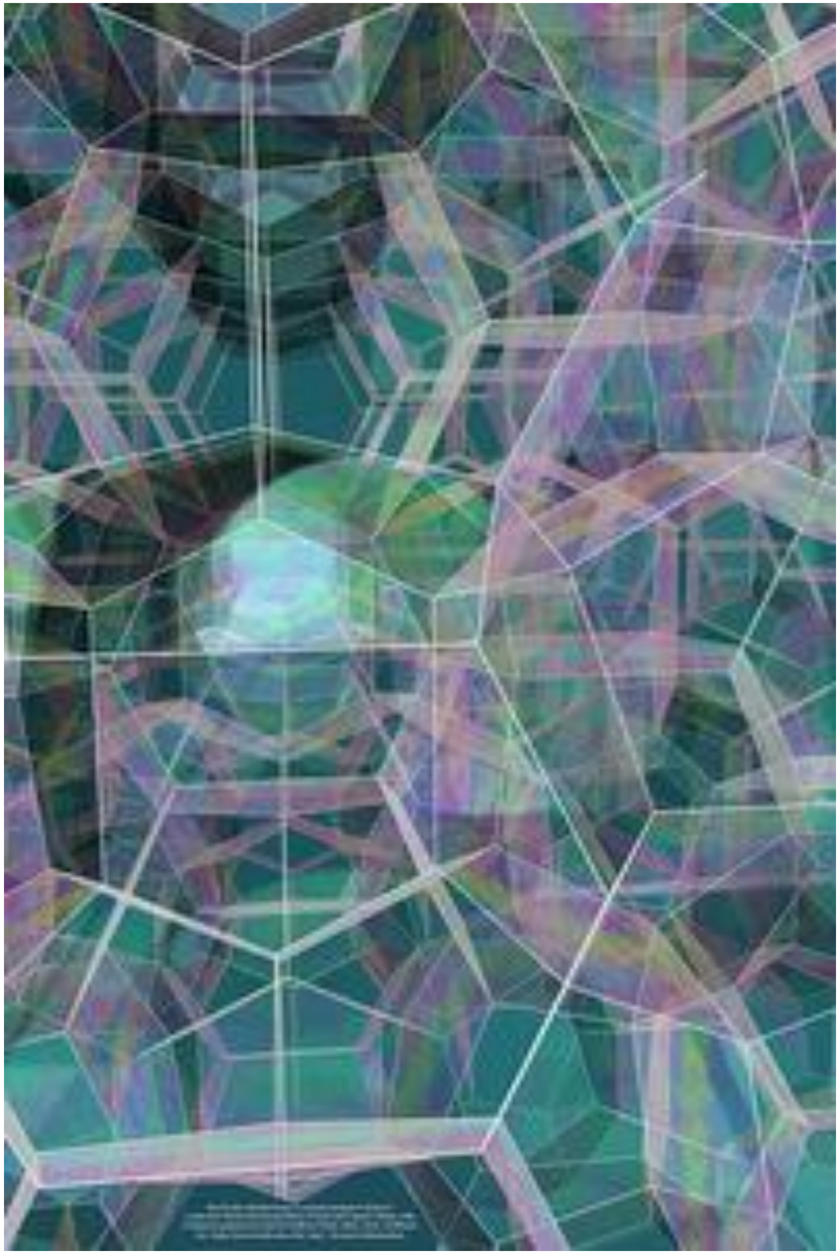
Model System

Soap froth under quasi-static conditions

Network of thin films (surfaces) that
divide space into polyhedral cells

Local geometry satisfies Plateau’s laws

Yield-stress fluid
    soft solid
    flows above the yield stress

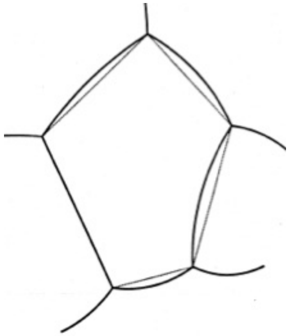


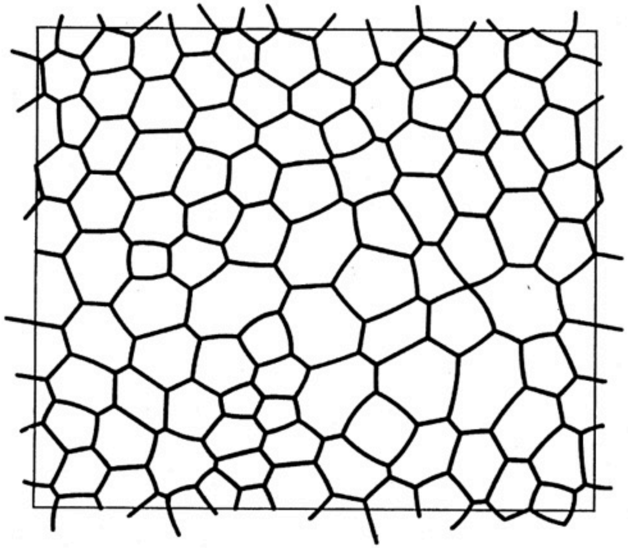
John Sullivan, TU Berlin

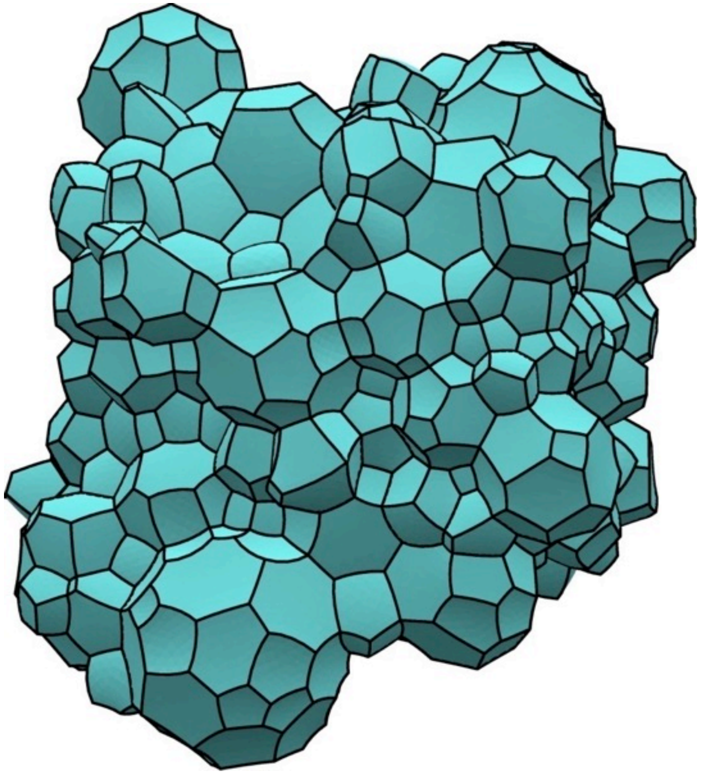
Foam at equilibrium is an idealization

Equilibrium is when all of the fast things have happened
…and all of the slow things have not.

Richard Feynman







We neglect all real non-equilibrium processes that eventually destroy foam.

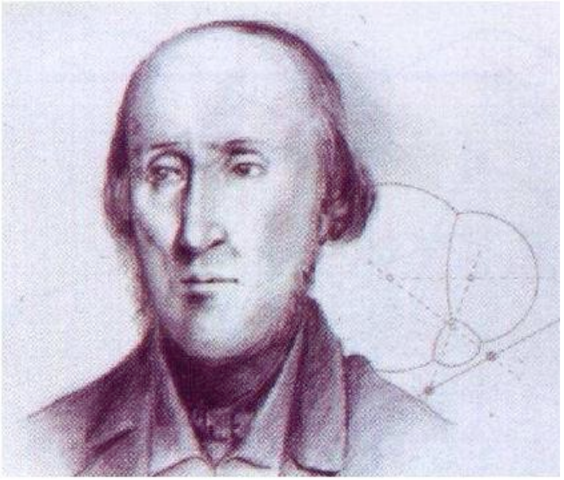
Plateau’s Laws for Equilibrium Structure

Ideal soap froth: Surfaces (constant surface tension σ) define trivalent polyhedral cells.

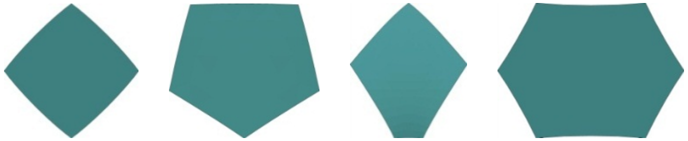
1) Each film has constant mean curvature
   Young-Laplace equation $\Delta P = 2\sigma(R_1^{-1}+R_2^{-1})$
2) Three films meet at 120° angles at cell edges
3) Four edges meet at tetrahedral angles: *acos*(-1/3) = 109.47°

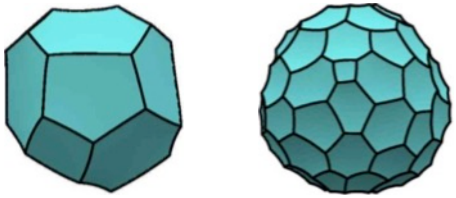
J.A.F. Plateau (1873) “Statique Experimentale et Theorique des Liquides Soumis aux Seules Forces Moleculaires,” Gauthier-Villard, Paris.

J.E. Taylor (1976) *Ann Math* **103**, 489.



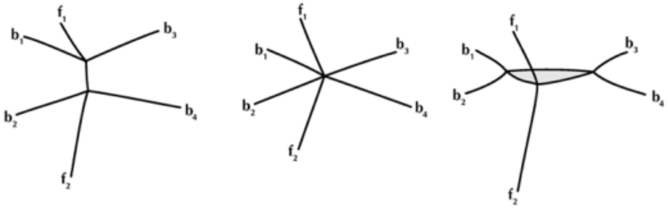
Courtesy of Weaire and Hutzler





No foam film can be a flat polygon with straight edges because the vertex angles must be tetrahedral angles.

Cell edges shrink to zero length during flow and provoke T1 topological transitions
– a fundamental mechanism of foam flow



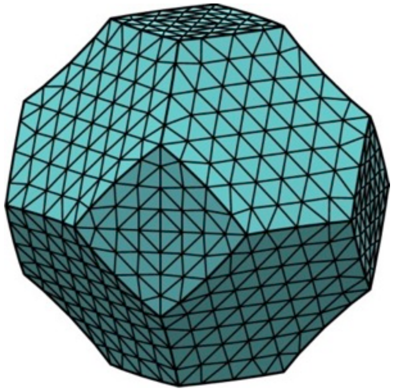
Surface Evolver

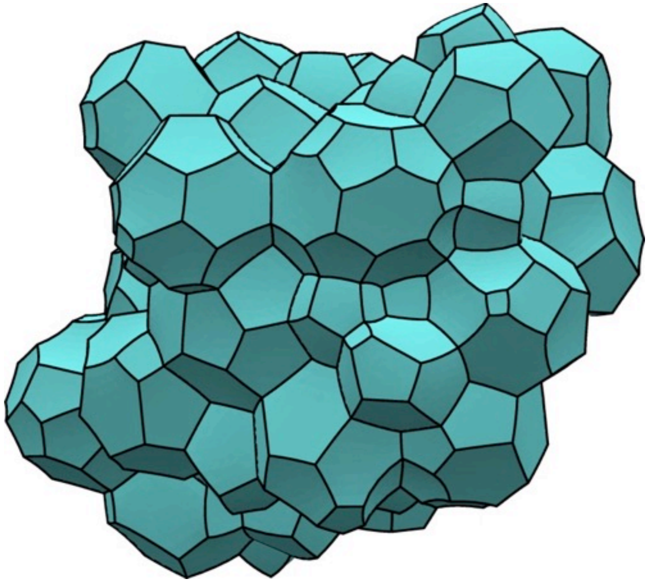
An interactive program for modeling liquid surfaces shaped by various forces (surface tension) and constraints (spatial periodicity and cell volumes). The surface evolves toward minimal energy by simulating the process of evolution by mean curvature.

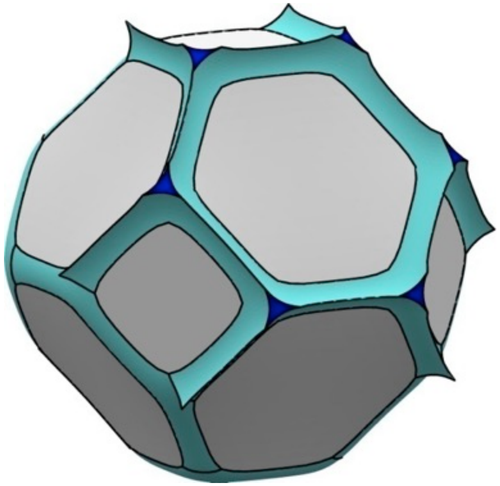
Developed by Ken Brakke, Mathematics Department, Susquehanna University

Free download — Google “Surface Evolver”

K.A. Brakke (1992) *Exp Math* **1**, 141.

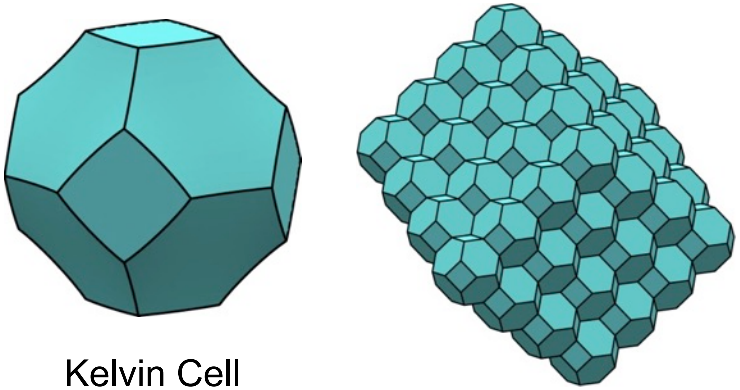




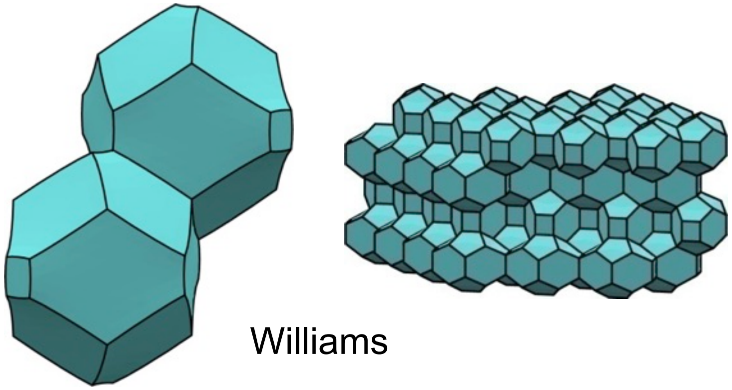


Quasi-static equilibrium, minimal surfaces, elastic-plastic rheology

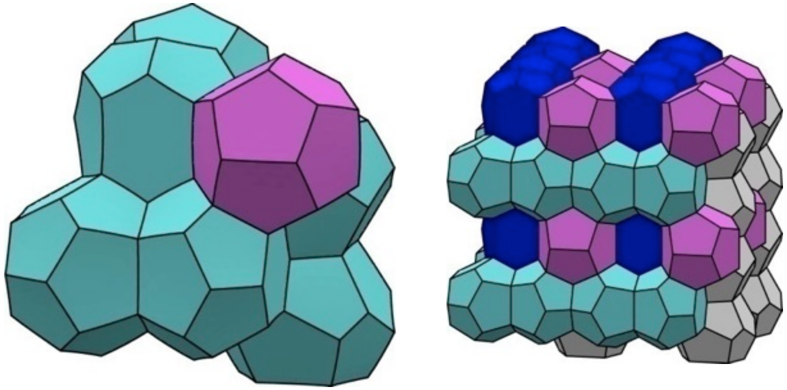
Ordered Foams



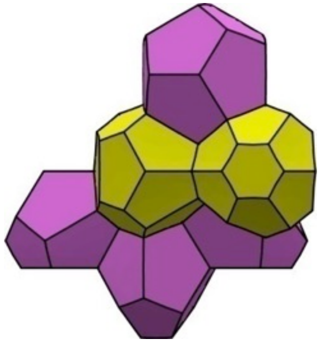
Kelvin Cell



Williams

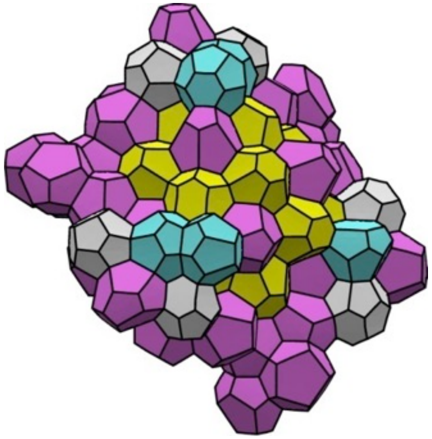


Weaire-Phelan (A15)



Friauf-Laves (C15)

Tetrahedrally Closest Packed (TCP)

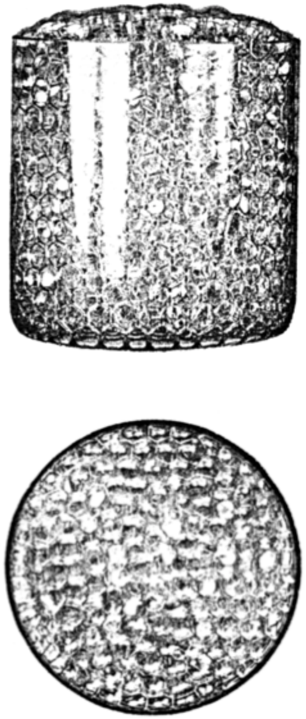


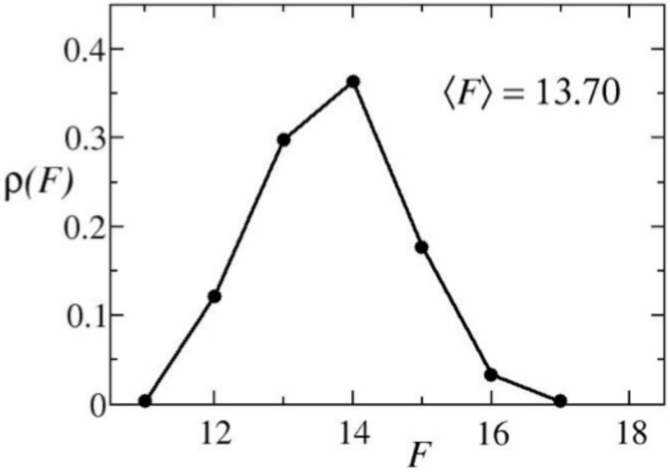
Bergman (T)

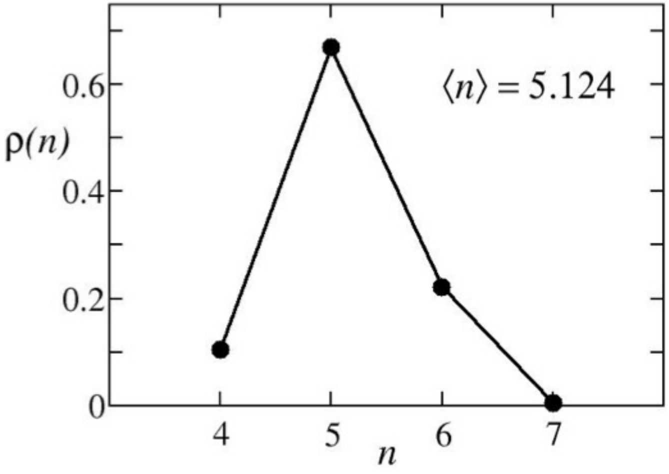
W. Thompson (Lord Kelvin) (1887) *Phil Mag* **24**, 503.
R.E. Williams (1968) *Science* **161**, 276.
D. Weaire & R. Phelan (1994) *Phil Mag Lett* **69**, 107.
N. Rivier (1994) *Phil Mag Lett* **69**, 297.

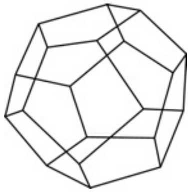
Shape of 600 cells in Monodisperse Soap Froth

E.B. Matzke (1946) *Am J Botany* **33**, 58.









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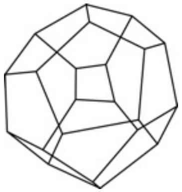
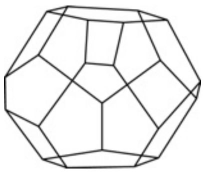
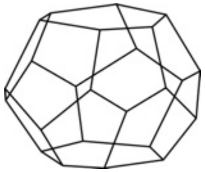


1-10-2

Matzke

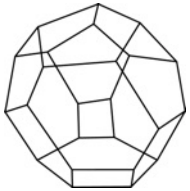


3-6-4



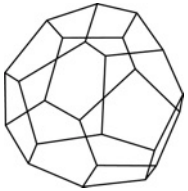
0-12-2

Goldberg



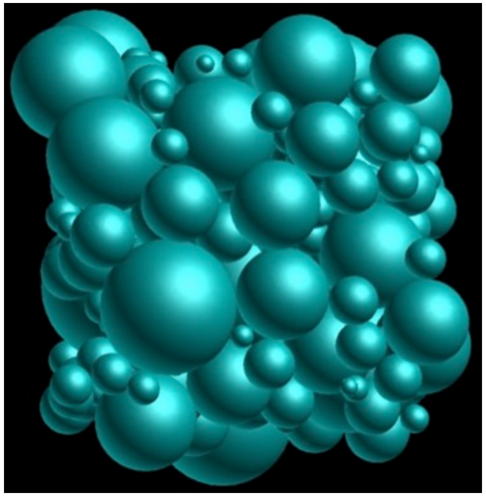
2-8-4

“Williams"



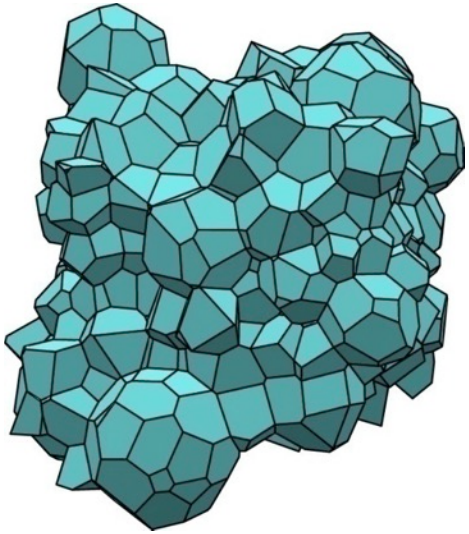
1-10-3

Simulating Random Polydisperse Soap Froth

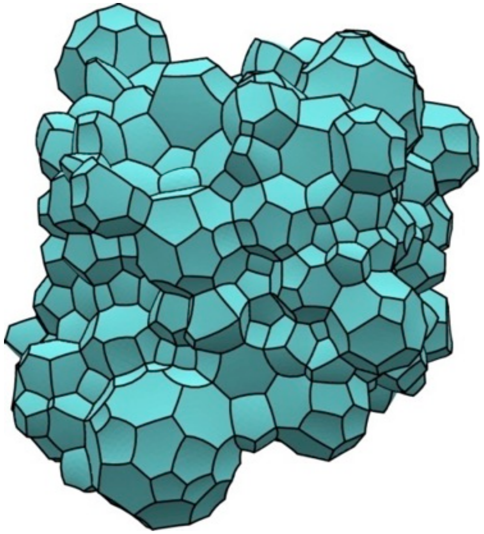


Molecular Dynamics

Random Close Packed (RCP) Spheres



Laguerre (Weighted Voronoi) Tessellation

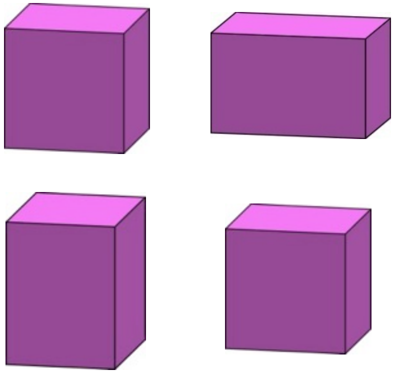


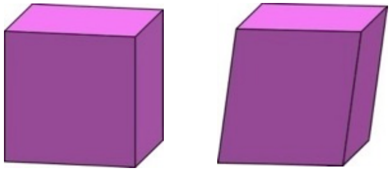
Surface Evolver

Relaxed Foam

Tension-Compression Cycles

Annealing

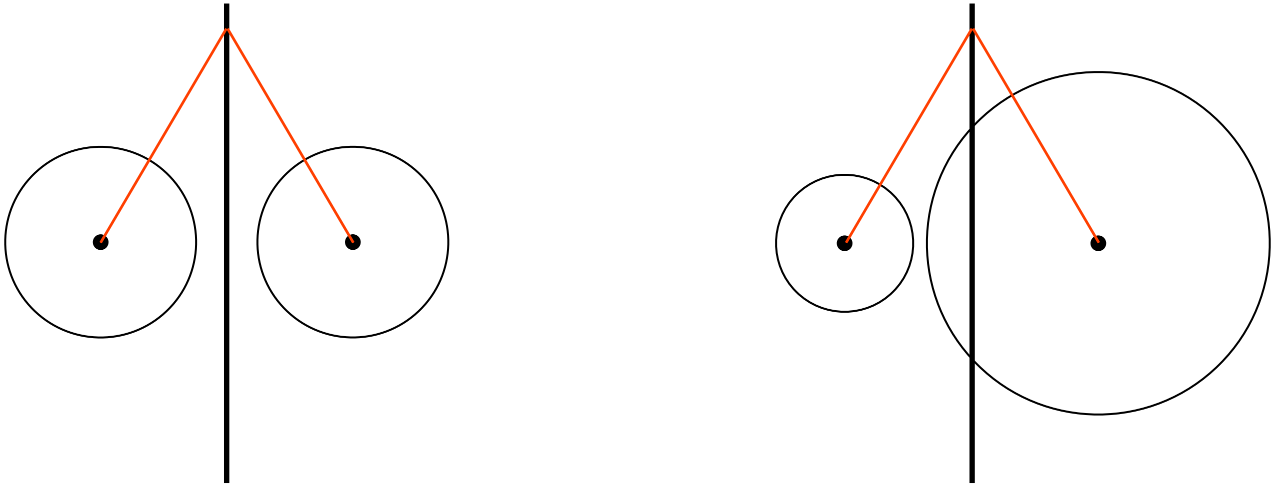




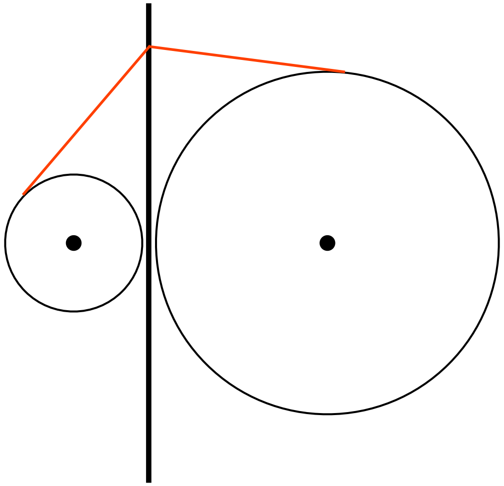
Relax the lattice to achieve isotropic stress

Elastic Recoil

Voronoi Tessellation

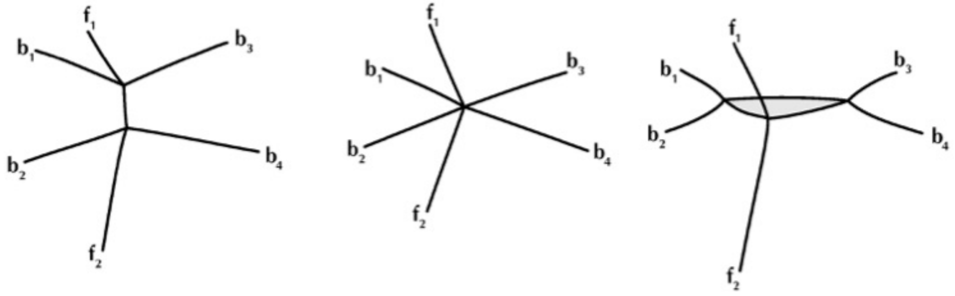


Weighted-Voronoi (Laguerre or radical) Tessellation

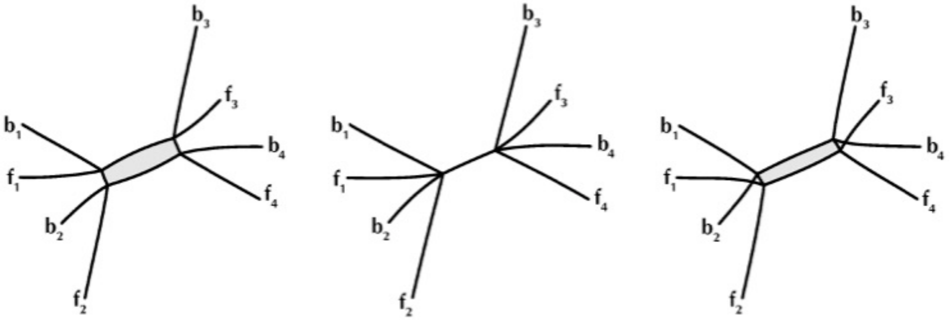


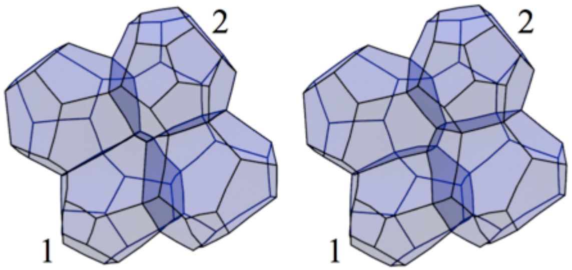
Topological (T1) transitions – bubble neighbor switching

Edge to Triangle Transition

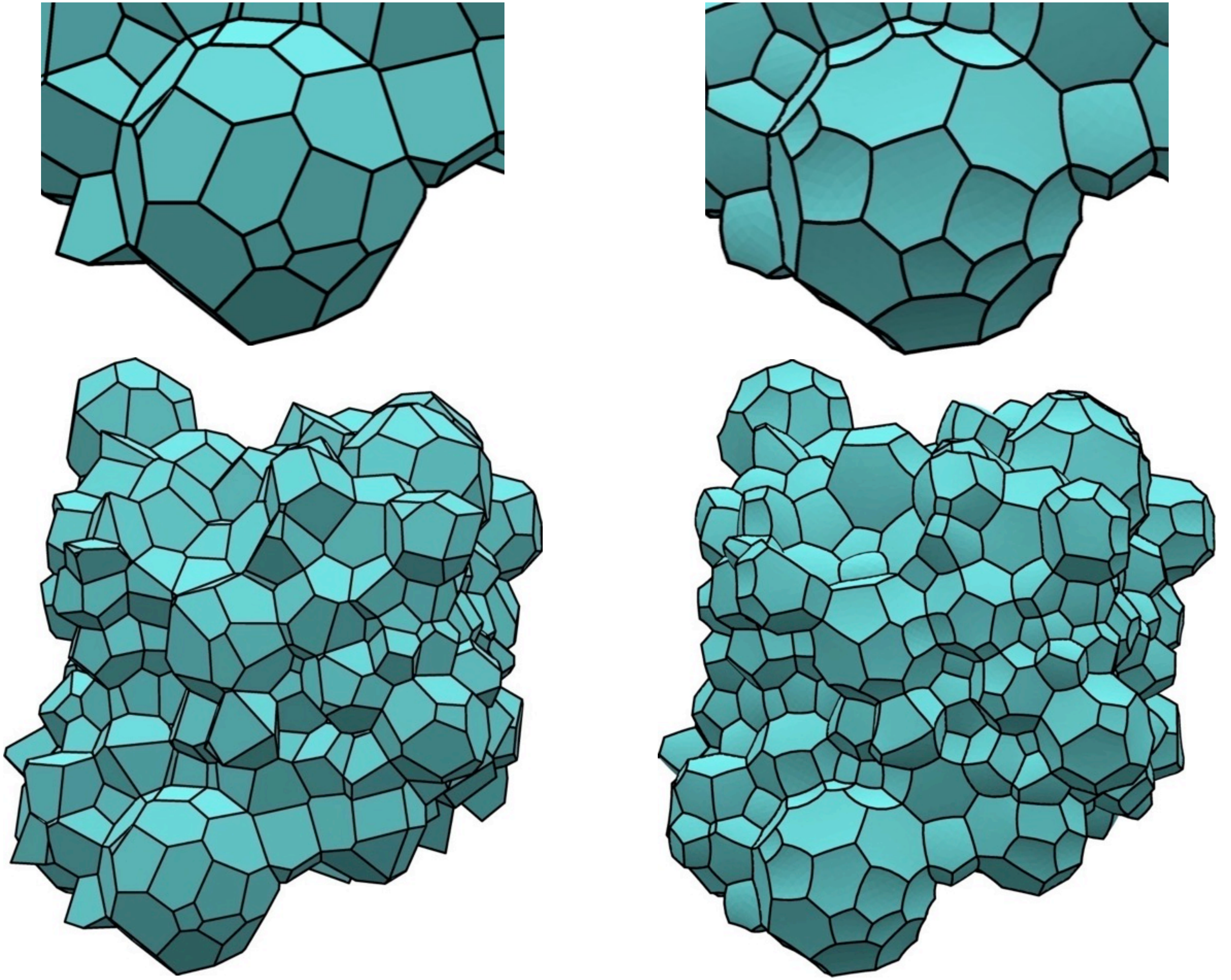


Quad to Quad Transition (quad flip)

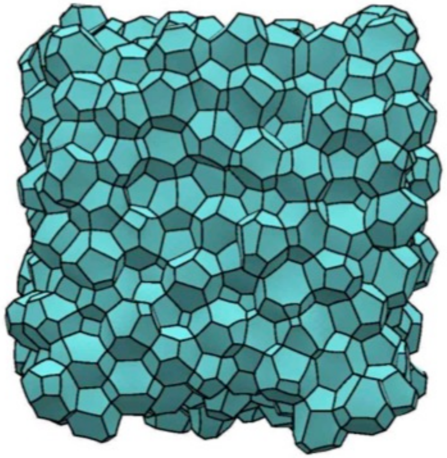




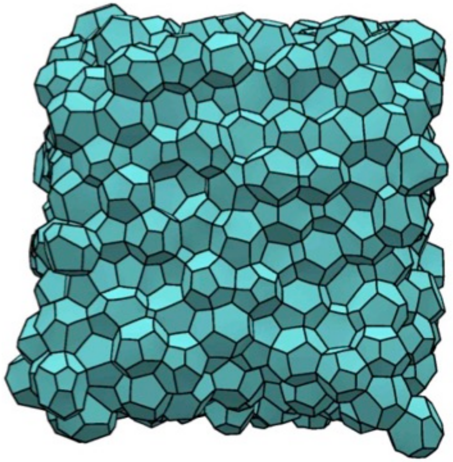
Laguerre Tessellation vs. Equilibrium Foam



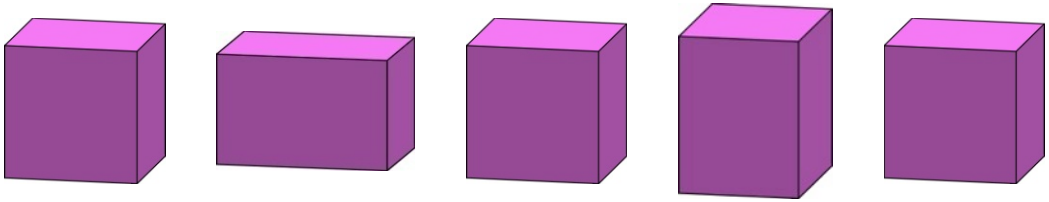
Structure evolution during annealing

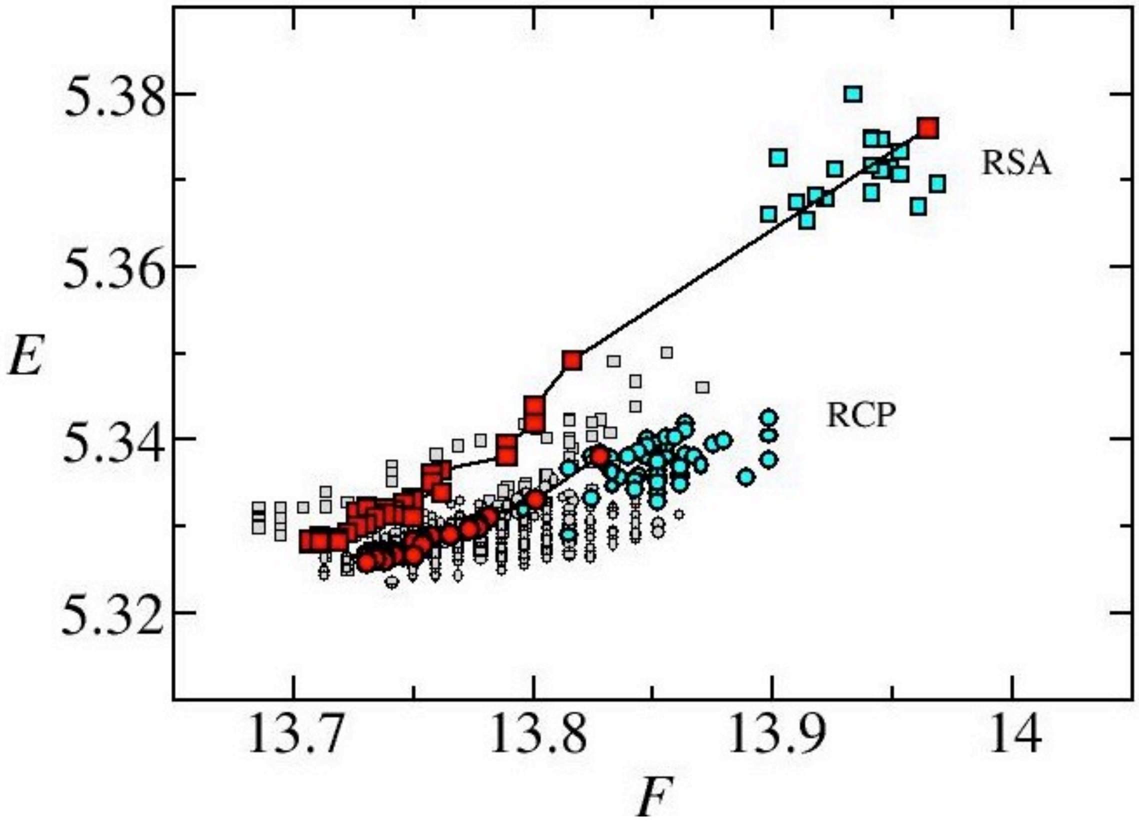


before annealing
E = 5.369



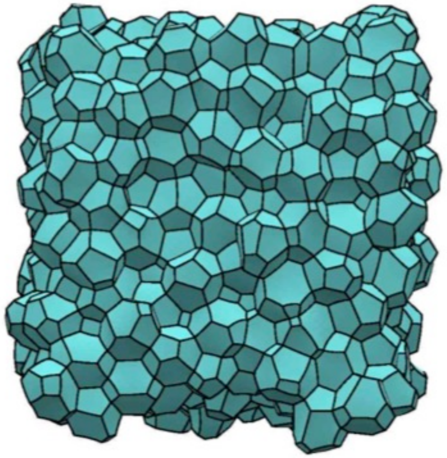
after annealing
E = 5.326



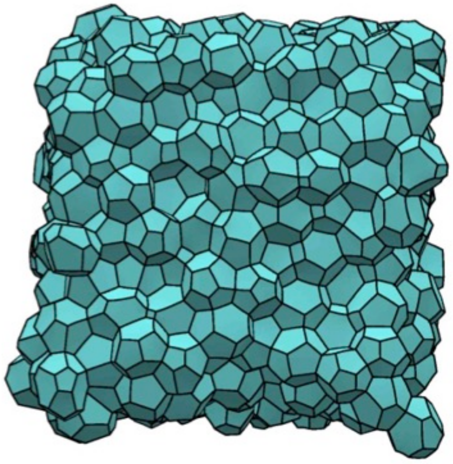


Cell shapes in random monodisperse soap froth

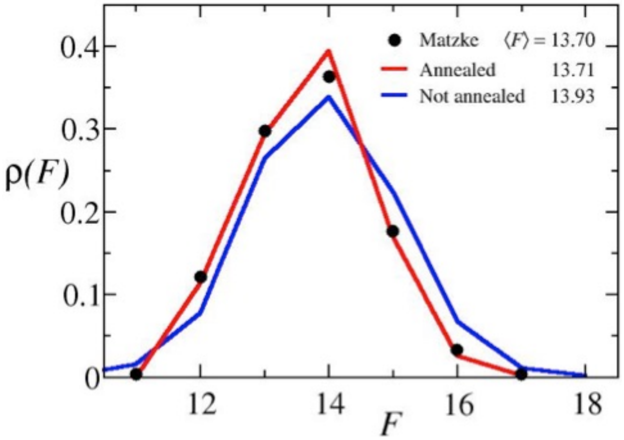
Kraynik, Reinelt & van Swol (2003) *Phys Rev E* **67**, 031403.

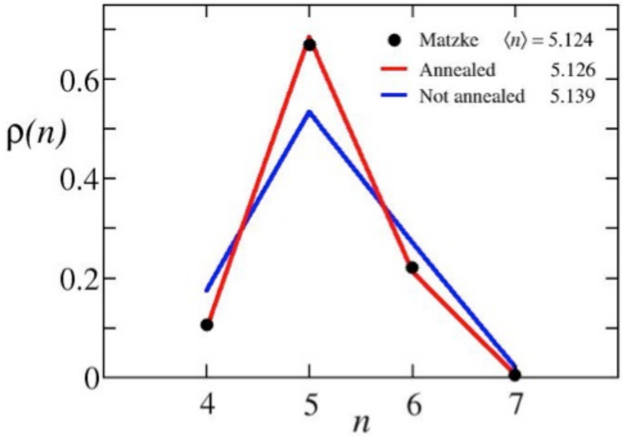


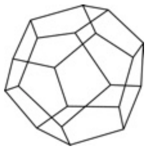
before annealing
E = 5.369



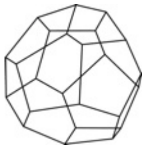
after annealing
E = 5.326





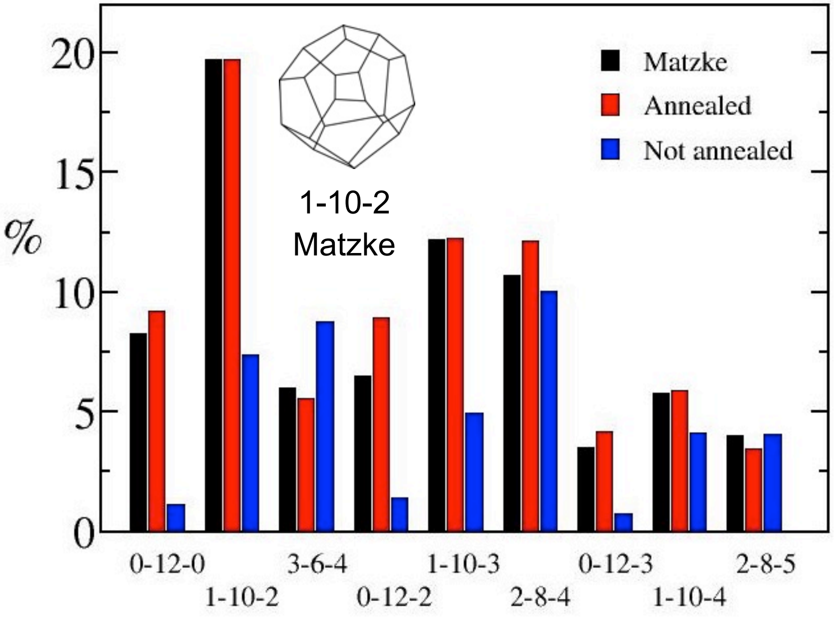


0-12-0



1-10-3

2-8-4



Macroscopic (Global) Stress

$$\Sigma_{ij} \;=\; -\frac{1}{V}\;\sum_k p_k V_k\;\delta_{ij} \;+\; \frac{\sigma}{V}\;\sum_k\;\iint_{S_k}(\delta_{ij}-n_i n_j)\;ds$$

isotropic terms $$=\; -\frac{1}{V}\;\sum_k p_k V_k\;\delta_{ij} \;+\; \frac{\sigma}{V}\;\sum_k\;\iint_{S_k}(\frac{2}{3}\delta_{ij})\;ds$$

non-isotropic terms $$+\;\frac{\sigma}{V}\;\sum_k\;\iint_{S_k}(\frac{1}{3}\delta_{ij}-n_i n_j)\;ds$$

equation of state $$-p_f \;=\; -\langle p_b\rangle \;+\; \frac{2}{3}\,\frac{\sigma\;S}{V} \;=\; -\langle p_b\rangle \;+\; \frac{2}{3}\;E_f$$

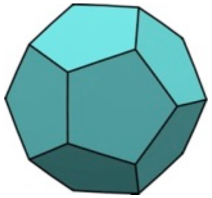
Cell Shape

cell shape tensor $$q_{ij} \;=\; V^{-2/3}\iint_S\;(\frac{1}{3}\delta_{ij}-n_i n_j)\;ds$$

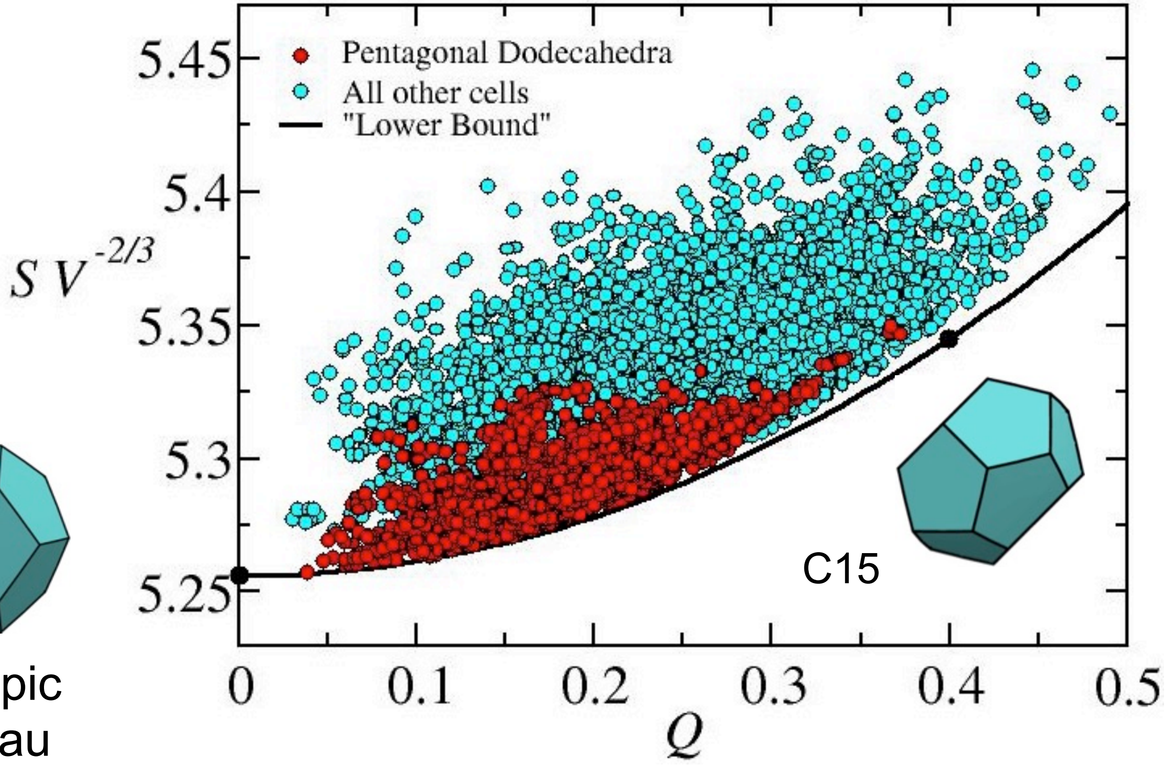
measure of cell distortion $$Q \;=\; (J_2)^{\frac{1}{2}} \;=\; (\frac{1}{2}q_{ij}\;q_{ij})^{\frac{1}{2}}$$

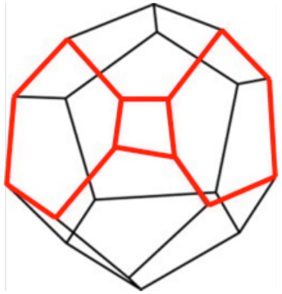
Geometrical Frustration in Random Monodisperse Soap Froth

Surface Area *vs* Cell Anisotropy

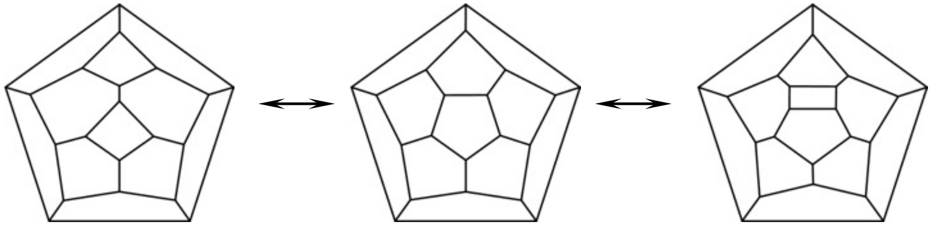


Isotropic
Plateau
Dodecahedron





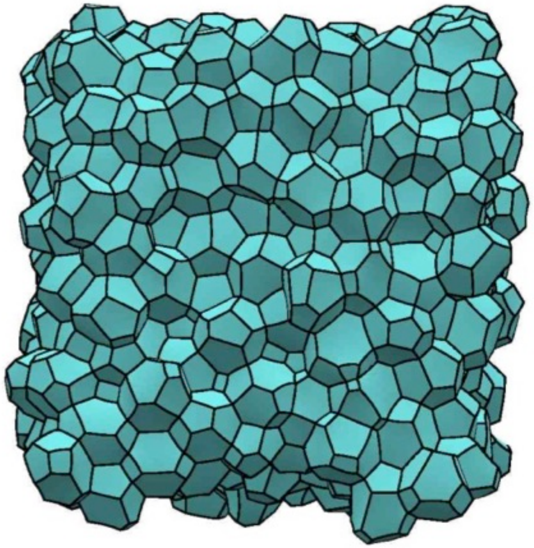
“Matzke”

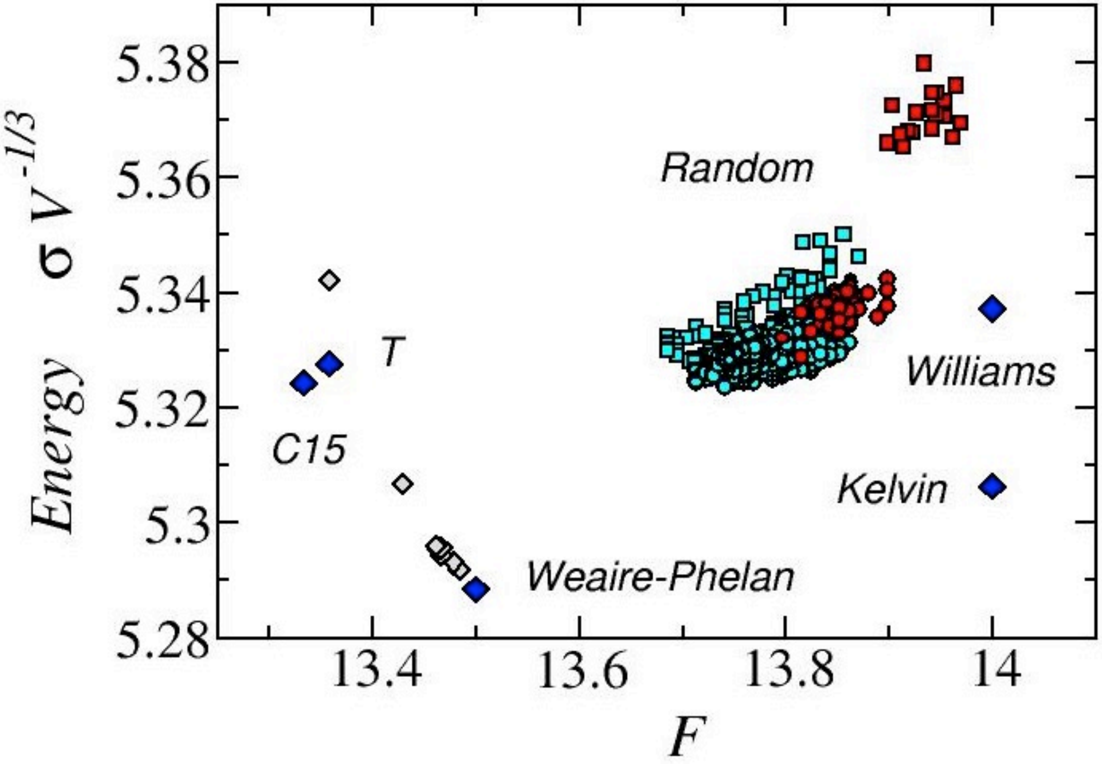


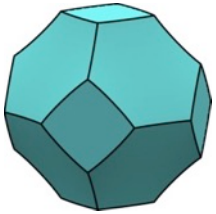
2-8-2 0-12-0 1-10-2

Monodisperse Soap Froth

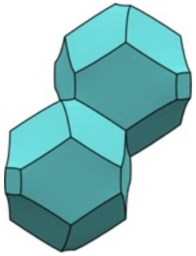
3D: $\langle F \rangle = \dfrac{12}{6 - \langle n \rangle}$



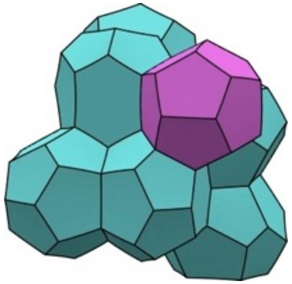




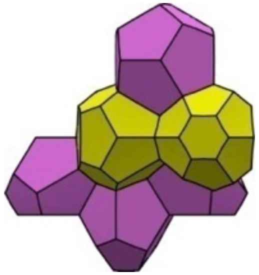
Kelvin



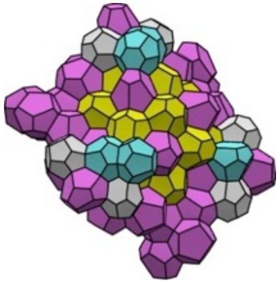
Williams



Weaire-Phelan
A15



Friauf-Laves
C15

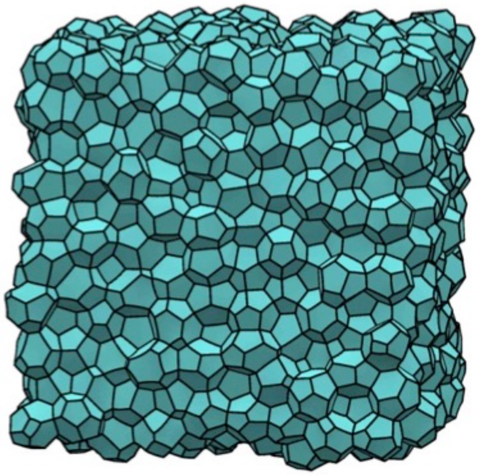


Bergman
T

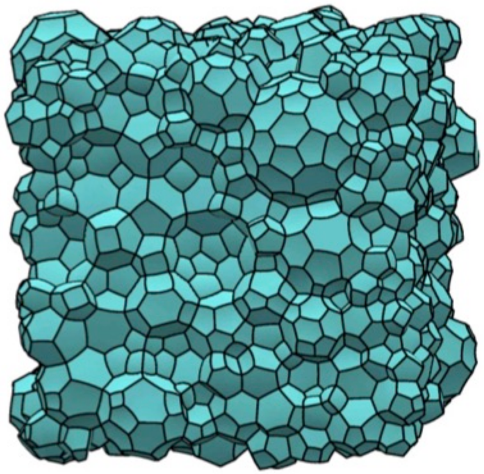
Random Soap Froth

Kraynik, Reinelt & van Swol (2003) *Phys Rev E* **67**, 031403;

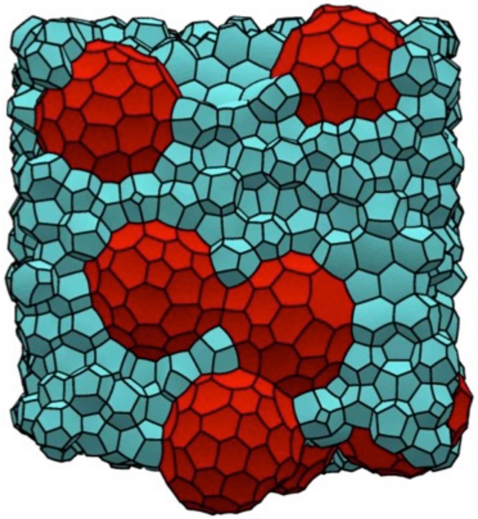
(2004) *Phys Rev Lett* **93**, 208301; (2005) *Colloids Surfaces A* **263** 11-17.



Monodisperse



Polydisperse



Bidisperse

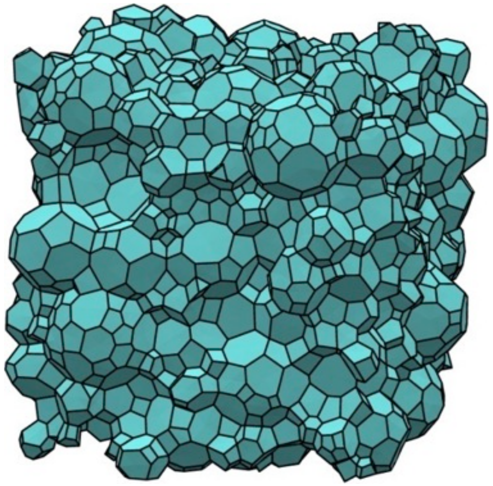
Spatially periodic structure

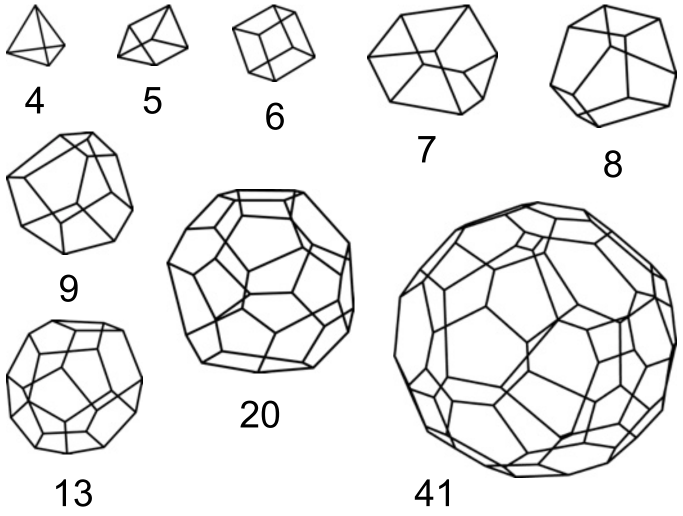
1728 cells

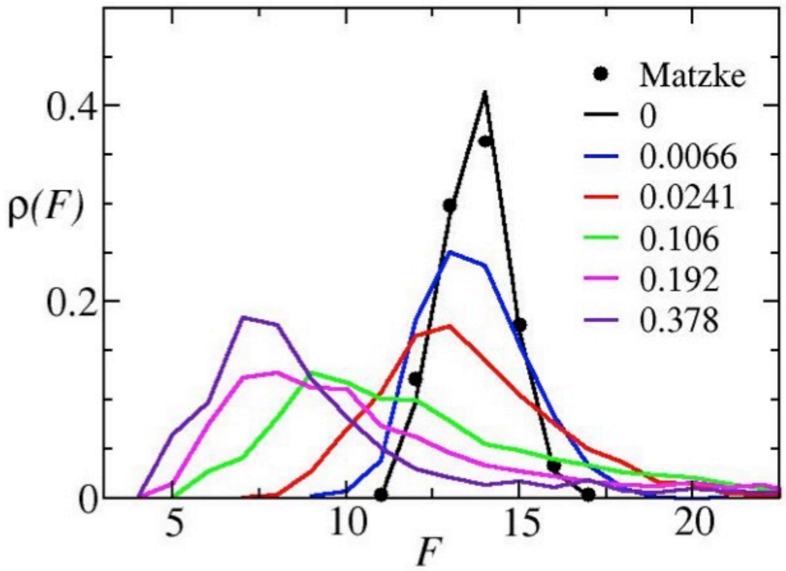
Cell volumes vary by three orders of magnitude

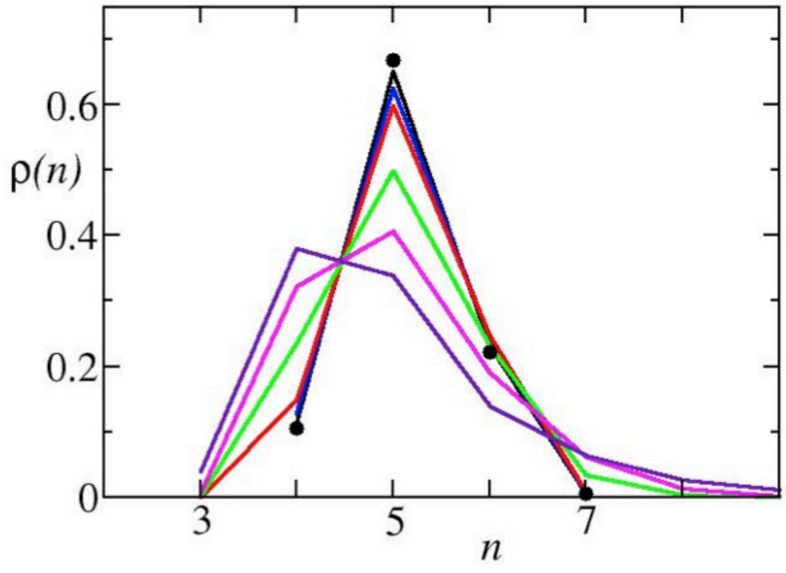
Topological Statistics of Random Polydisperse Soap Froth

Kraynik, Reinelt & van Swol (2004) Phys Rev Lett 93, 208301



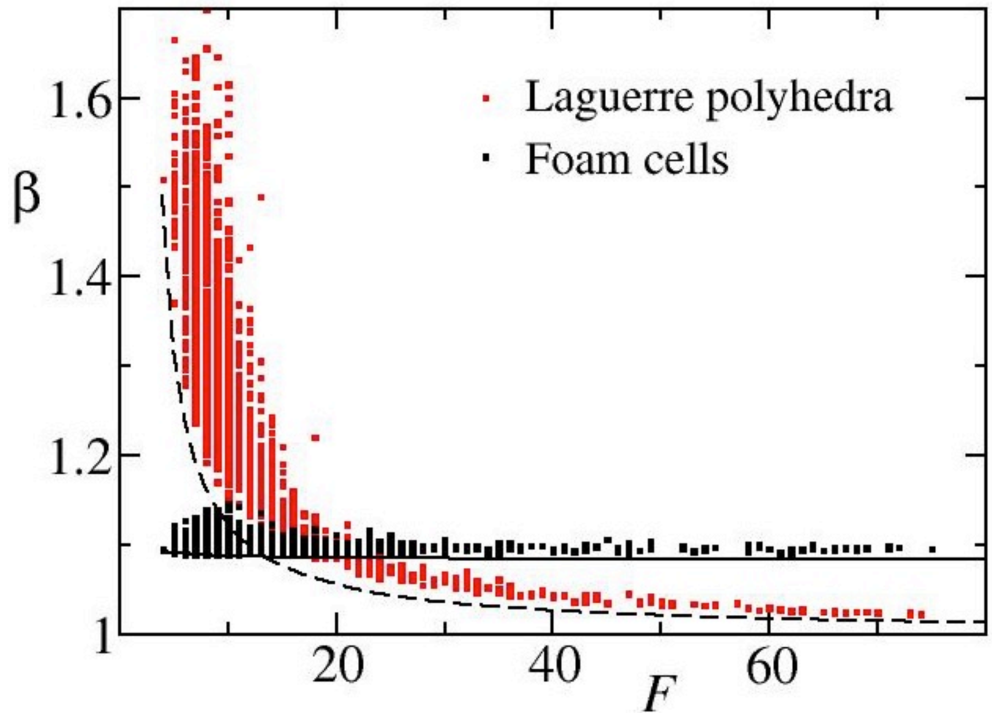


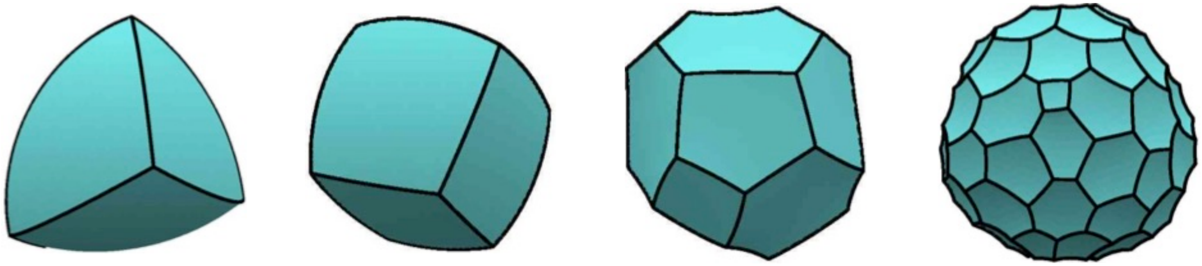




The surface area of a foam cell is about 10% greater than an equal-volume sphere

$$\beta = S\,(36\pi V^2)^{-1/3} = 1.101 \pm 0.006$$





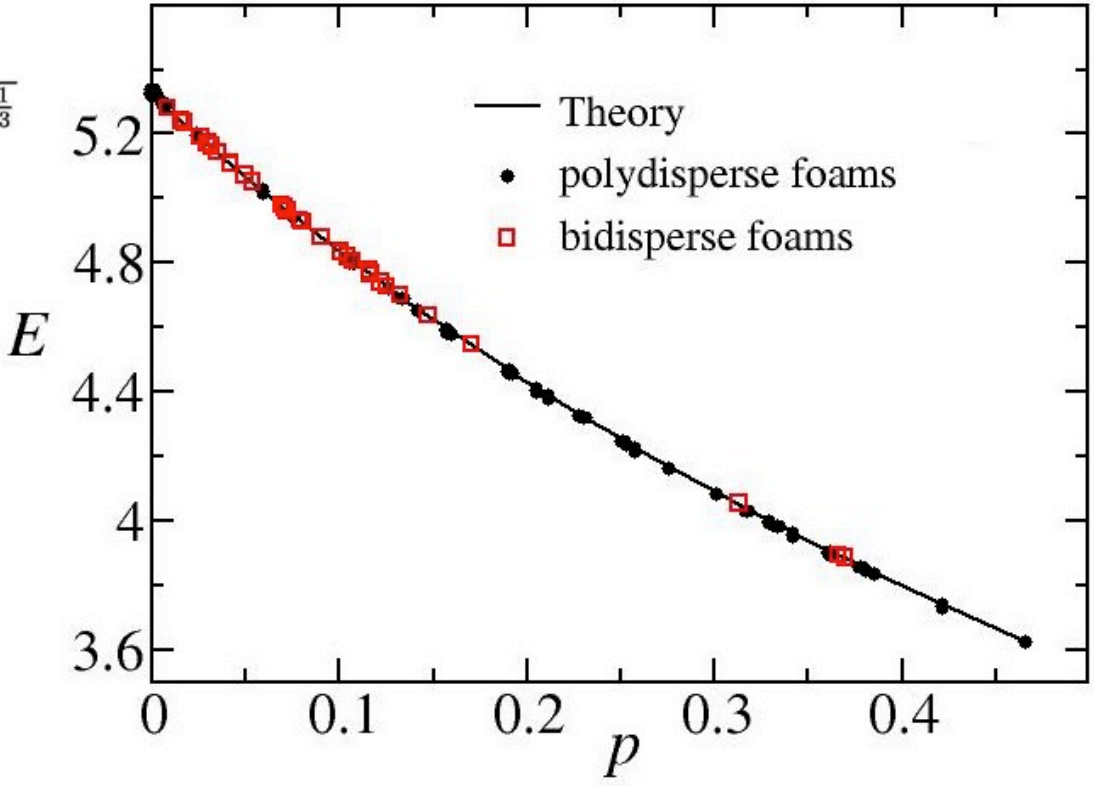
Foam Energy – Surface Free Energy Density

$$E = \sigma \frac{S_{foam}}{V_{foam}} = \sigma \frac{\sum S}{\sum V} = 3\beta\sigma \frac{\langle R^2 \rangle}{\langle R^3 \rangle} = 3\beta \frac{\sigma}{R_{32}} = \frac{\beta\,(36\pi)^{\frac{1}{3}}}{(1+p)} \frac{\sigma}{\langle V \rangle^{\frac{1}{3}}}$$

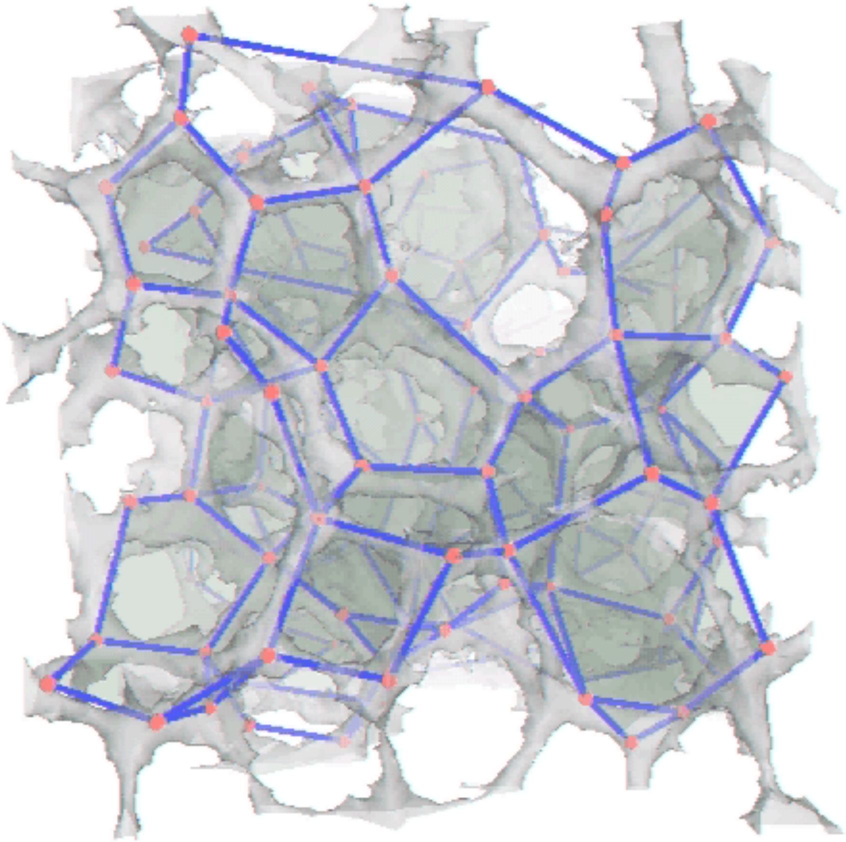
Sauter mean radius: $R_{32} = \langle R^3 \rangle / \langle R^2 \rangle$

polydispersity parameter: $p = R_{32}/\langle R^3 \rangle^{\frac{1}{3}} - 1 = \langle R^3 \rangle^{\frac{2}{3}}/\langle R^2 \rangle - 1 \geq 0$

$$E = 3.30 \frac{\sigma}{R_{32}} = \frac{5.32}{(1+p)} \frac{\sigma}{\langle V \rangle^{\frac{1}{3}}}$$



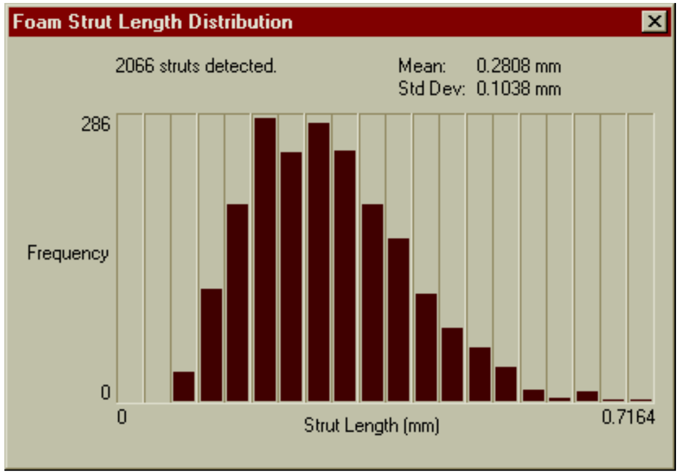
Polyurethane foam skeleton from image analysis of CT data



Matt Montminy, PhD thesis, U Minnesota (2001)

Montminy, Tannenbaum & Macosko,
The 3D structure of real polymer foams,
J. Coll. Int. Sci. **280** 202-211 (2004).

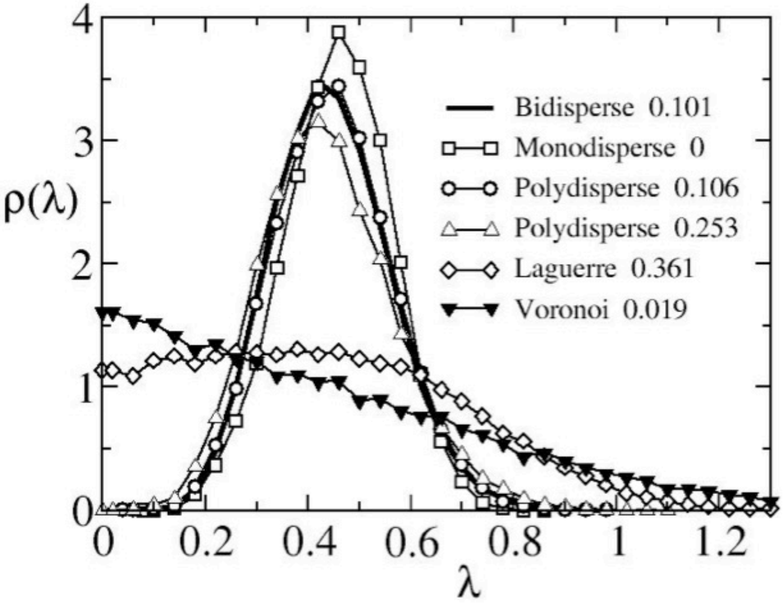
Strut length distribution



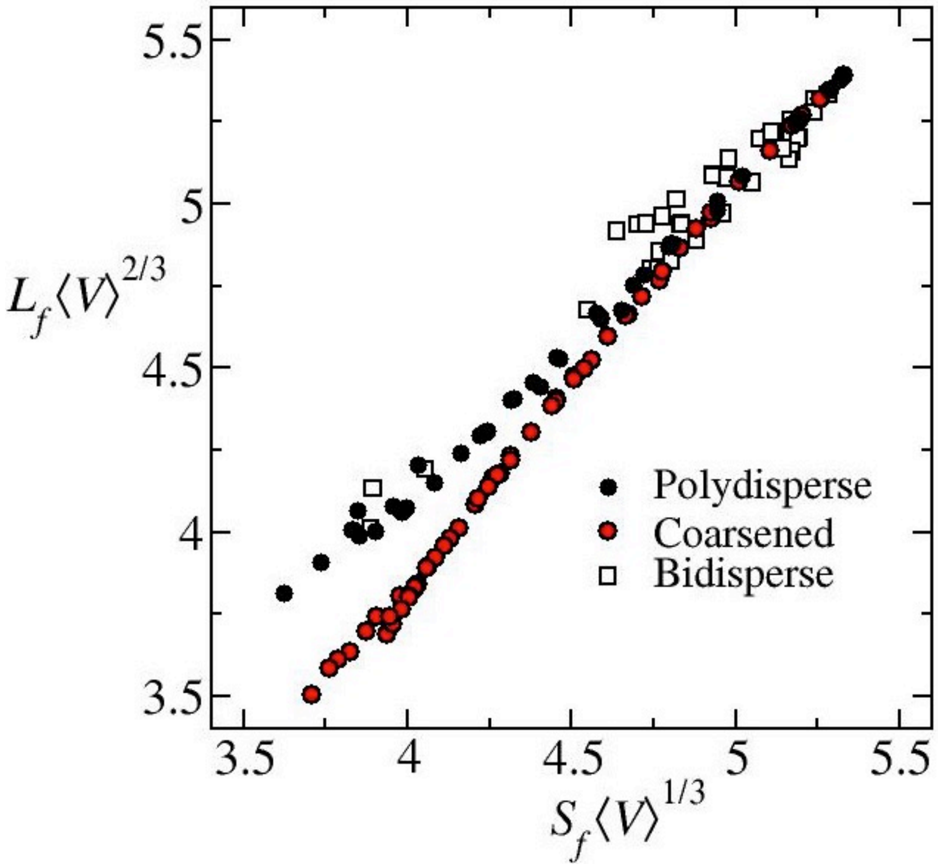
Foam Strut Length Distribution

2066 struts detected.

Mean: 0.2808 mm
Std Dev: 0.1038 mm



The total edge length and surface area of soap froth are approximately equal when both are normalized by the average cell volume.

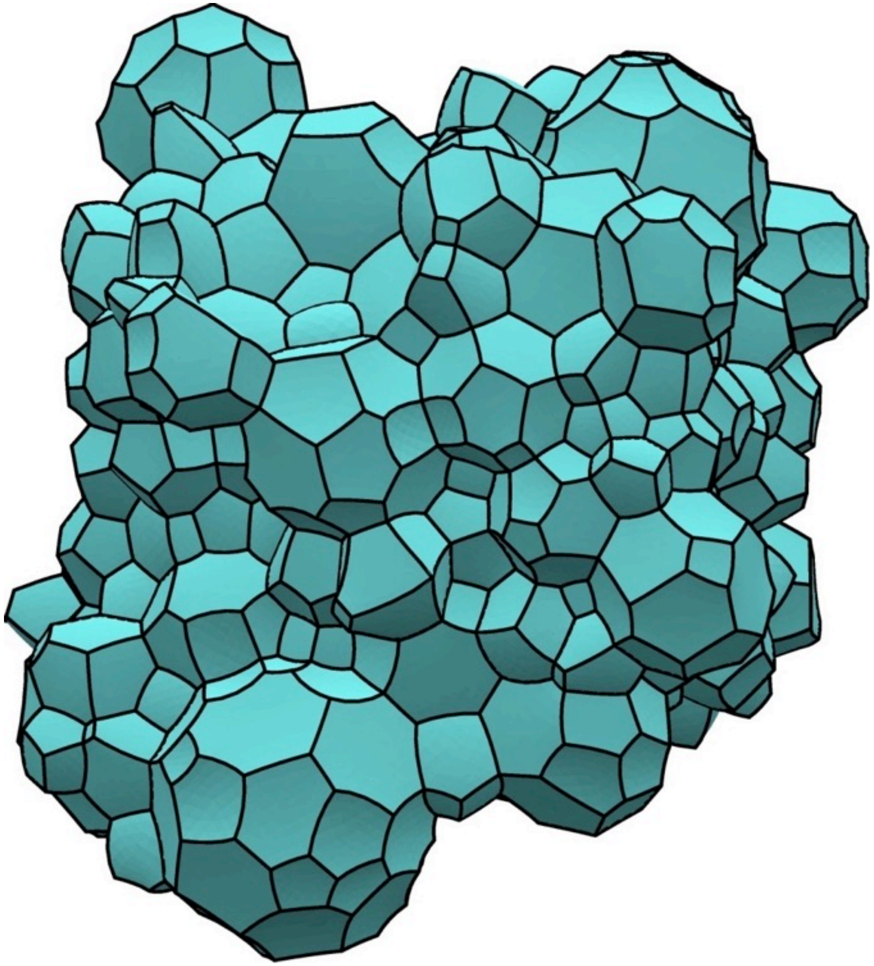


S_f = total surface area per unit volume of foam

L_f = total edge length per unit volume of foam

V = cell volume

A foam cell is composed of faces with different shapes and curvatures and the faces **are not** spherical caps.

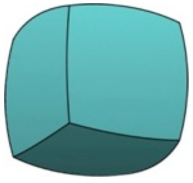


Isotropic Plateau Polyhedra (IPP)

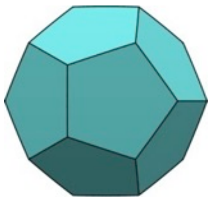
Hilgenfeldt, Kraynik, Reinelt & Sullivan (2004) Europhysics Lett **67**, 484.

IPP are idealized foam cells that have F identical (regular) spherical-cap faces and satisfy Plateau's laws

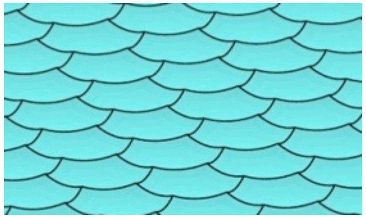
Tetrahedron
F=4



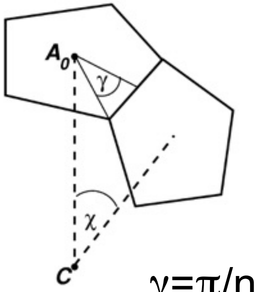
Cube
F=6



Dodecahedron
F=12

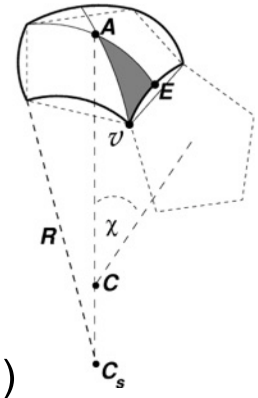


"Golf Ball"
F = infinity

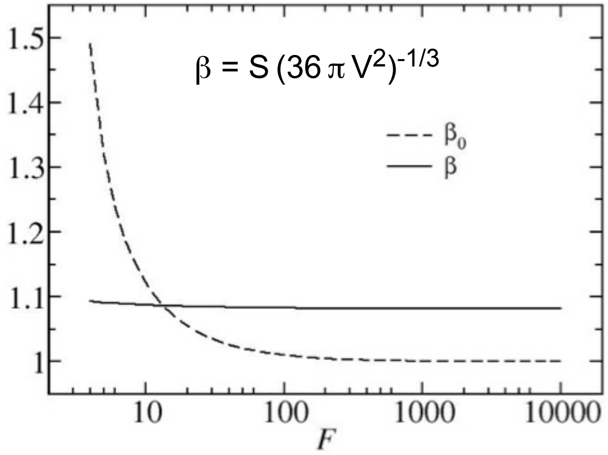


$\gamma=\pi/n$

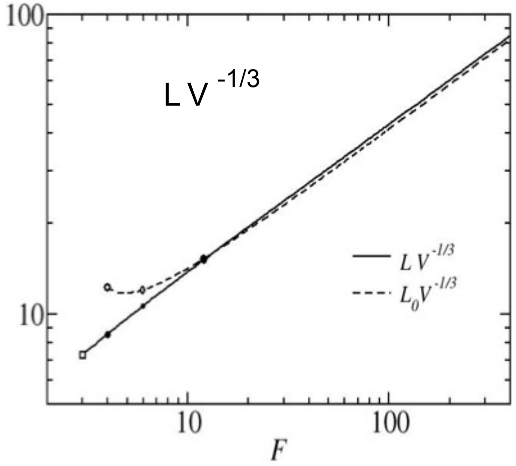
$F=12/(6-n)$



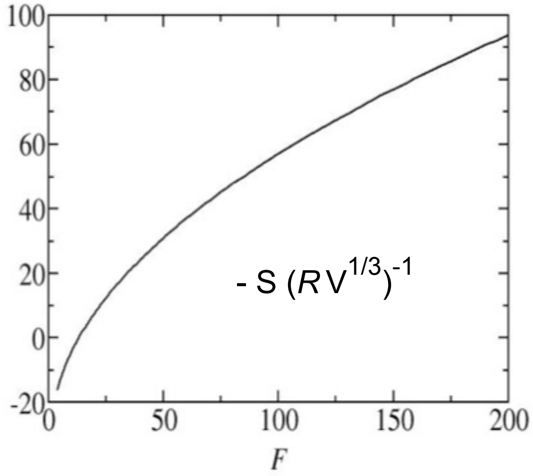
Normalized Surface Area



Edge Length

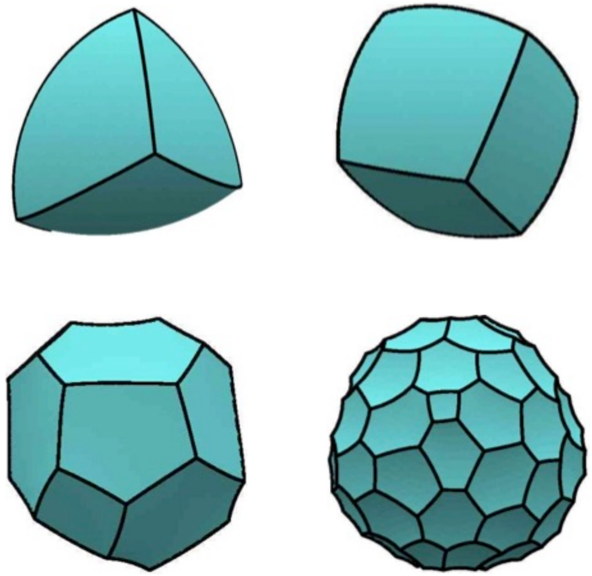


Diffusive Growth Rate



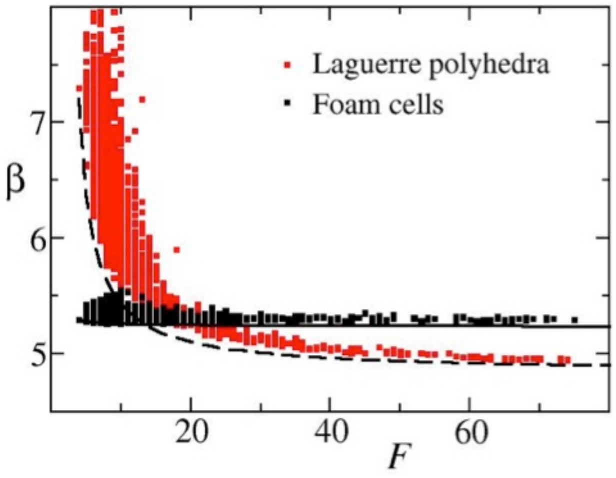
Dashed curves refer to isotropic polyhedra with flat faces.

IPP theory captures cell geometry – with no adjustable parameters



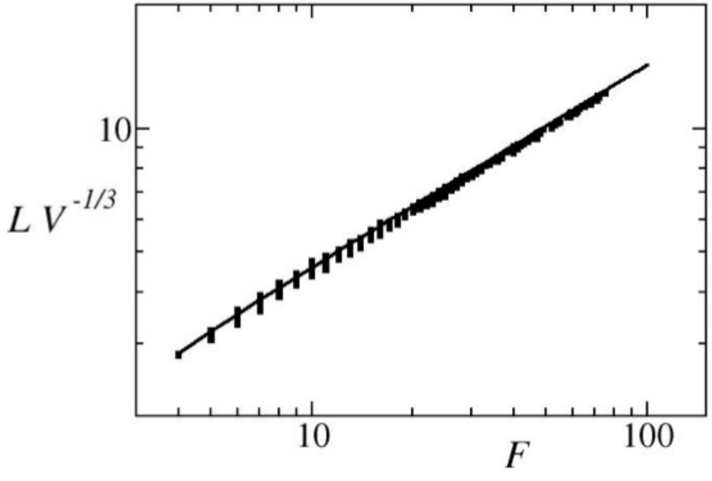
Surface Area

$$\beta = S\,(36\pi V^2)^{-1/3} = 1.101 \pm 0.006$$

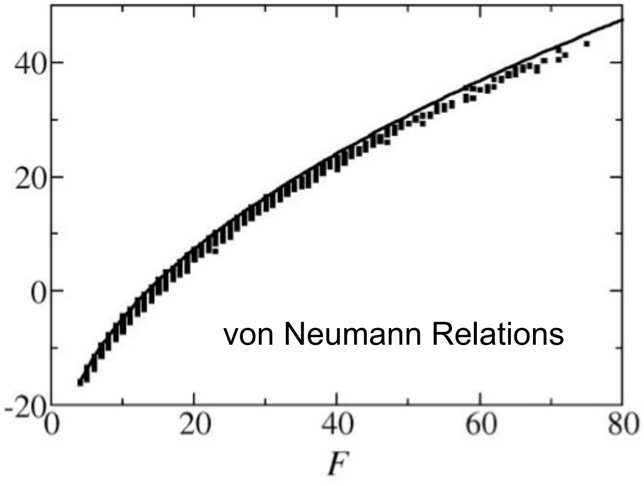


$L\,,\ \mathcal{G} \sim F^{1/2}$

Edge Length



Diffusive Growth Rate



von Neumann Relations

Shear Modulus: measurements for foams and emulsions

H.M. Princen & A.D. Kiss (1986) *J Coll Int Sci* **112** 427.

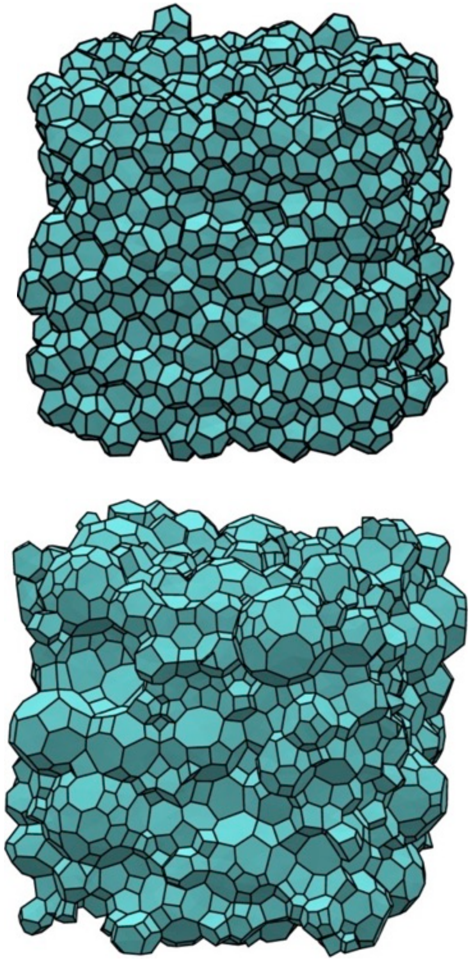
Shear modulus of highly concentrated liquid-liquid emulsions

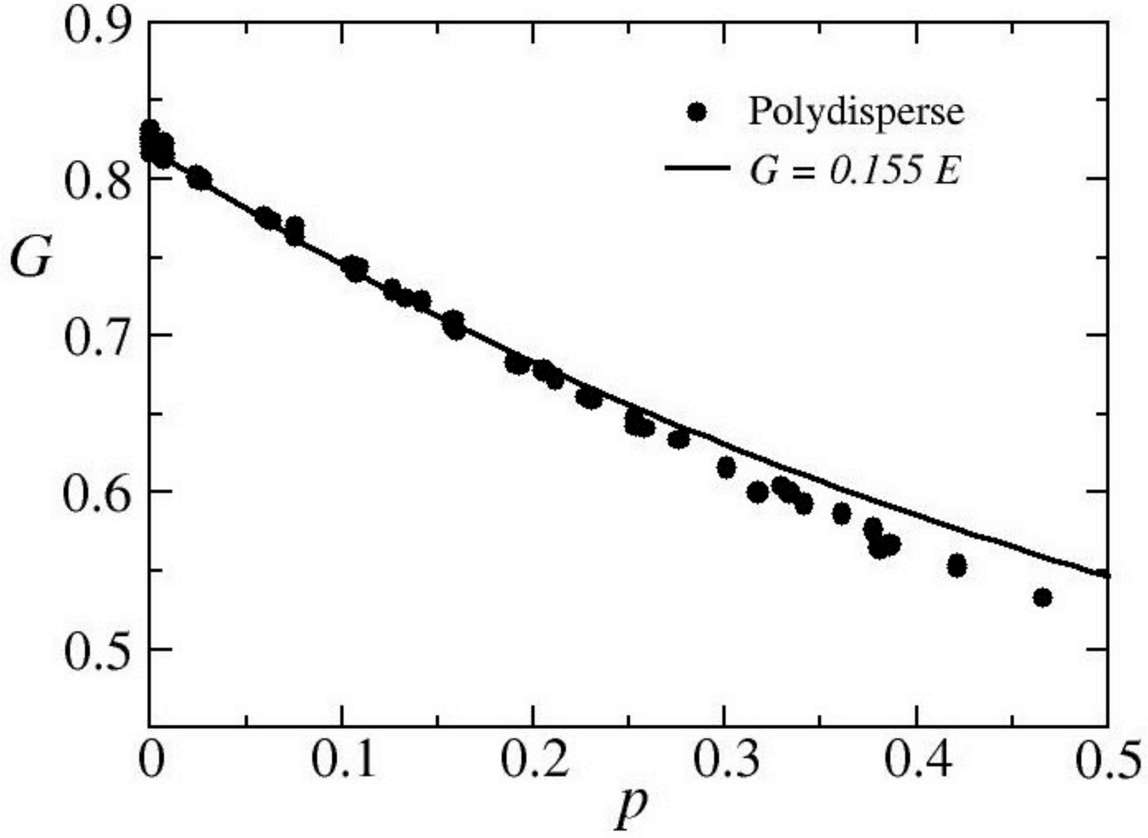
$$G \sim \sigma R_{32}^{-1} \phi^{1/3} (\phi - \phi_c)$$

$$R_{32} = \langle R^3 \rangle / \langle R^2 \rangle$$

“dry” limit ($\phi=1$) $\boxed{G = 0.51\, \sigma R_{32}^{-1}}$

Shear Modulus of Random Soap Froth





$$G \;\approx\; 0.155\; E \;=\; \boxed{0.512\,\frac{\sigma}{R_{32}}}$$

$$E \;=\; 3.30\,\frac{\sigma}{R_{32}} \;=\; \frac{5.32}{(1+p)}\,\frac{\sigma}{\langle V\rangle^{\frac{1}{3}}}$$

Shear Modulus: measurements for foams and emulsions

H.M. Princen & A.D. Kiss (1986) *J Coll Int Sci* **112** 427.

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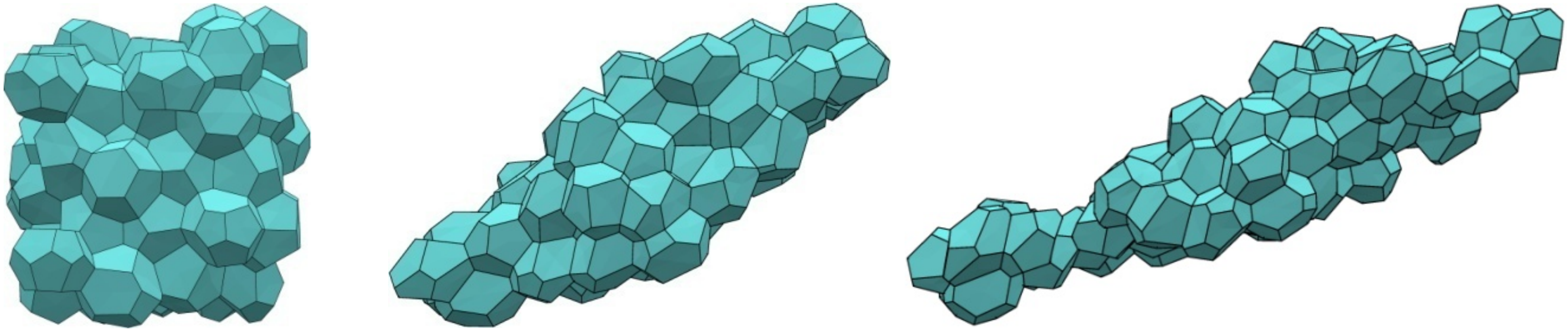
“dry” limit ($\phi=1$) $\boxed{G = 0.51\, \sigma R_{32}^{-1}}$

Quasi-static Simple Shearing Flow

Elastic deformations punctuated by cascades of topological transitions

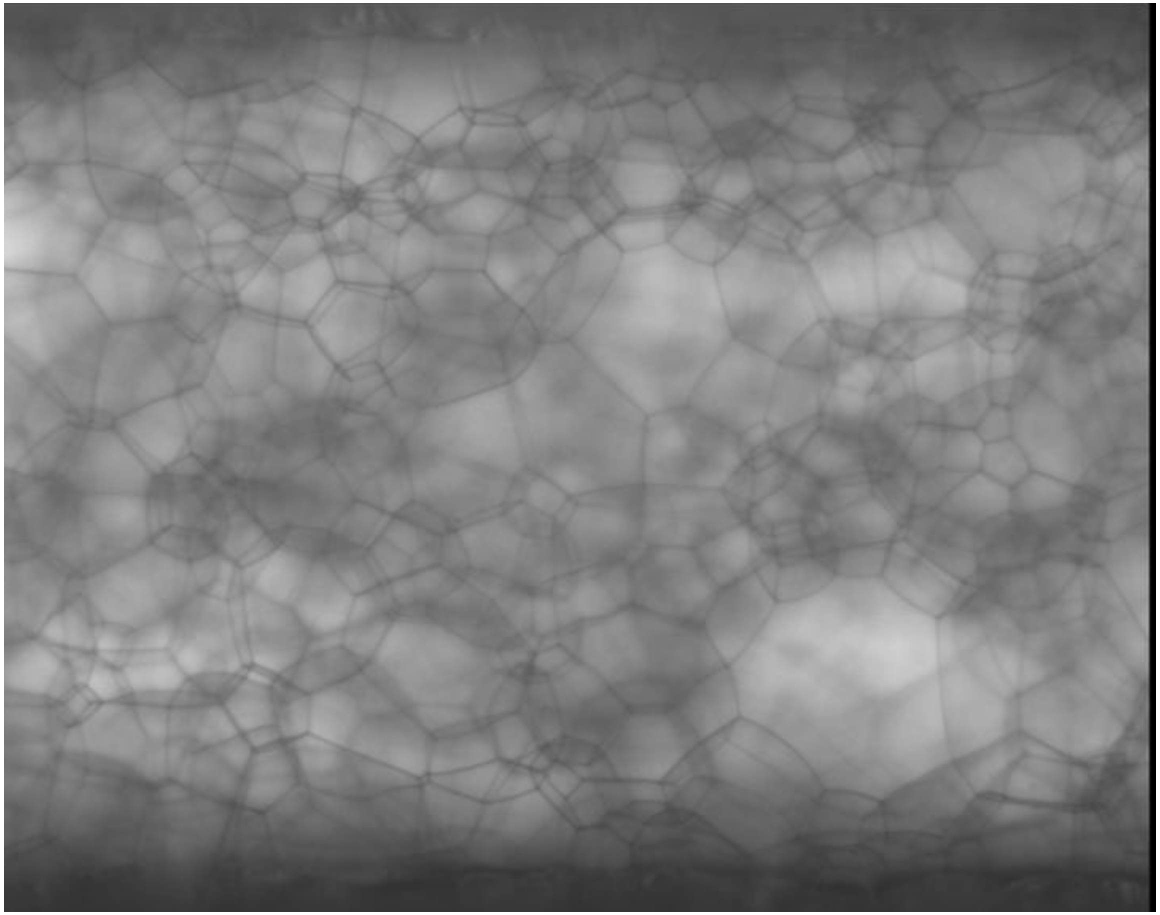


A.D. Gopal & D.J. Durian (1999) *J Coll Int Sci* **213**, 169.

Diffusing-Wave Spectroscopy (DWS) measurements of a solid-like regime
where foam flows by discrete rearrangements

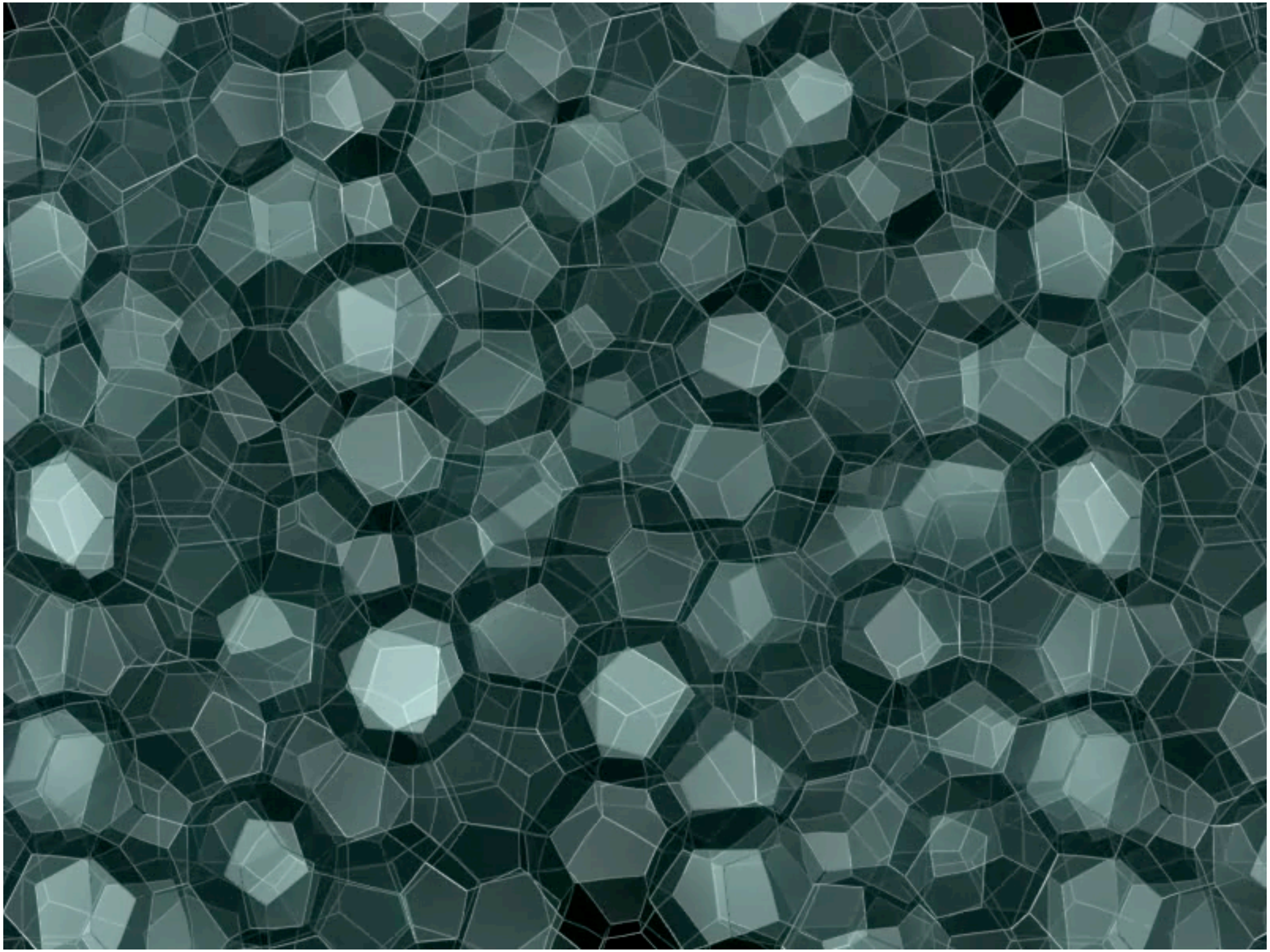
Quasi-static Simple Shearing Flow

Elastic deformations punctuated by cascades of topological transitions



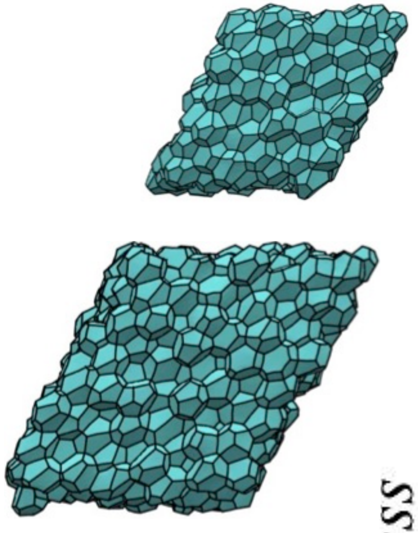
F. Rouyer, S. Cohen-Addad, M. Vignes-Adler & R. Hohler (2003) *Phys Rev E* **67**, 021405.
Video microscopy of foam under shear

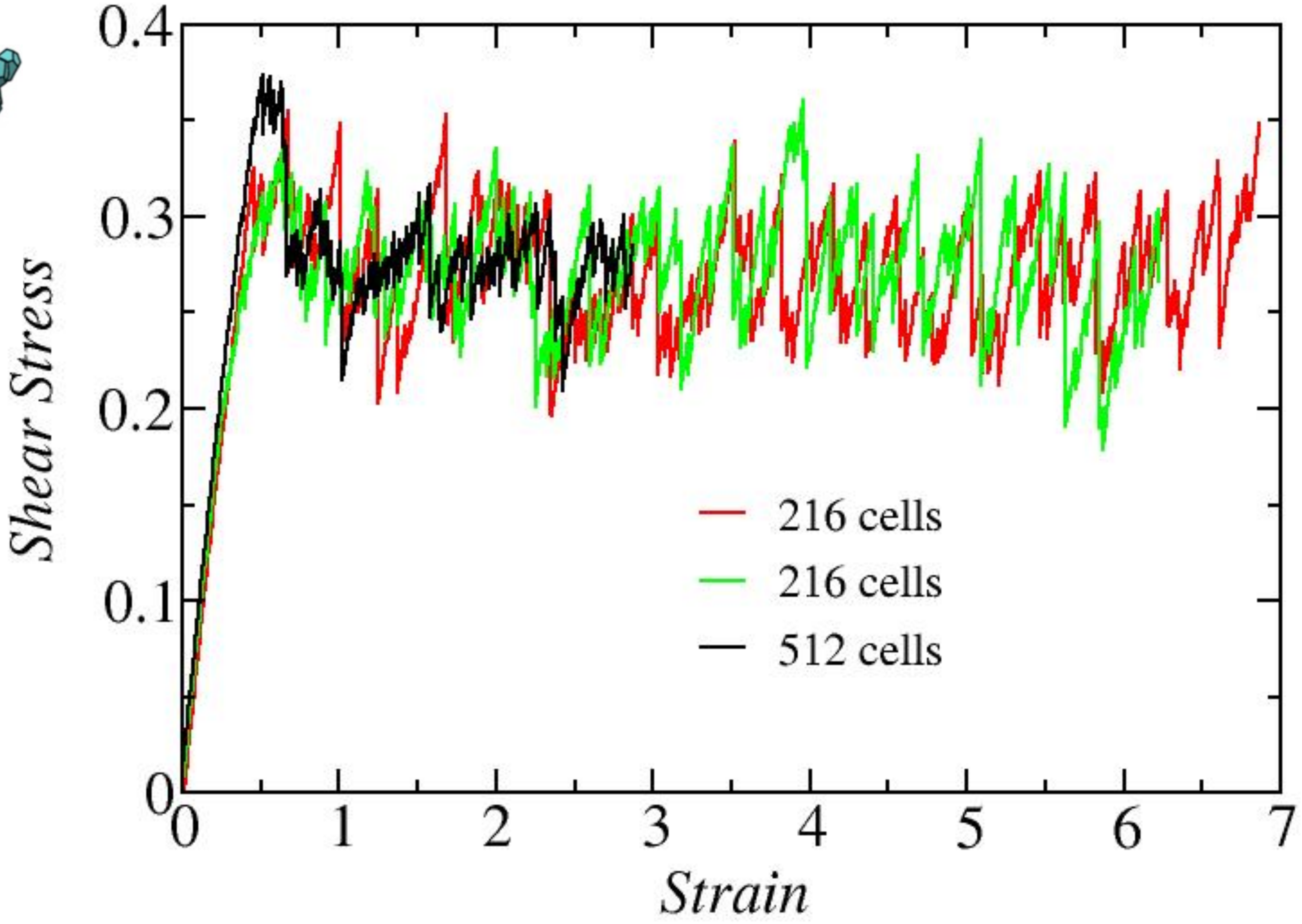
Evans, Kraynik, Reinelt, Mecke & Schroeder-Turk, Phys. Rev. Lett. 111, 138301 (2013).



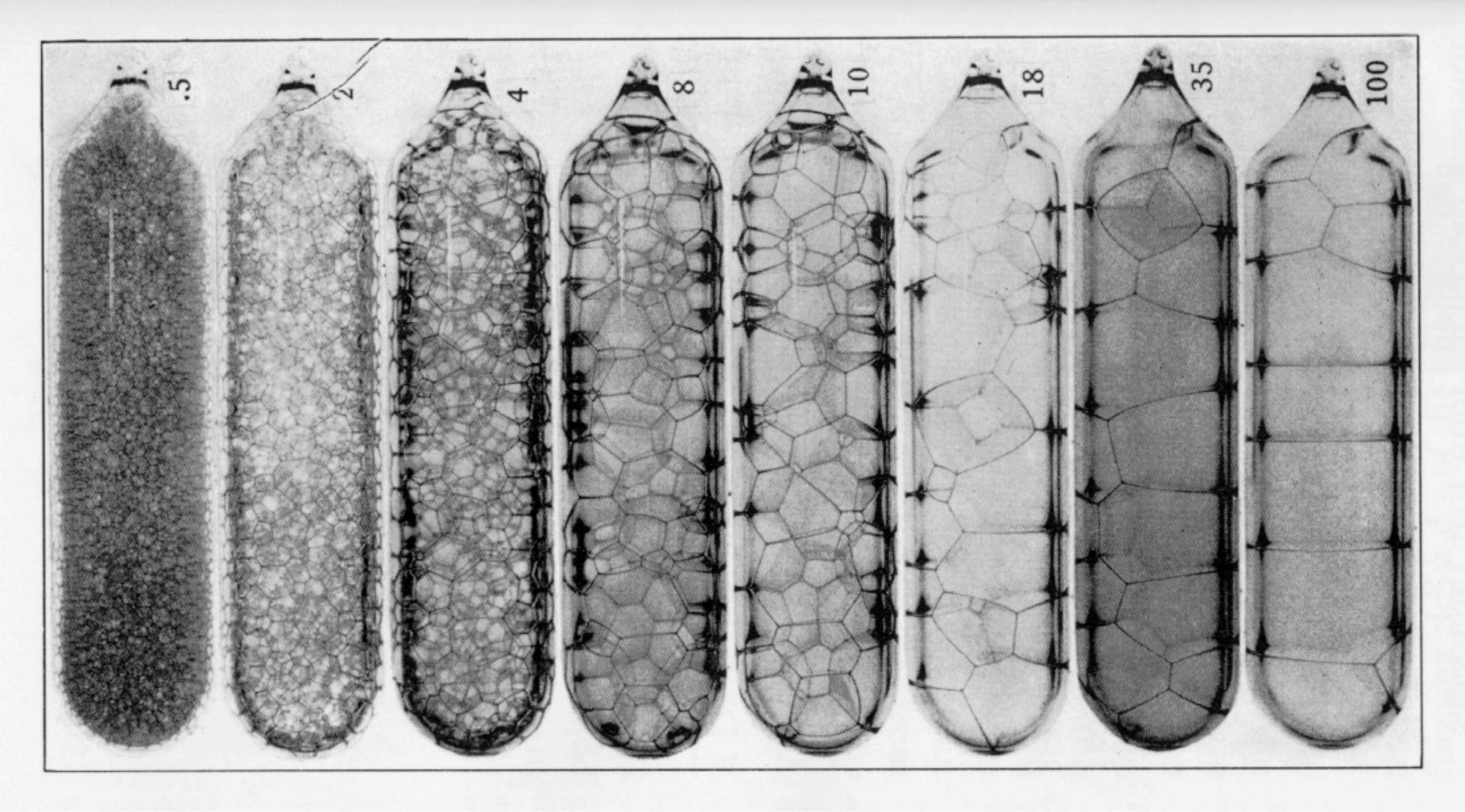
Myfanwy Evans

Simple Shearing Flow

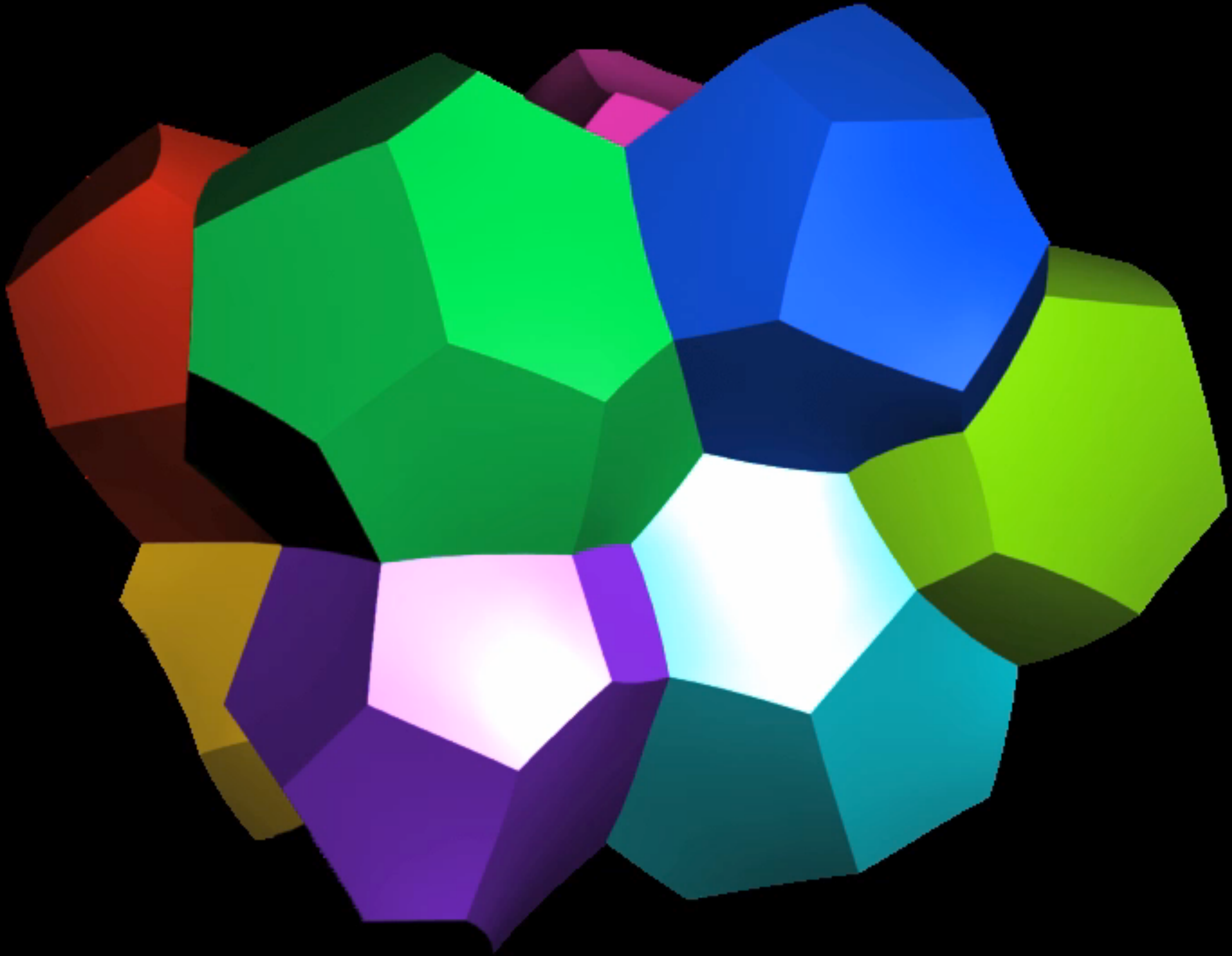


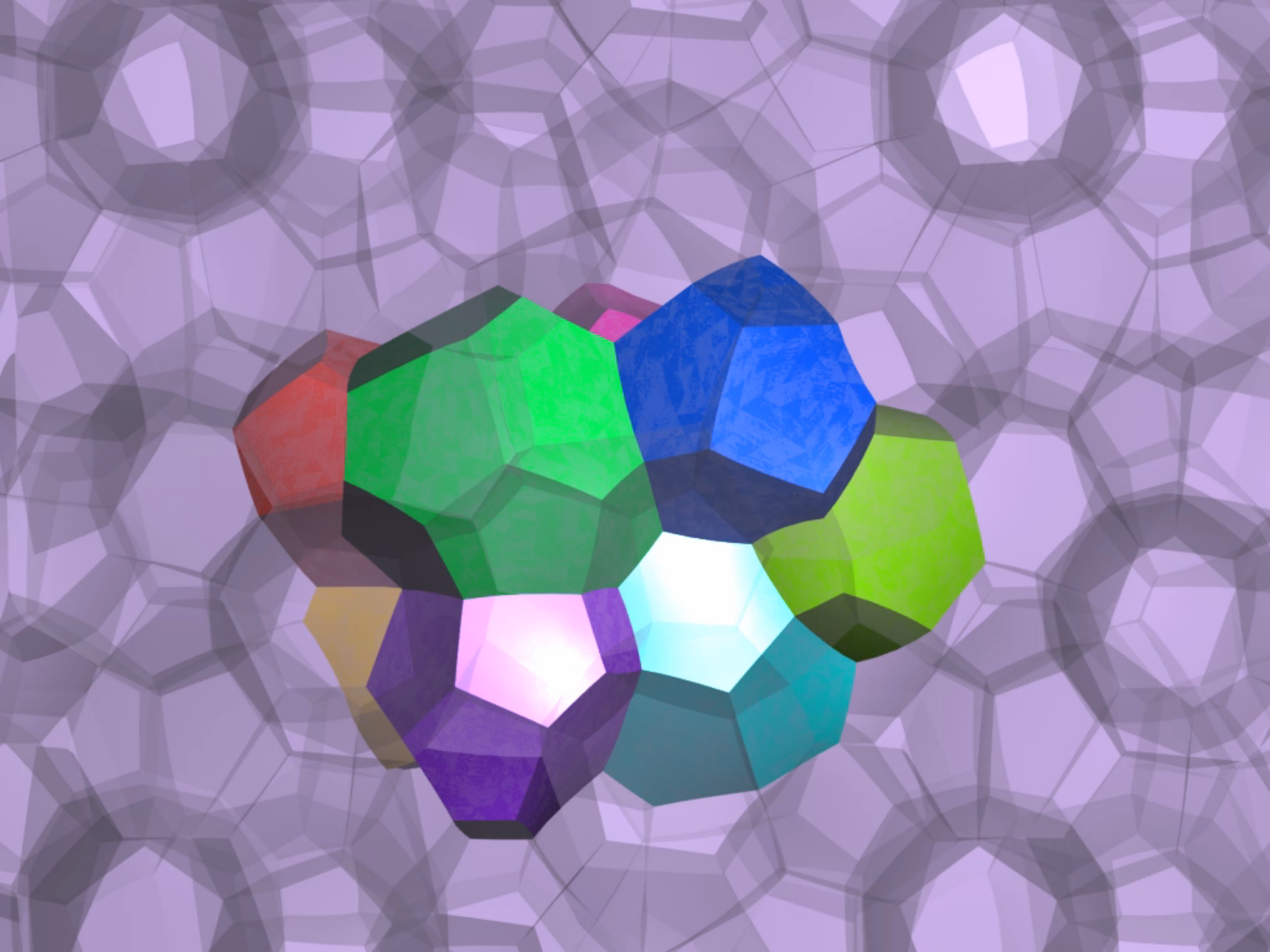


Diffusive coarsening of foam

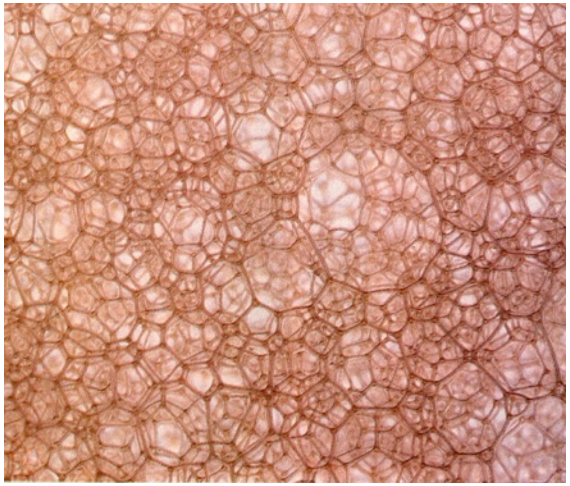
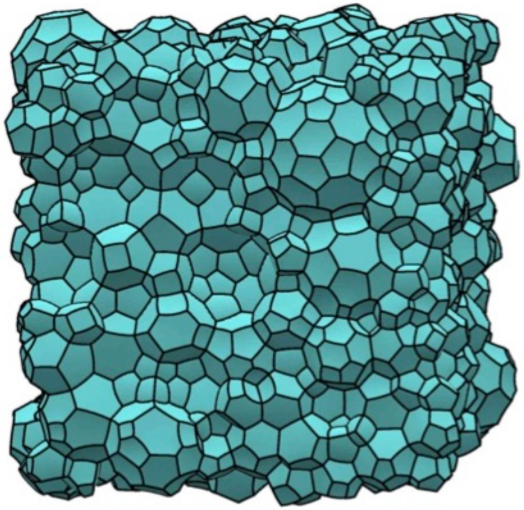
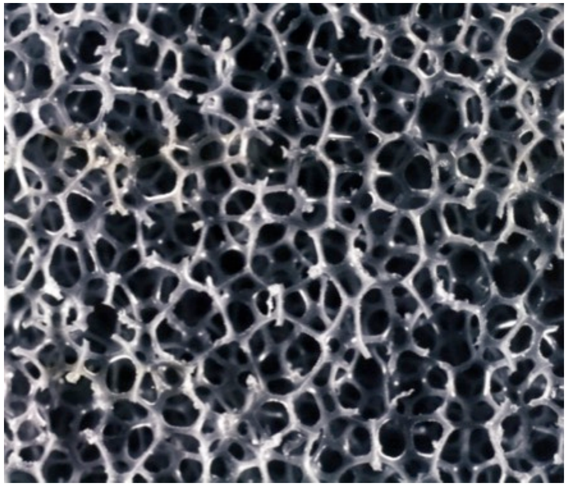
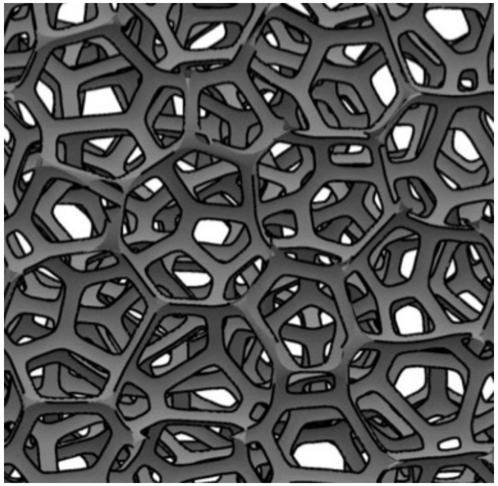


Cyril Stanley Smith (1981) *A Search for Structure: Selected Essays on Science, Art and History*, MIT Press, Cambridge, MA.





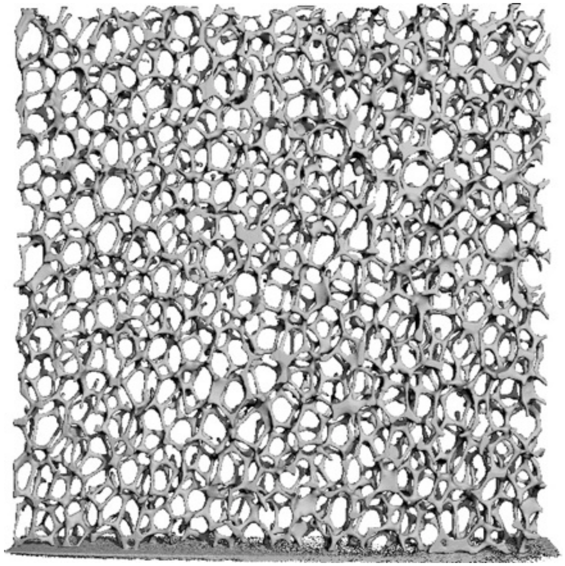
Low-density foams with random structure

	real materials	Surface Evolver models
soap froth		
solid foam with open cells		

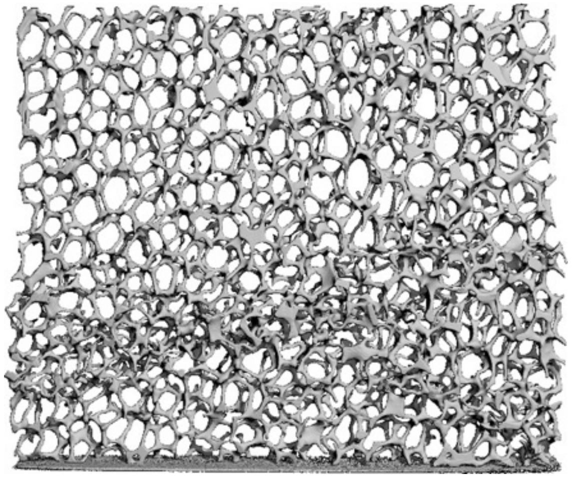
Crushing Aluminum Foam – X-ray CT

Jang, Kyriakides & Kraynik, Int. J. Solids Structures (2010)

Localized bands of crushed cells develop and gradually spread throughout the domain.



⓪



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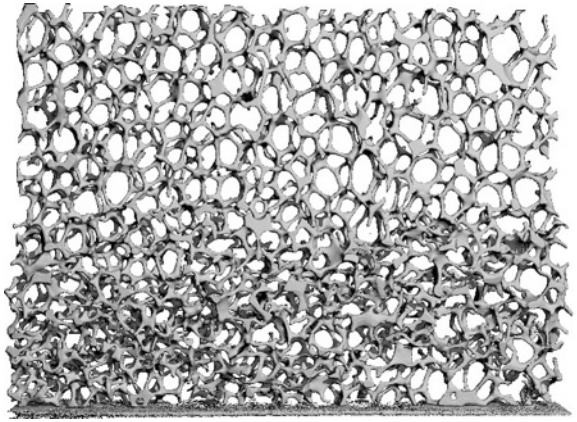

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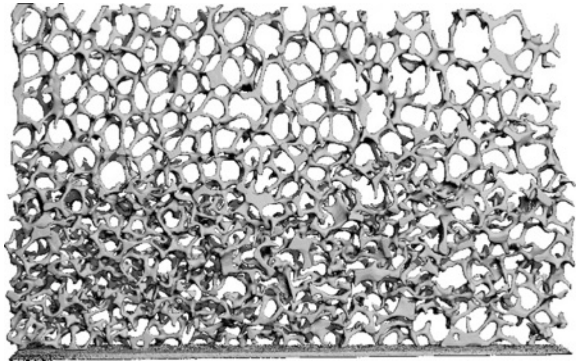

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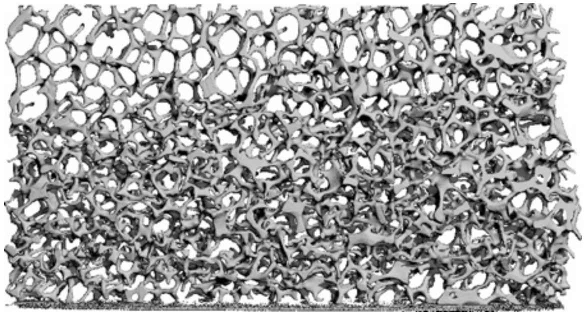

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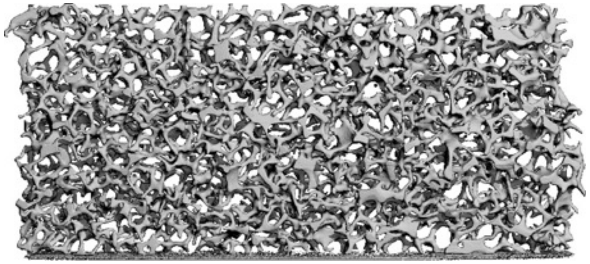
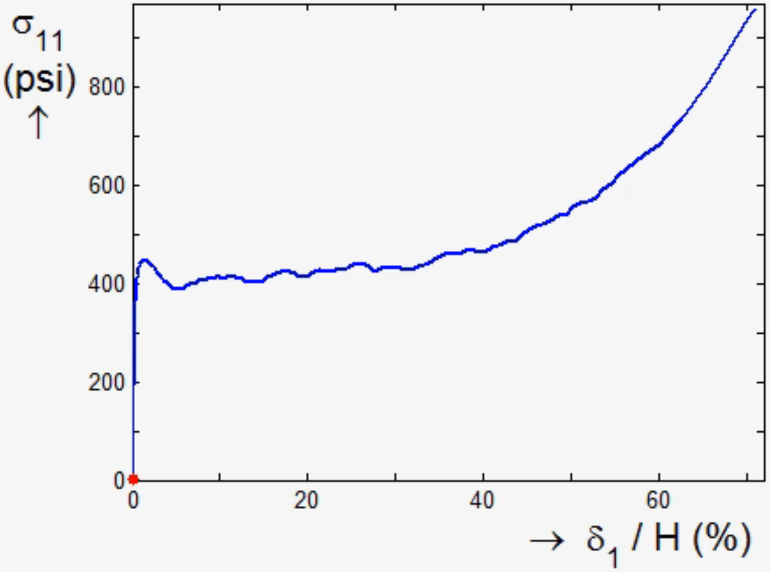


5

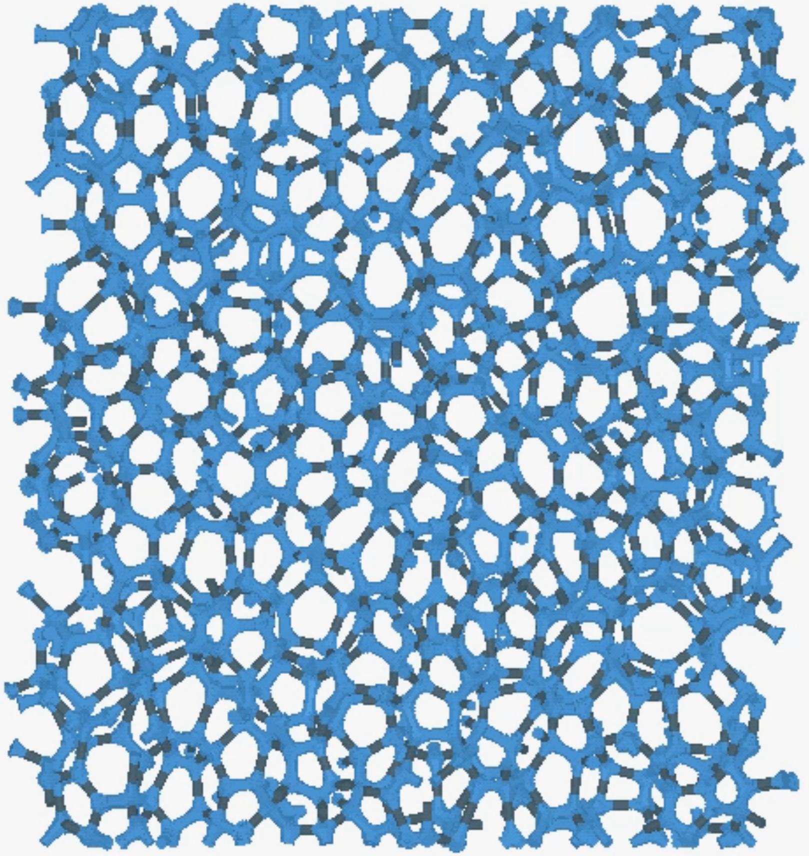
Crushing Aluminum Foam - LS Dyna simulations

Gaitanaros, Kyriakides & Kraynik, IJSS (2012)

LS-DYNA
elastic-plastic material (J2 flow)
anisotropic cells
non-uniform strut shape
shear deformable beams
beam contact

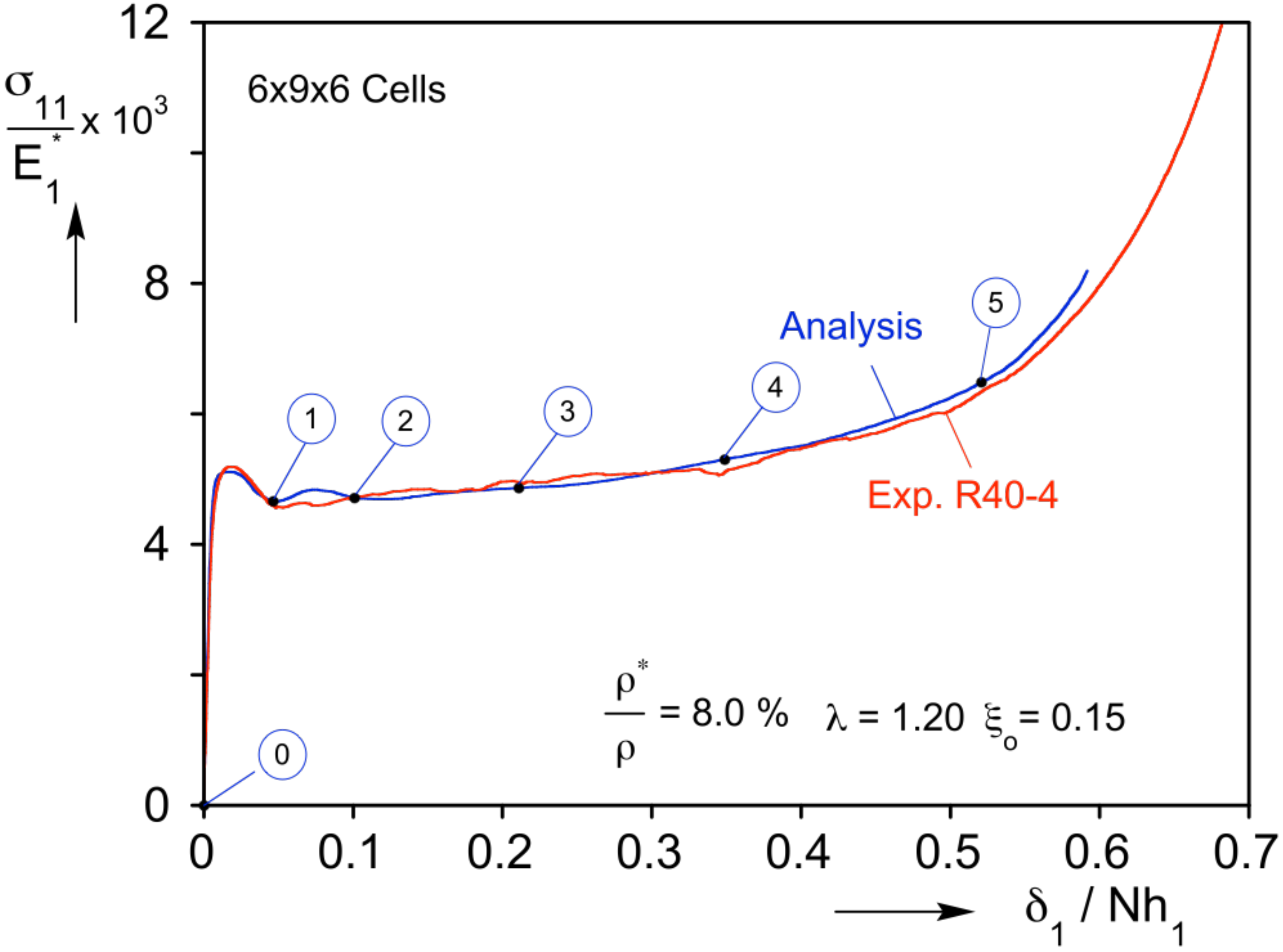


strain localization

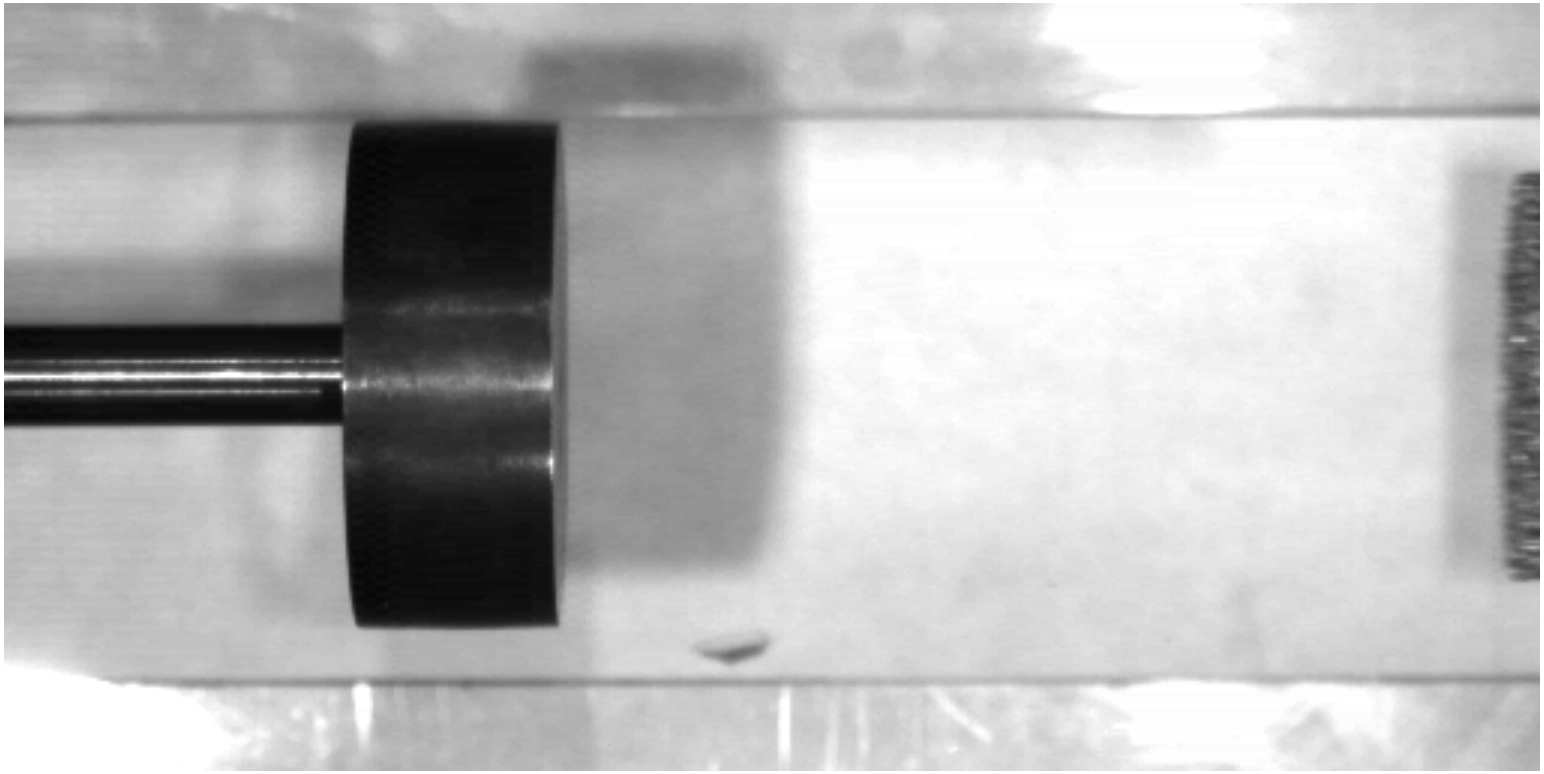


Crushing Aluminum Foam – Experiment and Simulation

Gaitanaros, Kyriakides & Kraynik, IJSS (2012)



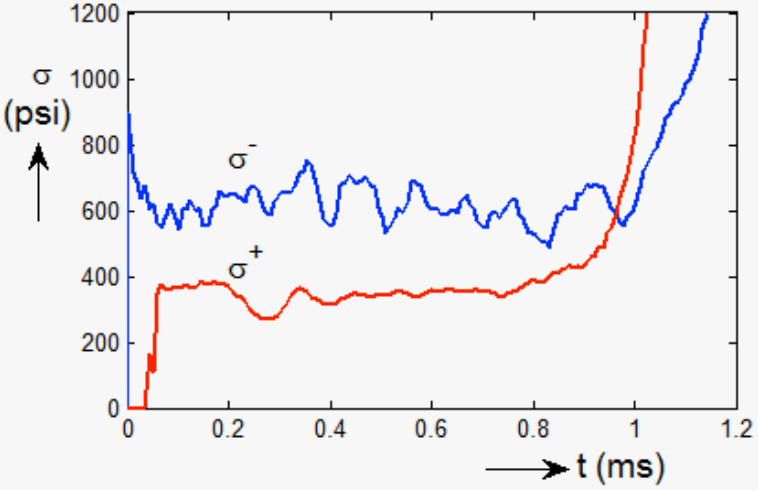
Direct Impact Test: V_i = 90 m/s

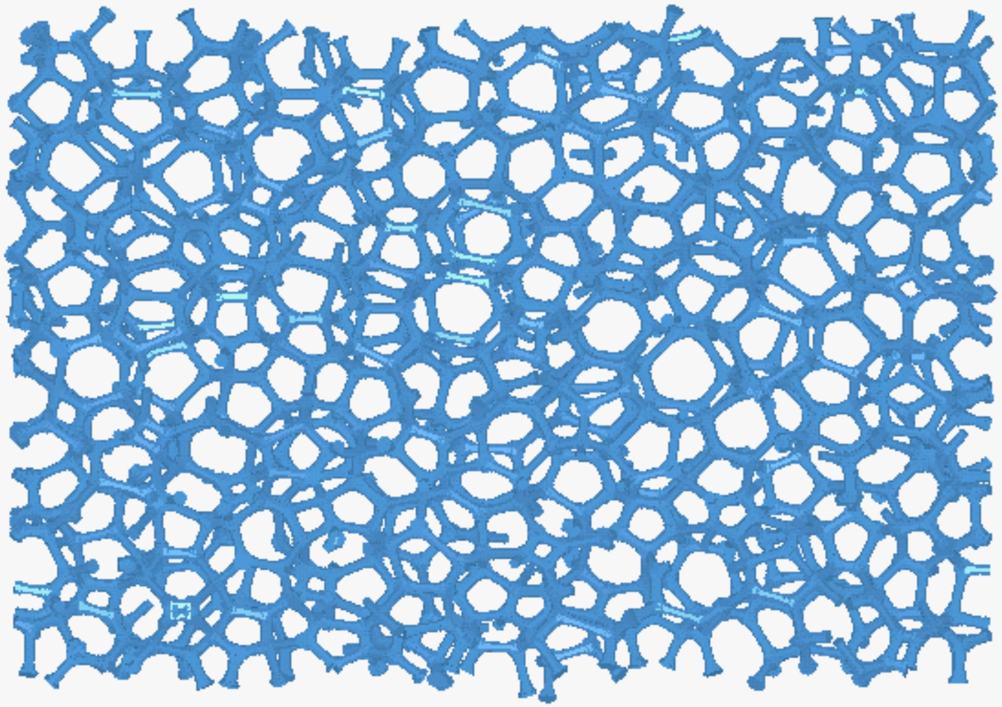


Frame rate: 40000/s
Exposure Time: 1/40000 s
Movie frame rate: 15 fps

Barnes, Ravi-Chandar, Kyriakides & Gaitanaros,
Dynamic crushing of aluminum foams: Part I – Experiments, IJSS (2014)

Direct Impact V_i = 90 m/s





shock formation

elapsed time ~ 1 ms

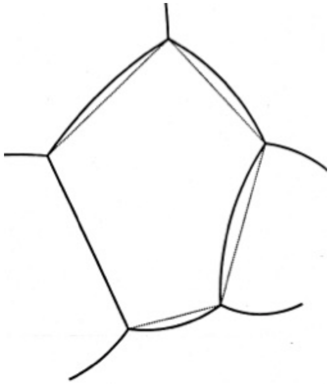
Gaitanaros & Kyriakides, Dynamic crushing of aluminum foams: Part II – Analysis, IJSS (2014)

Diffusive cell growth rate

J. von Neumann (1952) in *Metal Interfaces*, Am. Soc. Metals, Cleveland, OH, 108.

$$\frac{dA}{dt} = \mathcal{D}_{vn}\,(n-6)$$

Exact !

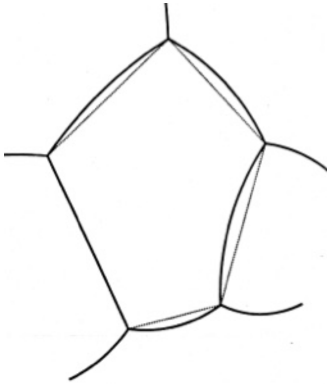


Diffusive cell growth rate

J. von Neumann (1952) in *Metal Interfaces*, Am. Soc. Metals, Cleveland, OH, 108.

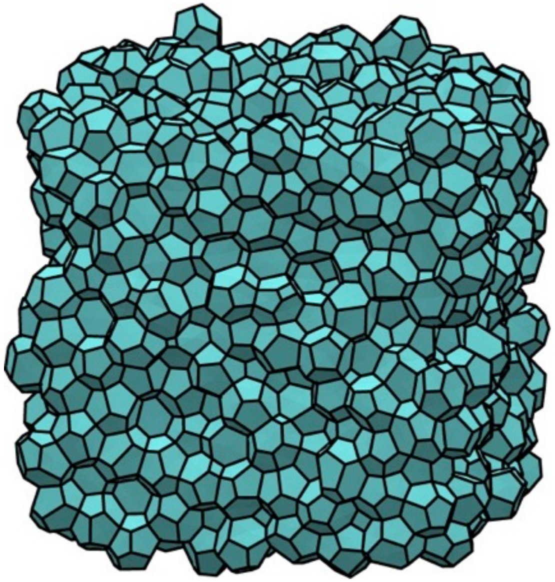
$$\frac{dA}{dt} = \mathcal{D}_{vn}\,(n-6)$$

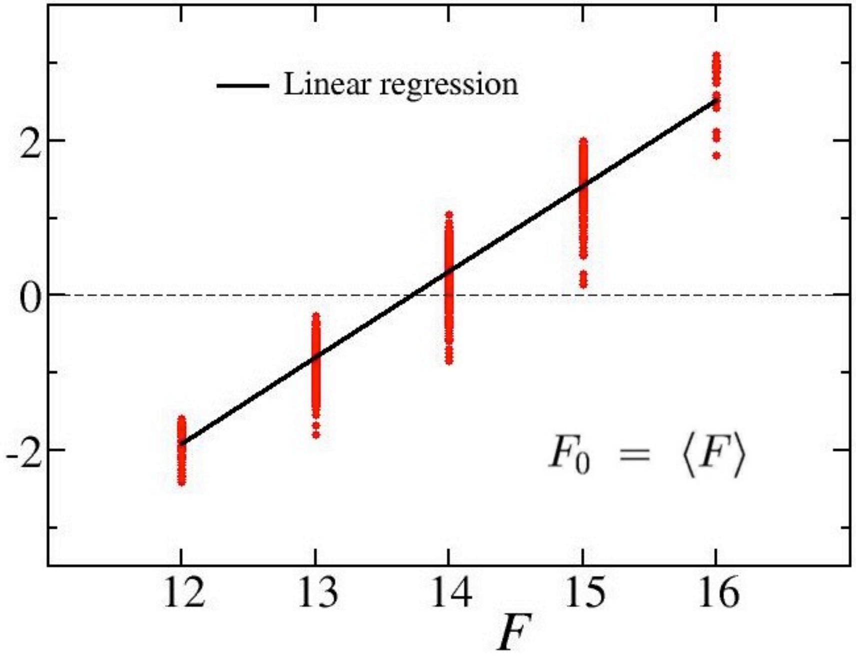
Exact !



Surface Evolver simulations of random monodisperse foam

$$V^{-1/3}\,\frac{dV}{dt} = \mathcal{D}\,(F - \langle F \rangle)$$





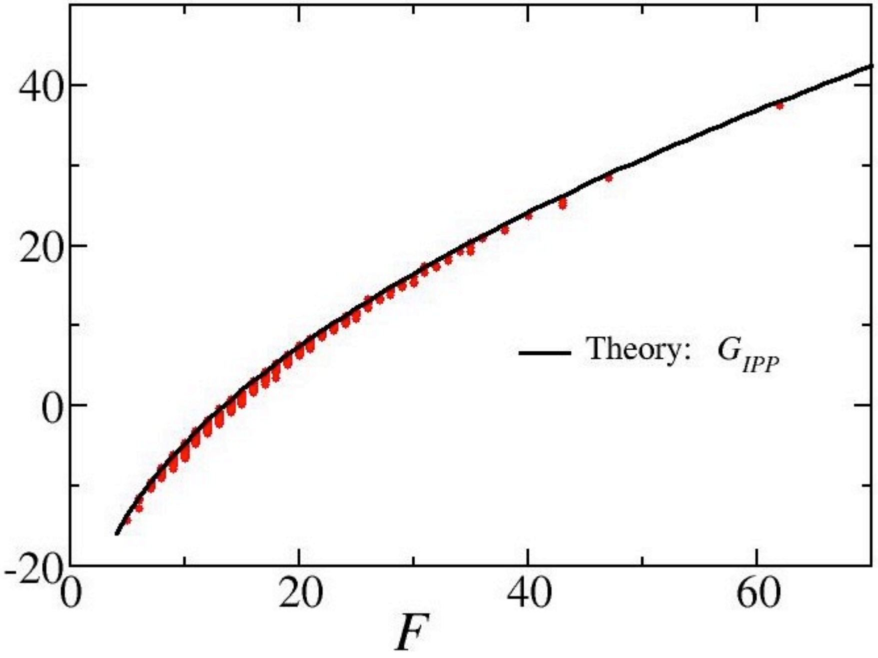
Theory for Diffusive Growth Rate

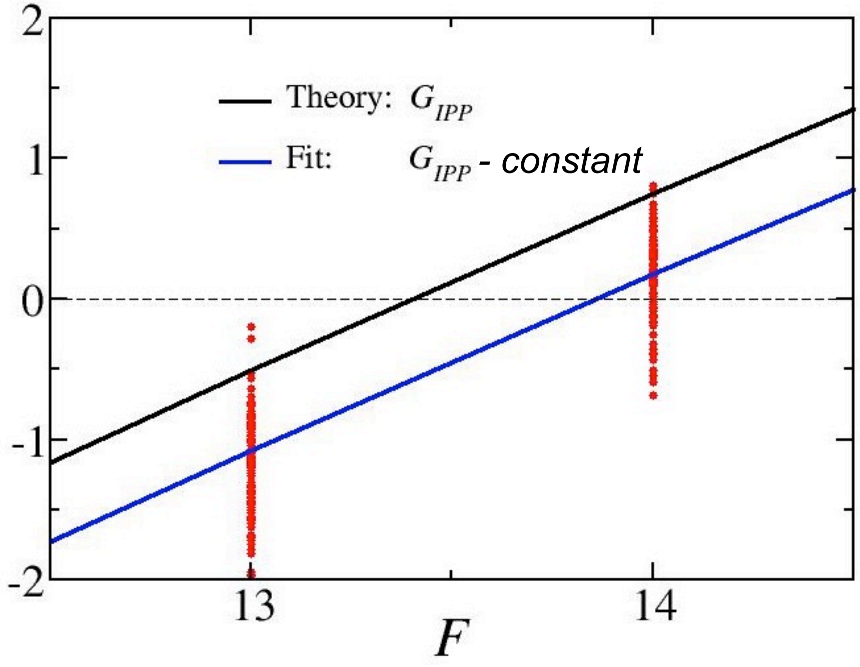
S. Hilgenfeldt, A.M. Kraynik, S.A. Koehler & H. Stone (2001) *PRL,* **86**, 2685.

$$V^{-1/3}\,\frac{dV}{dt} = \mathcal{D}\,\mathcal{G}(structure) \sim F^{1/2} + \ldots$$

S. Hilgenfeldt, A.M. Kraynik, D.A. Reinelt & J.M. Sullivan (2004) *EPL*, 67, 484.

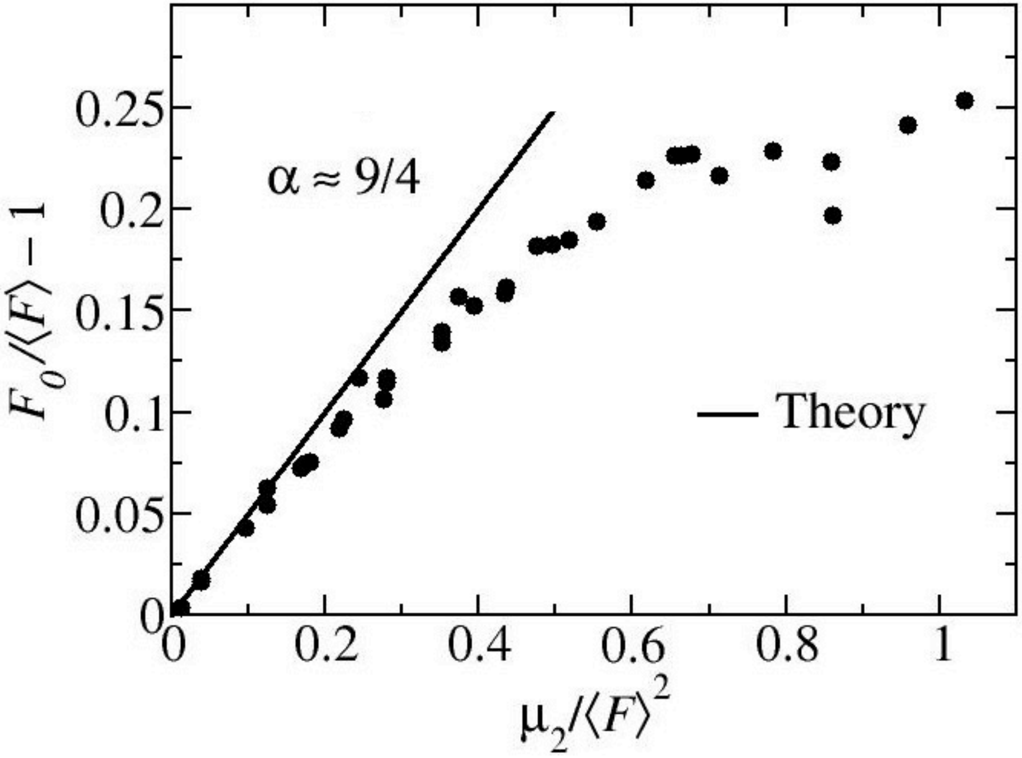
$$\mathcal{G}_{IPP}(F) \sim 2.213\,F^{1/2} - 7.778 + \ldots$$





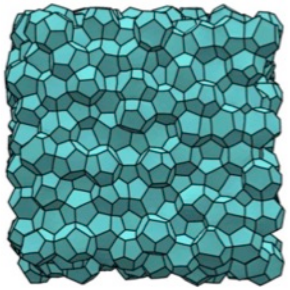
Theory and simulation for the cross-over in growth rate

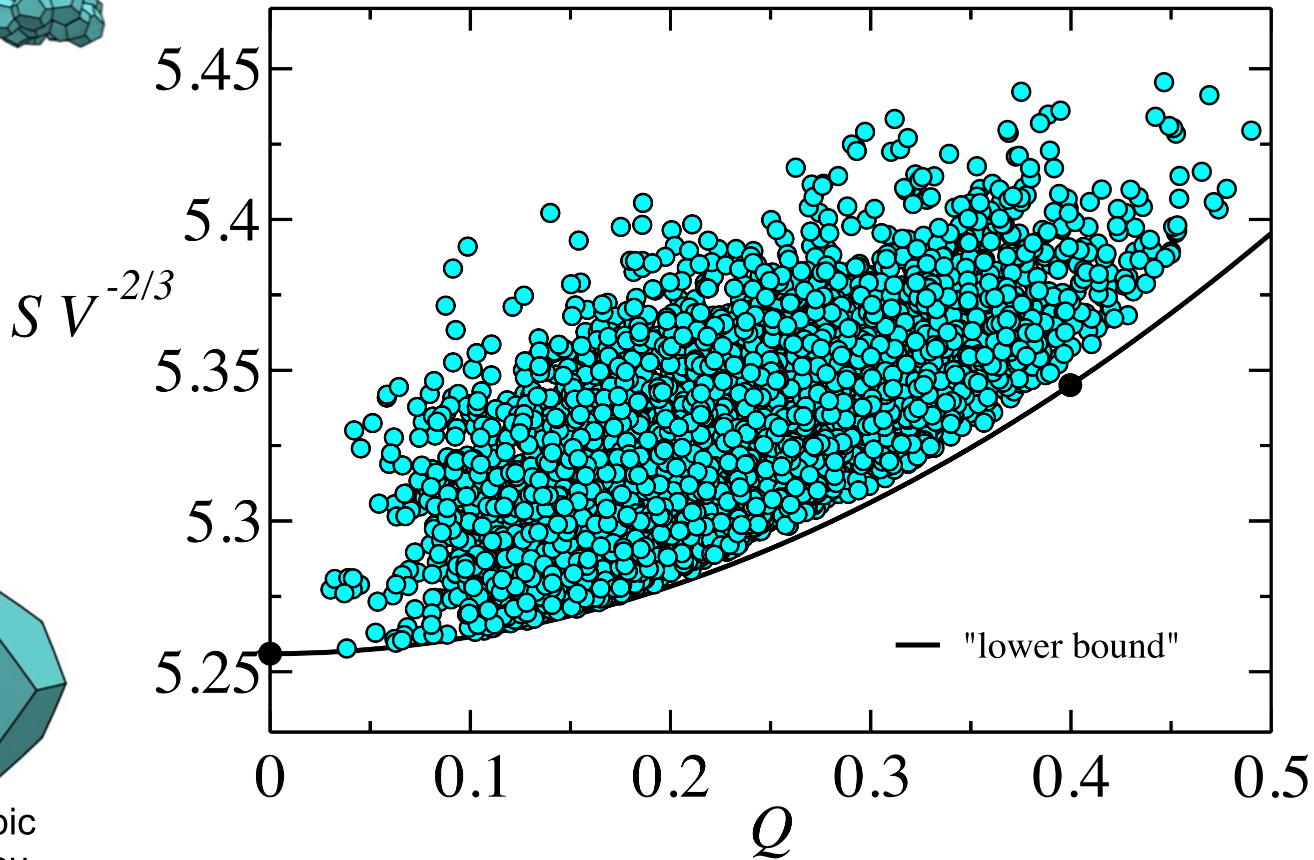
$$F_0 = \langle F \rangle \left[1 + \left(\frac{\alpha}{3} - \frac{1}{4} \right) \frac{\mu_2}{\langle F \rangle^2} \right]$$

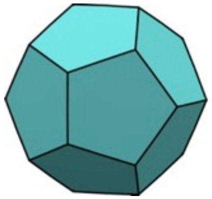


Local measures of cell shape

surface area (S) and distortion (Q)







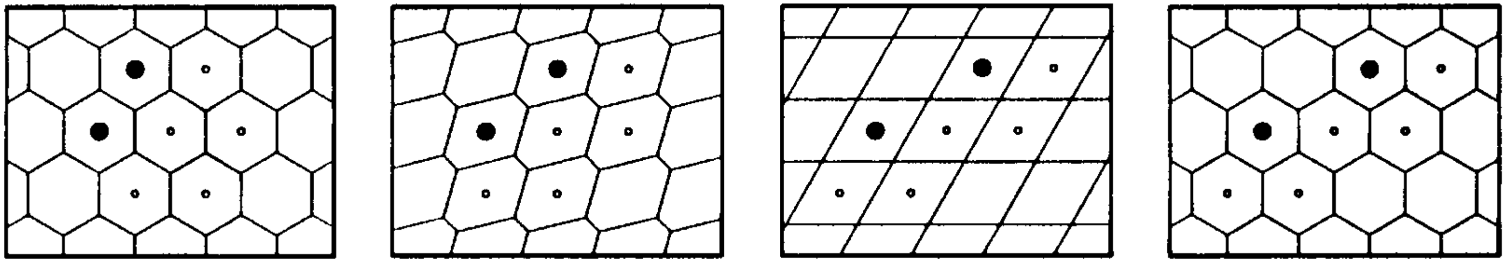
Isotropic
Plateau
Dodecahedron

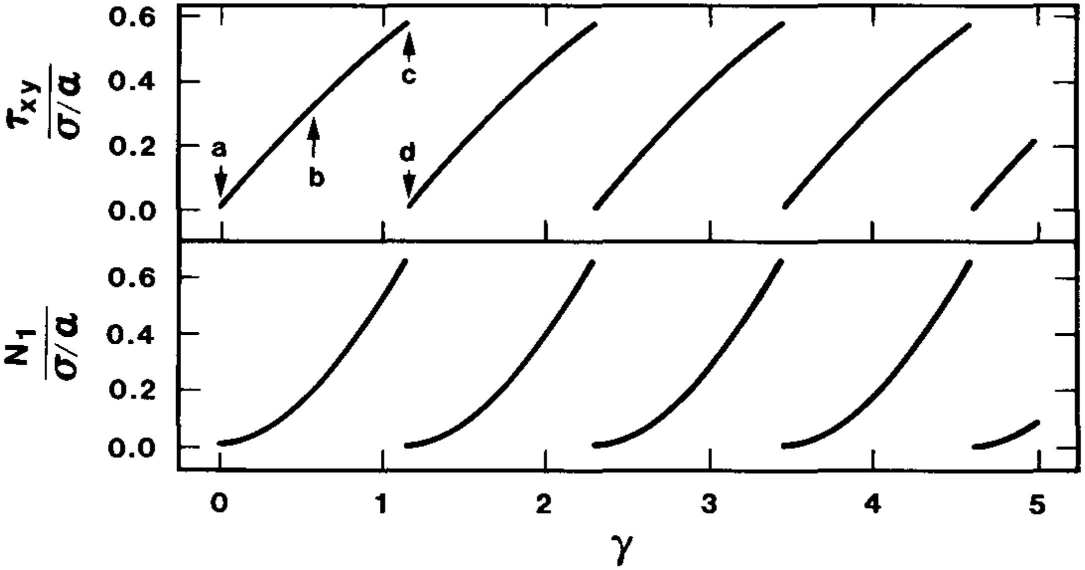
C15

<Q> = 0.24

Quasistatic simple shear of a liquid honeycomb

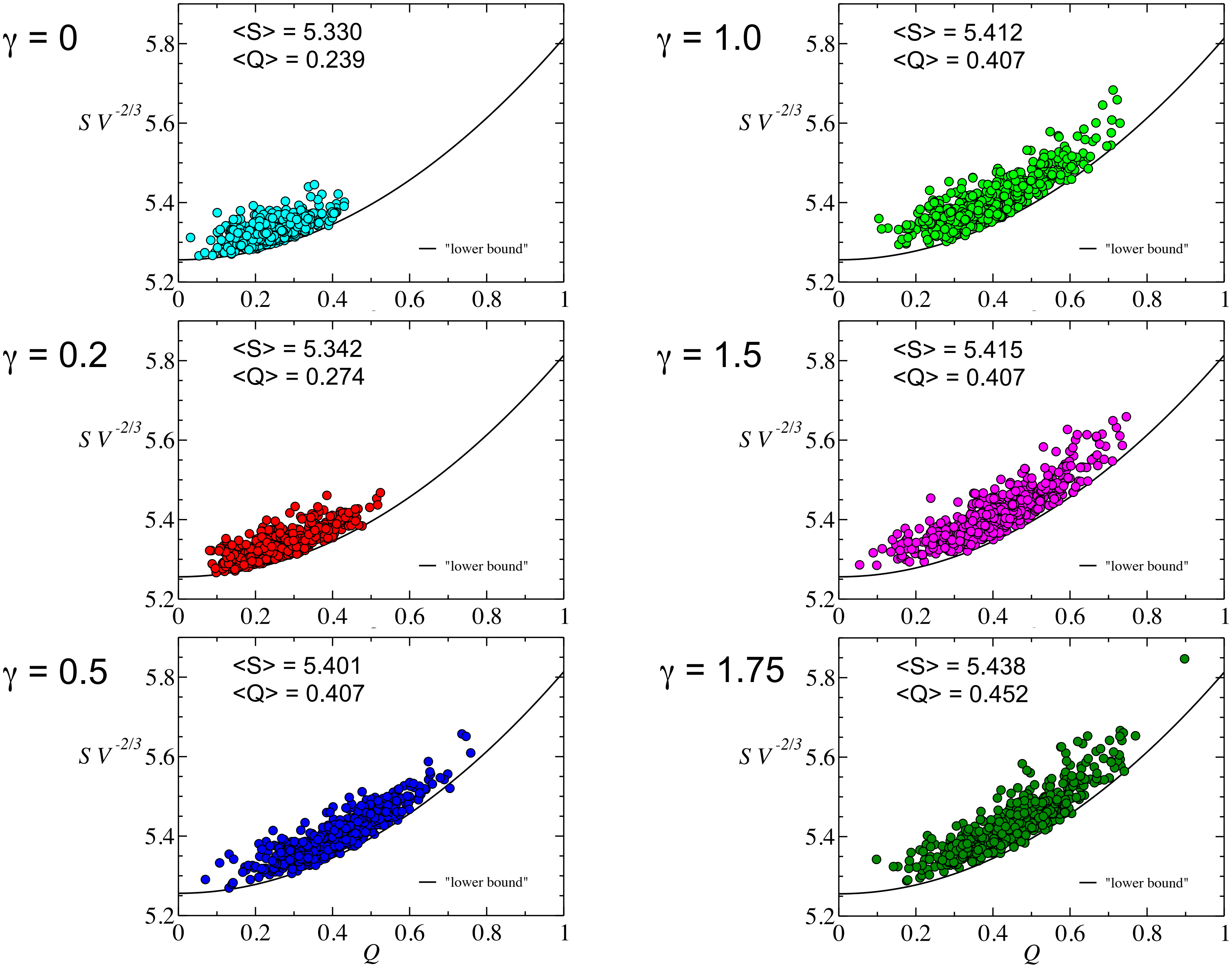
Princen (1983) *JCIS* **91** 160-175.



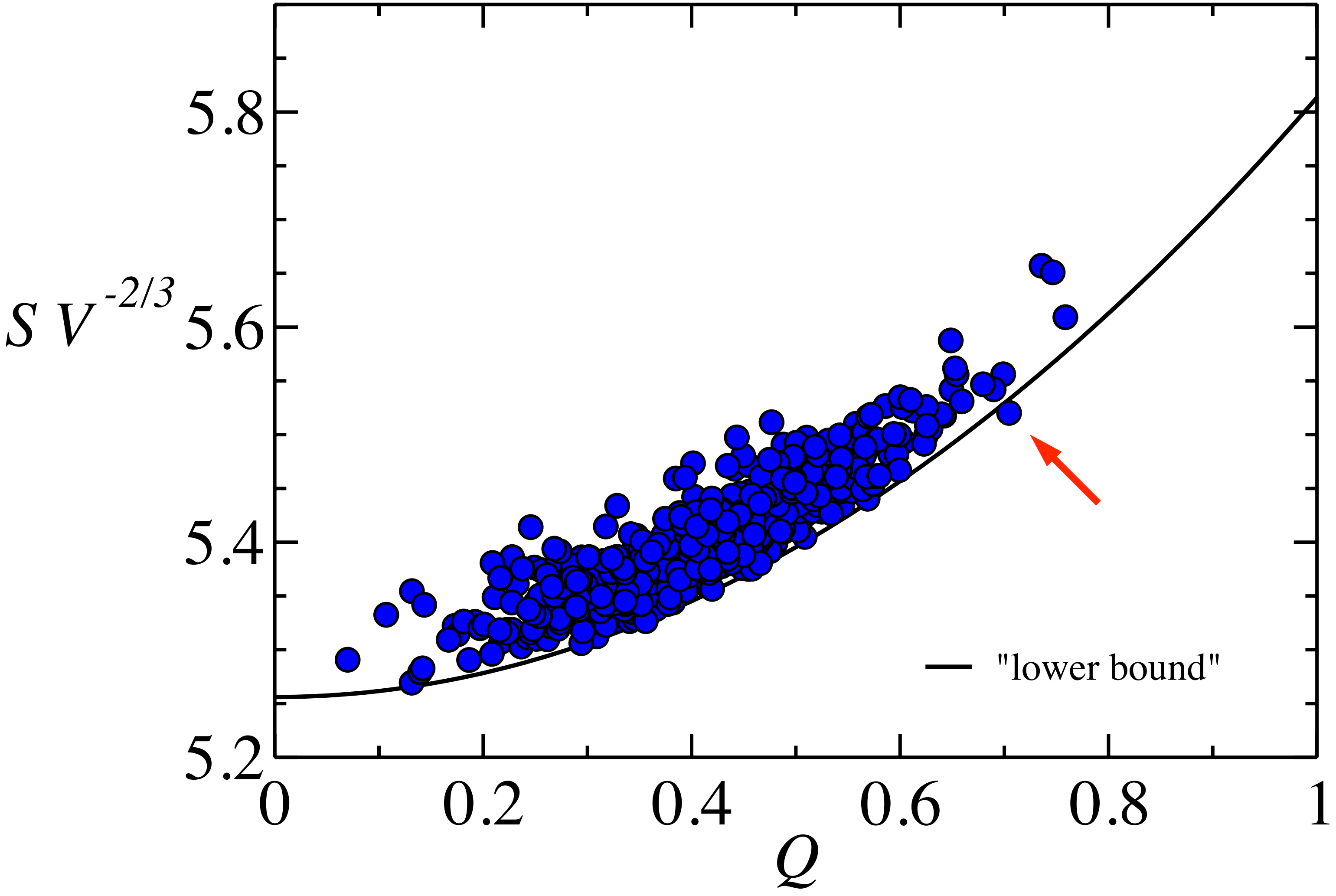


$\gamma_p = 2/3^{1/2}$

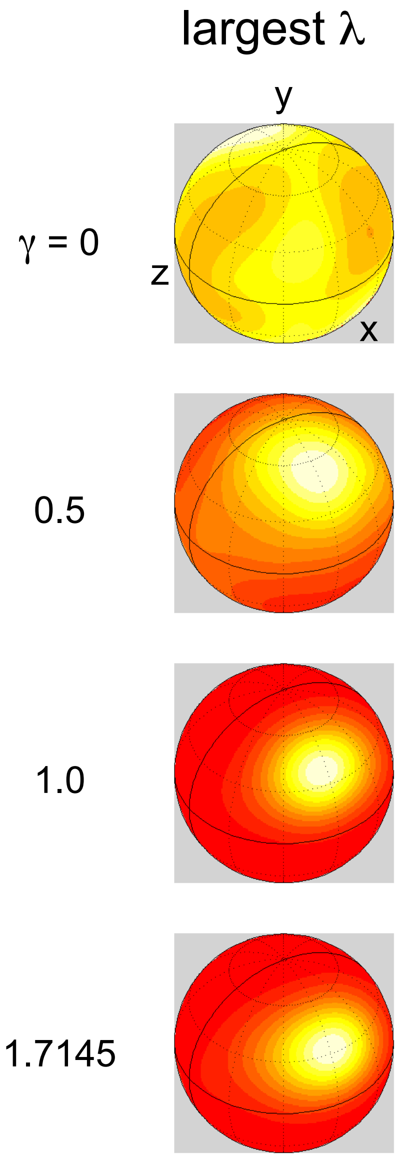
Simple Shear – local measures S and Q



Simple Shear – local measures S and Q



Eigenvectors of q_{ij} – orientation distribution



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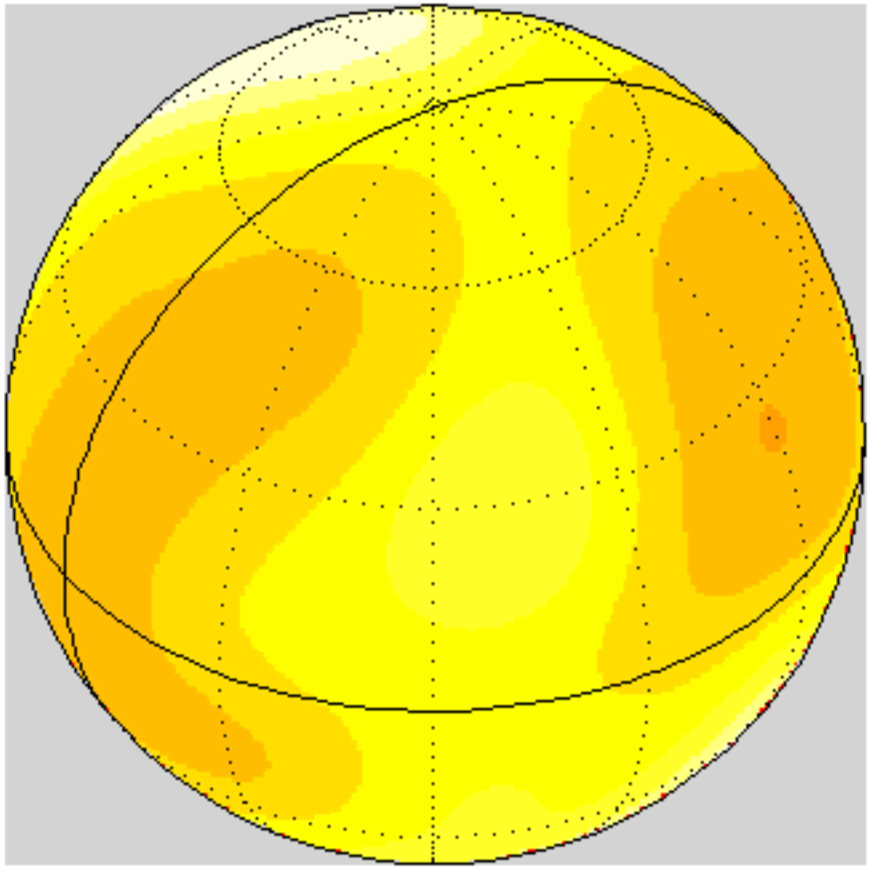
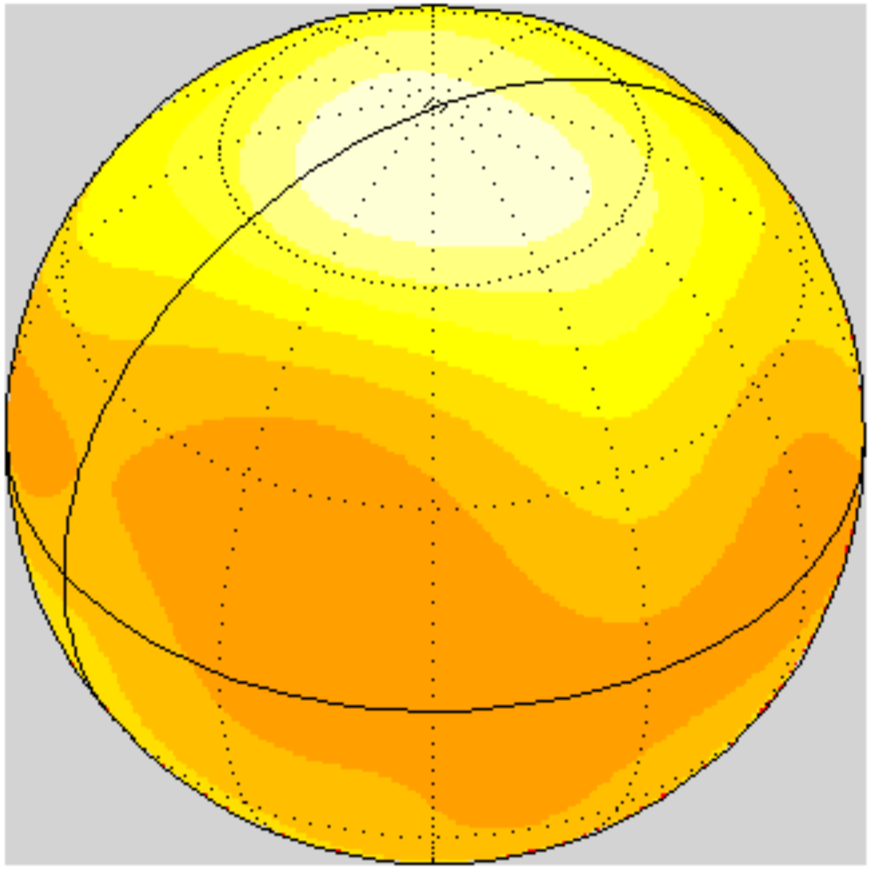
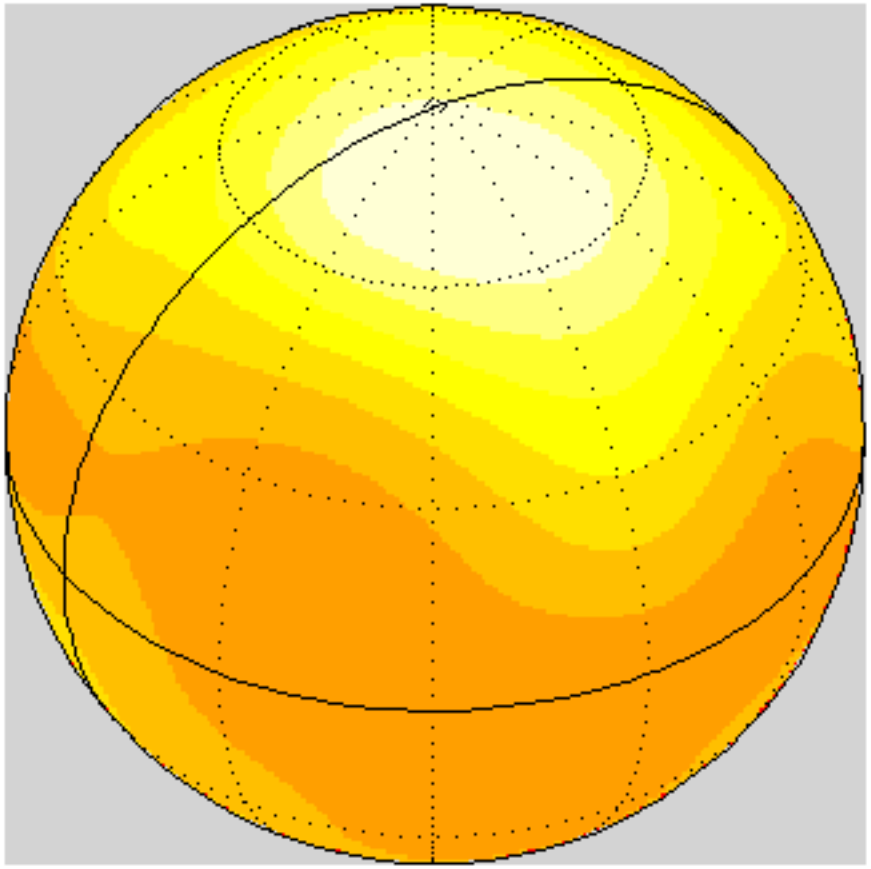


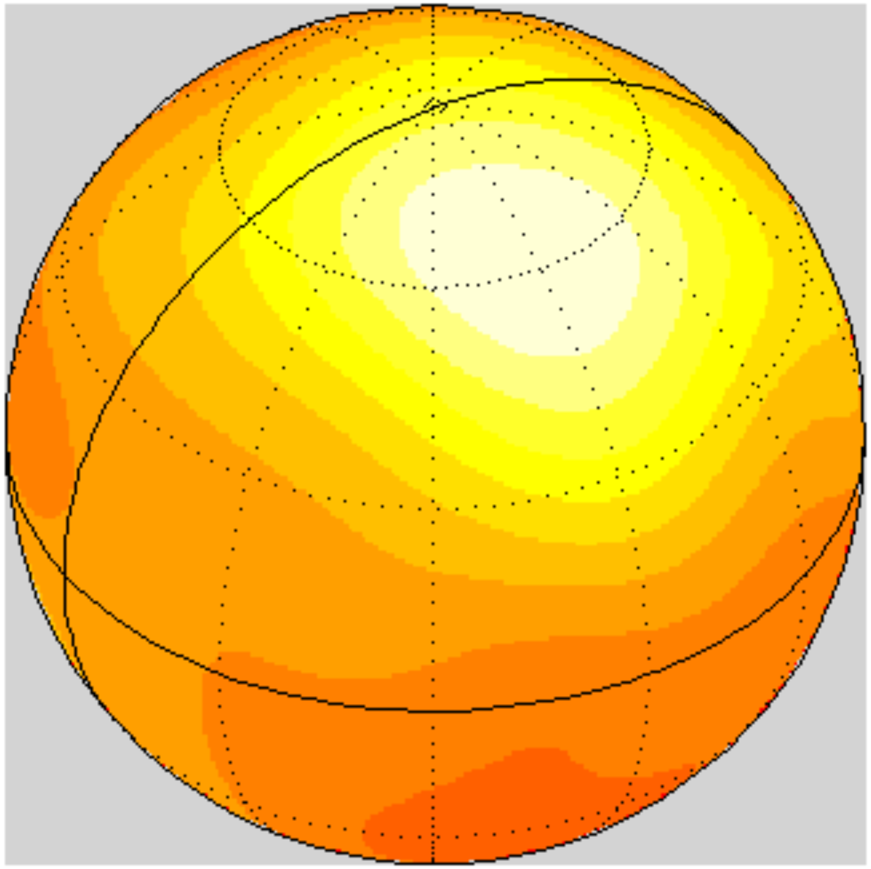
$\gamma = 0$



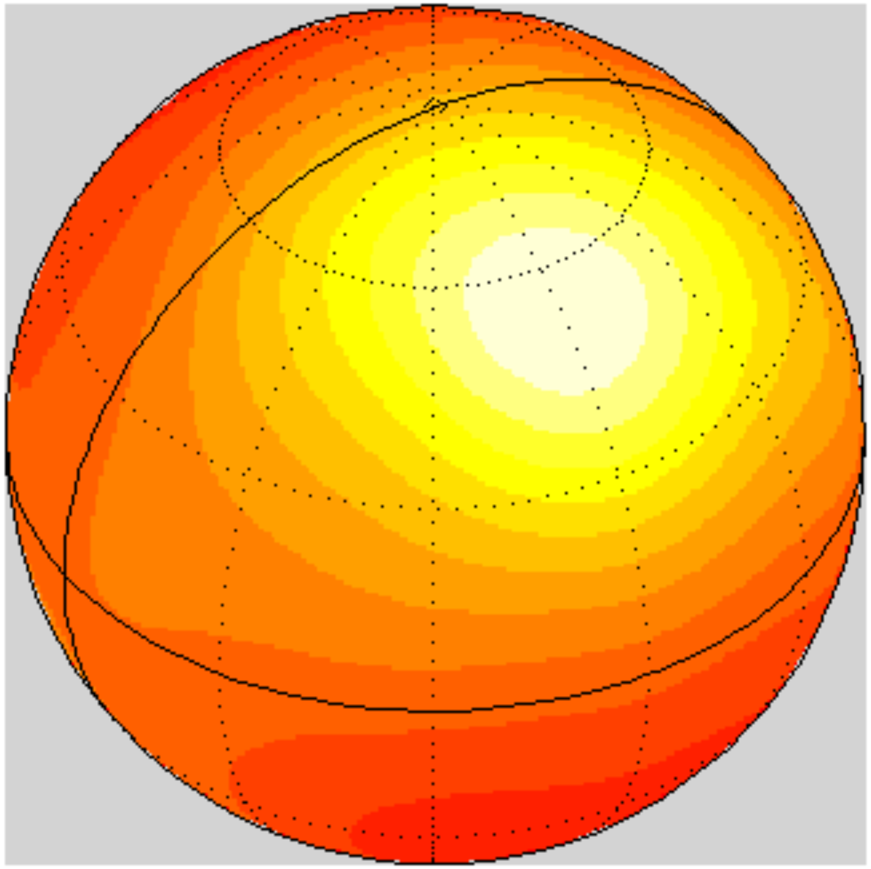
γ = 0.01



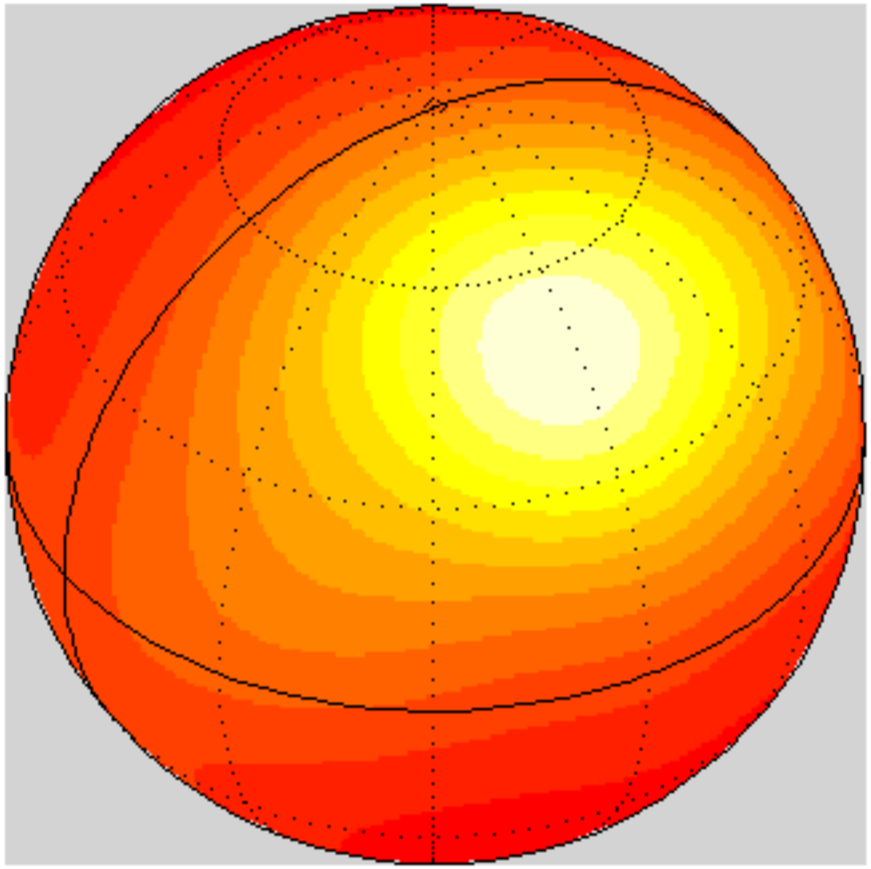
$\gamma = 0.02$



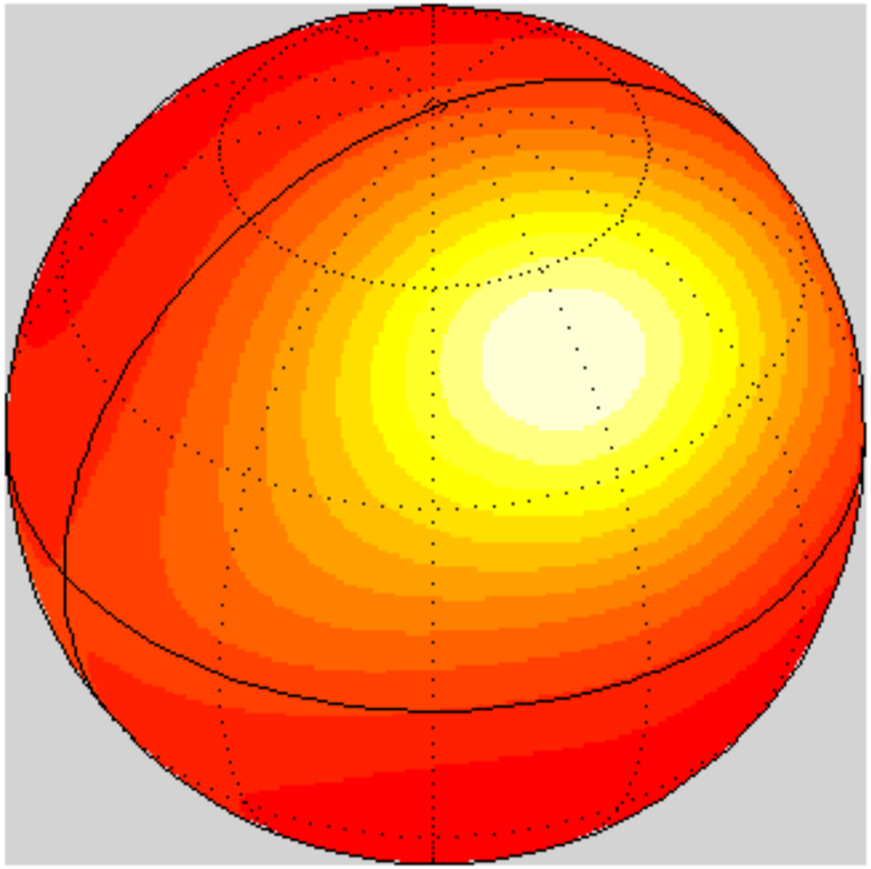
γ = 0.05



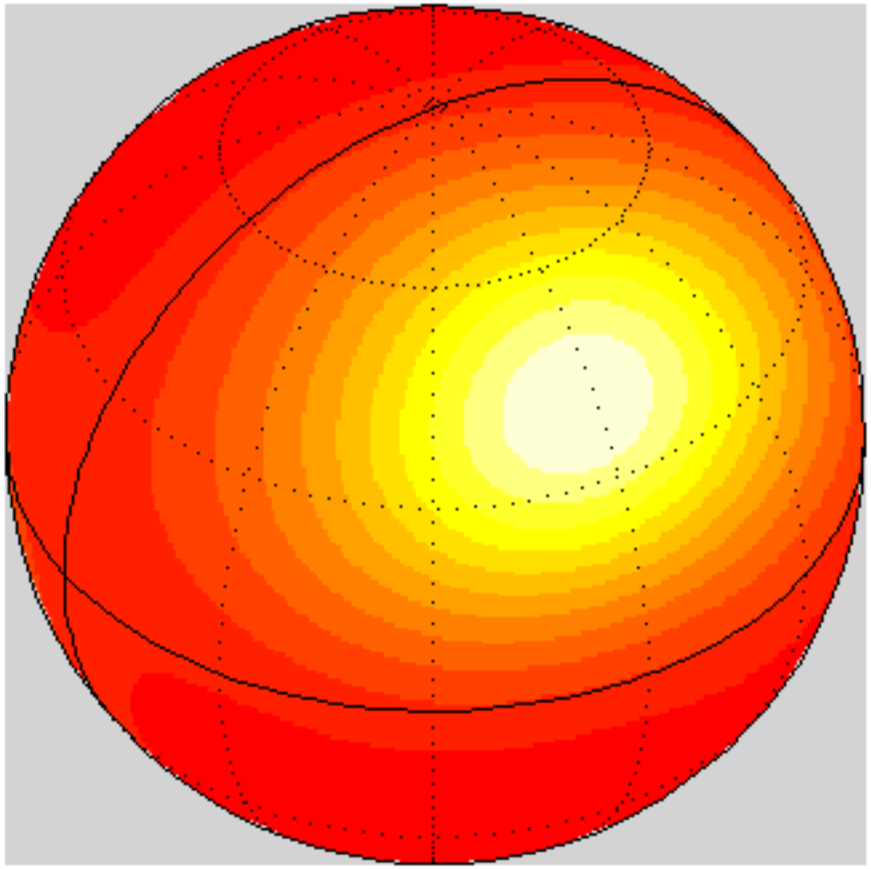
γ = 0.1



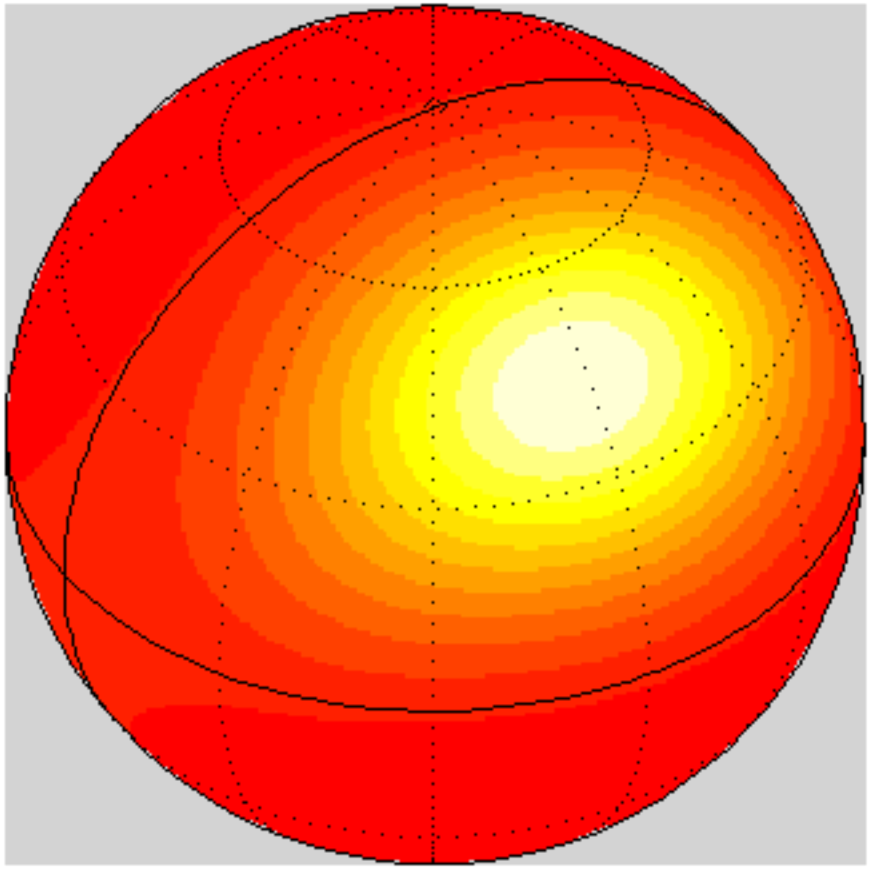
γ = 0.15



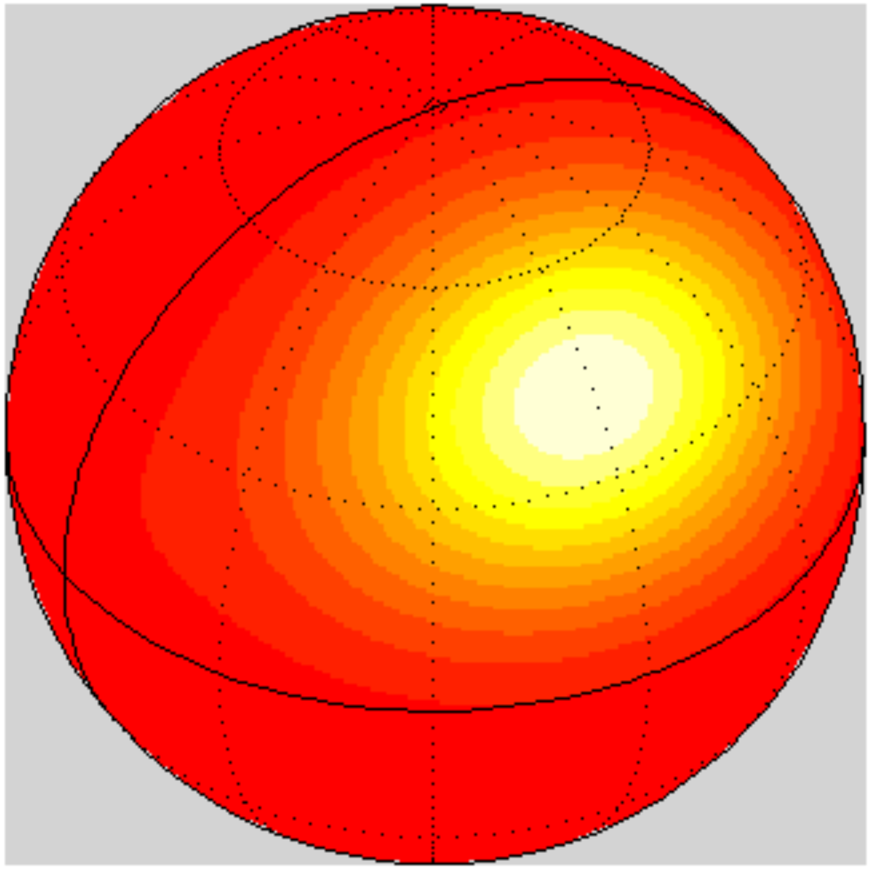
$\gamma = 0.2$



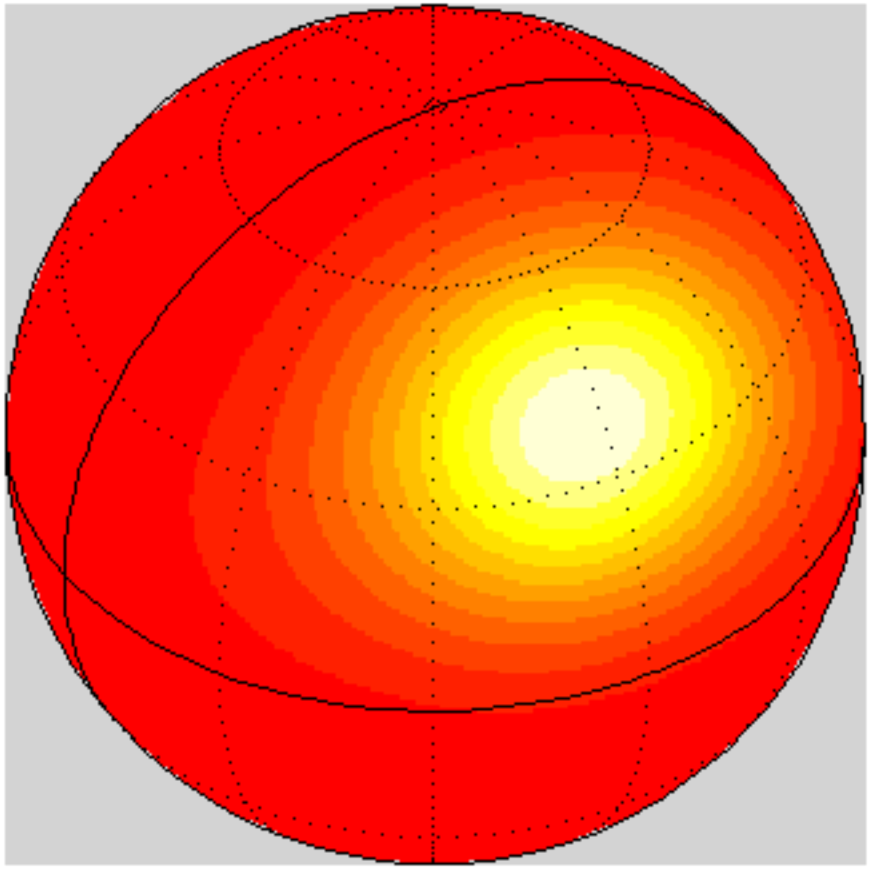
γ = 0.25



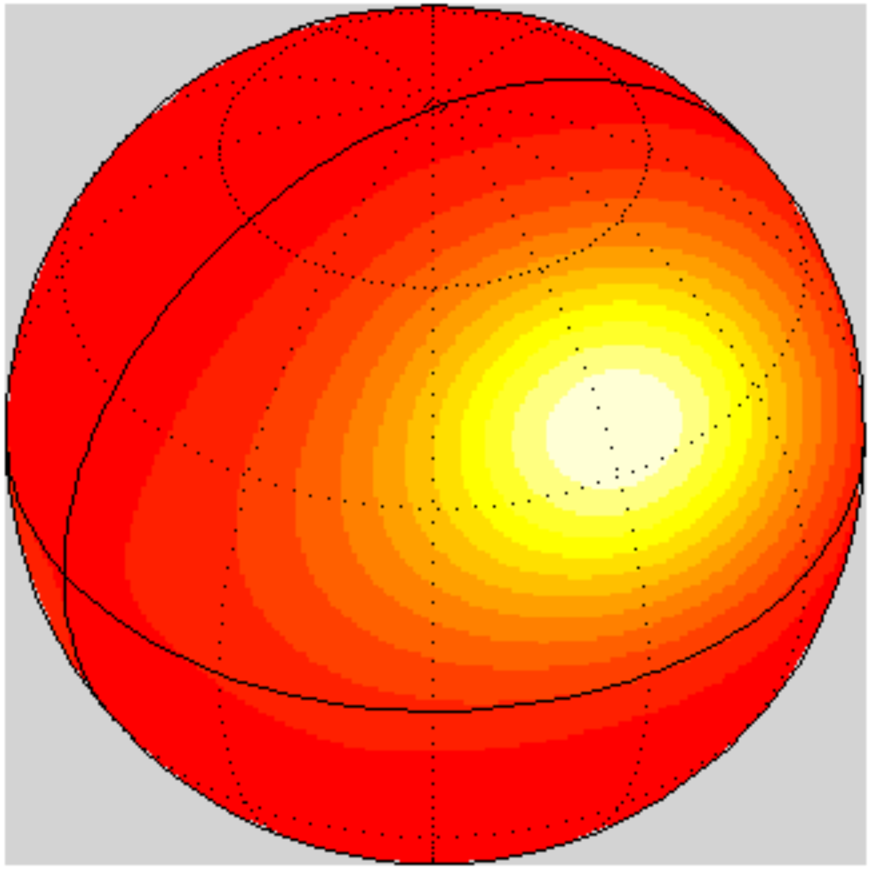
γ = 0.3



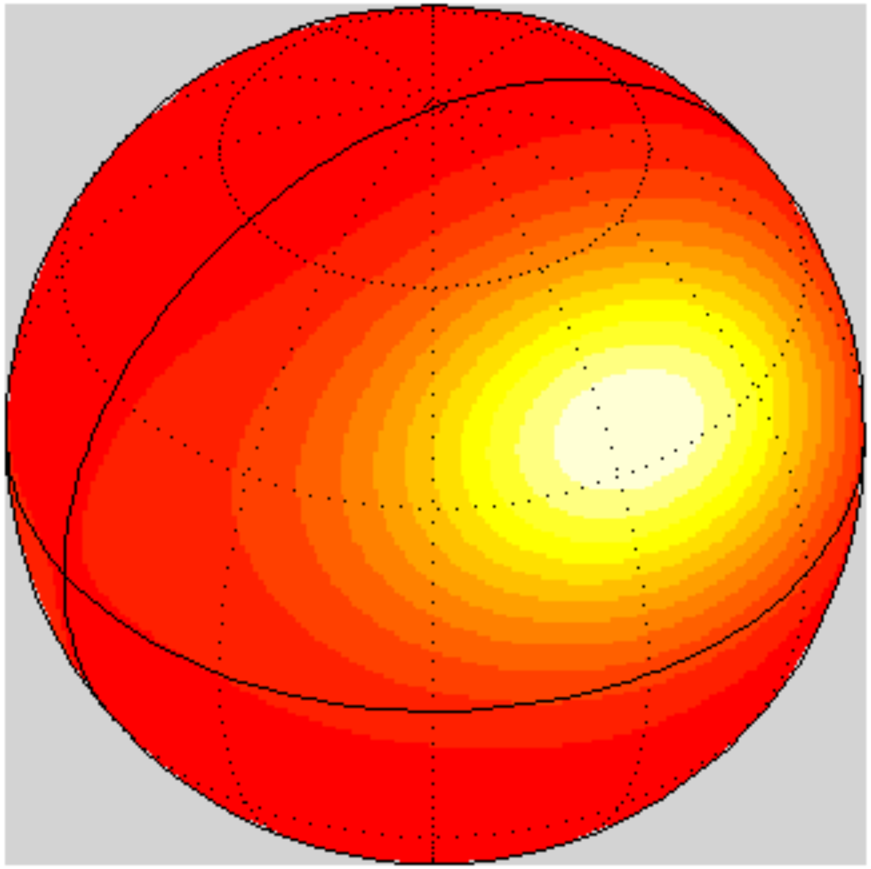
γ = 0.4



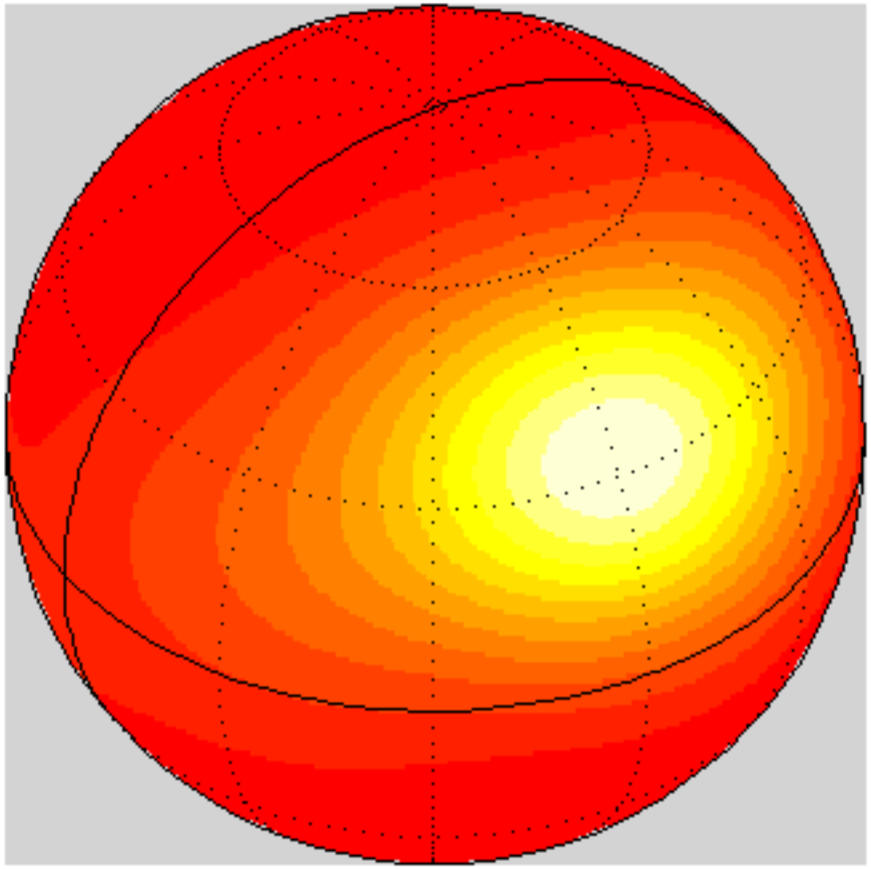
γ = 0.5



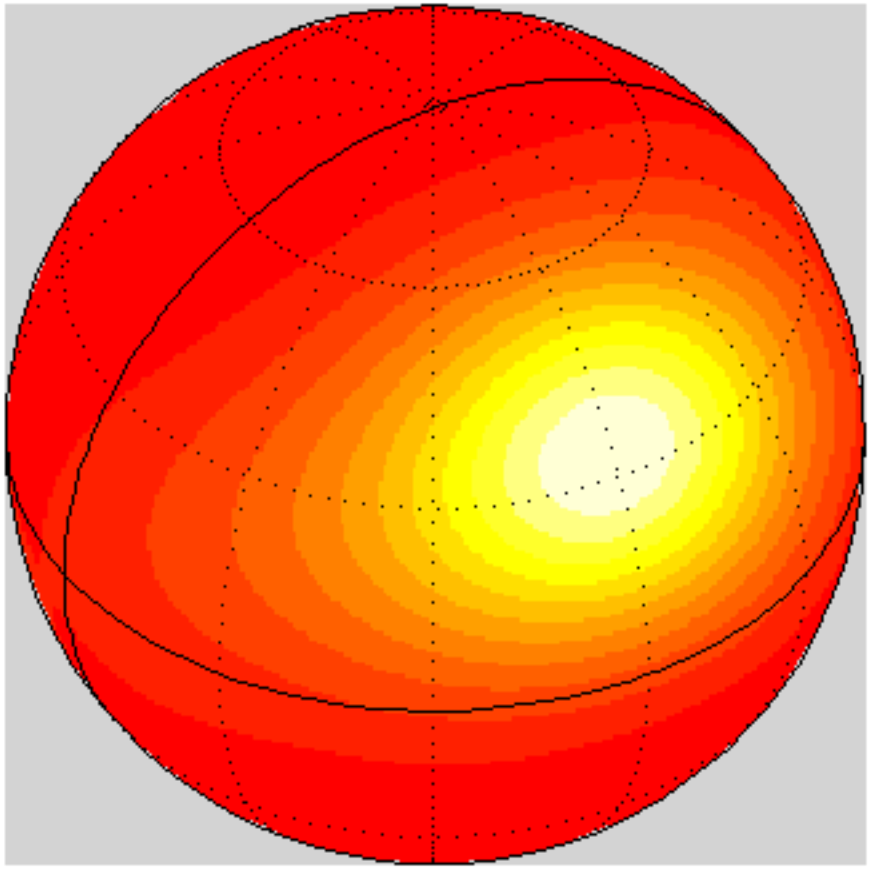
γ = 0.75



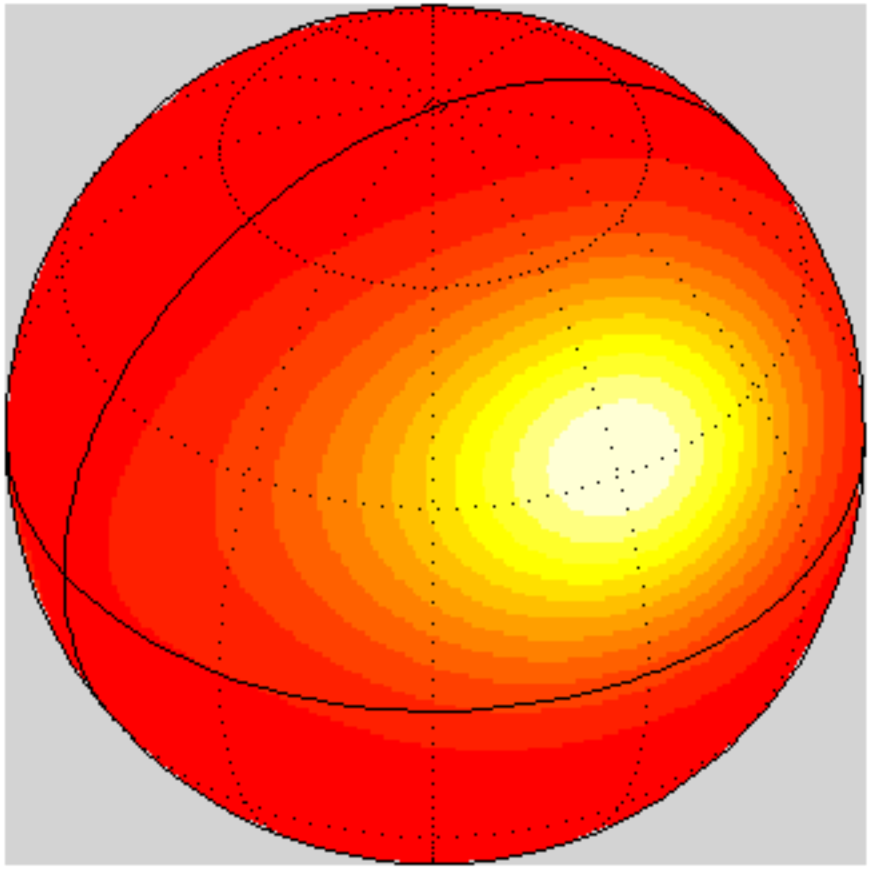
γ = 1.0



γ = 1.25



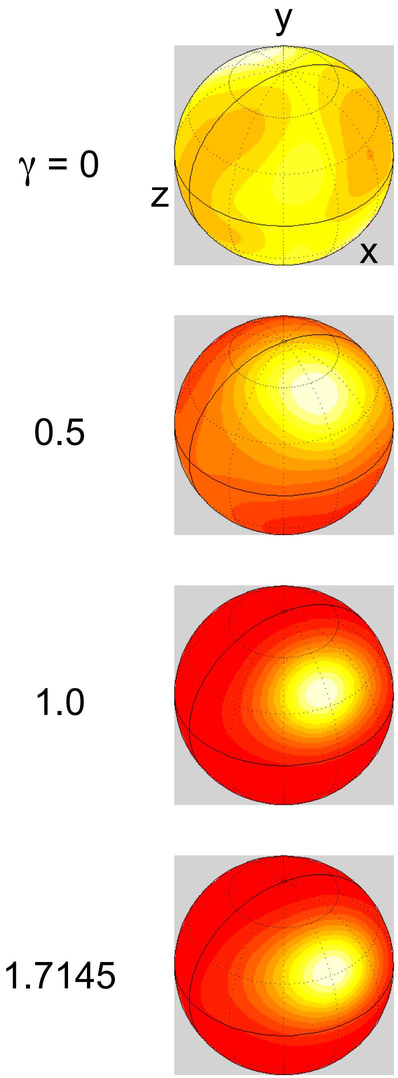
γ = 1.5



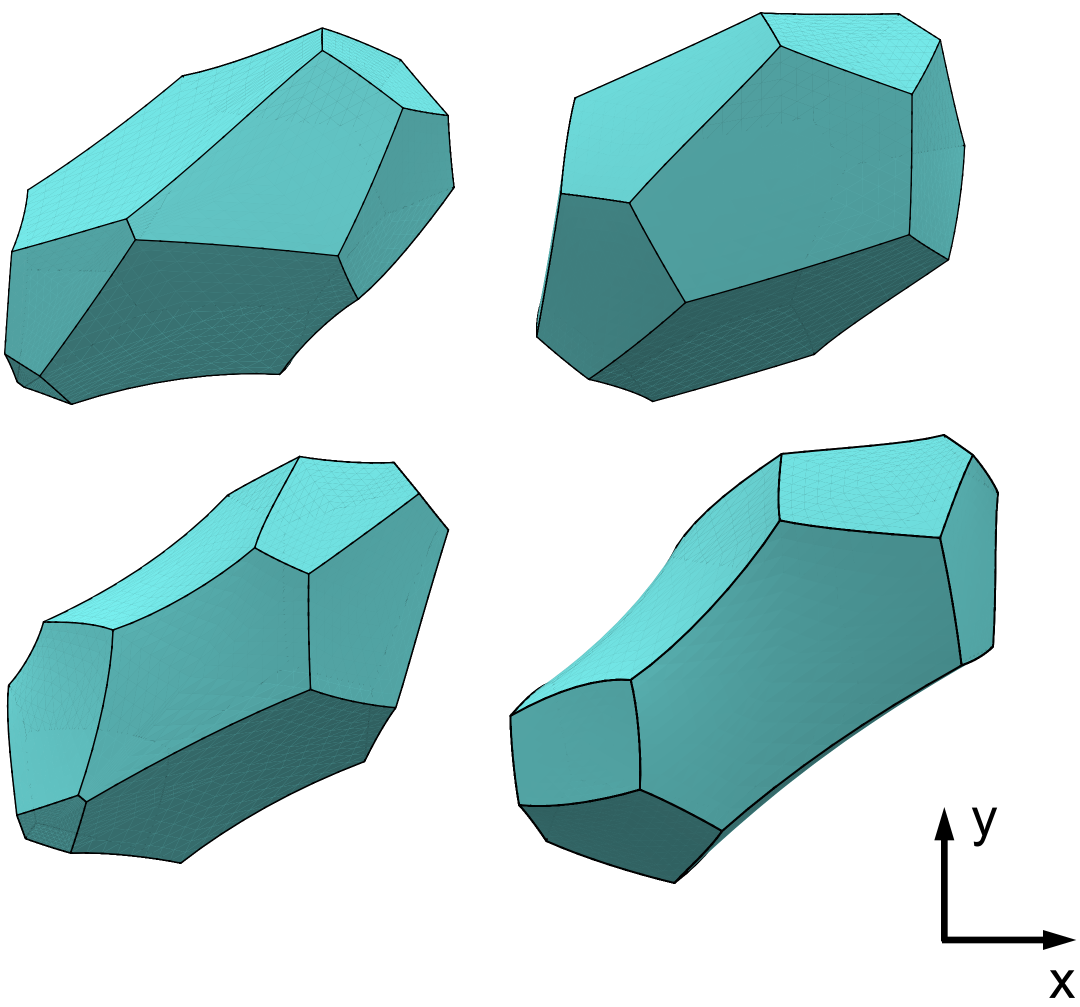
$\gamma = 1.745$

Eigenvectors of q_{ij} – orientation distribution

largest λ



Cells are stretched and aligned at large shear strains (γ > 1)



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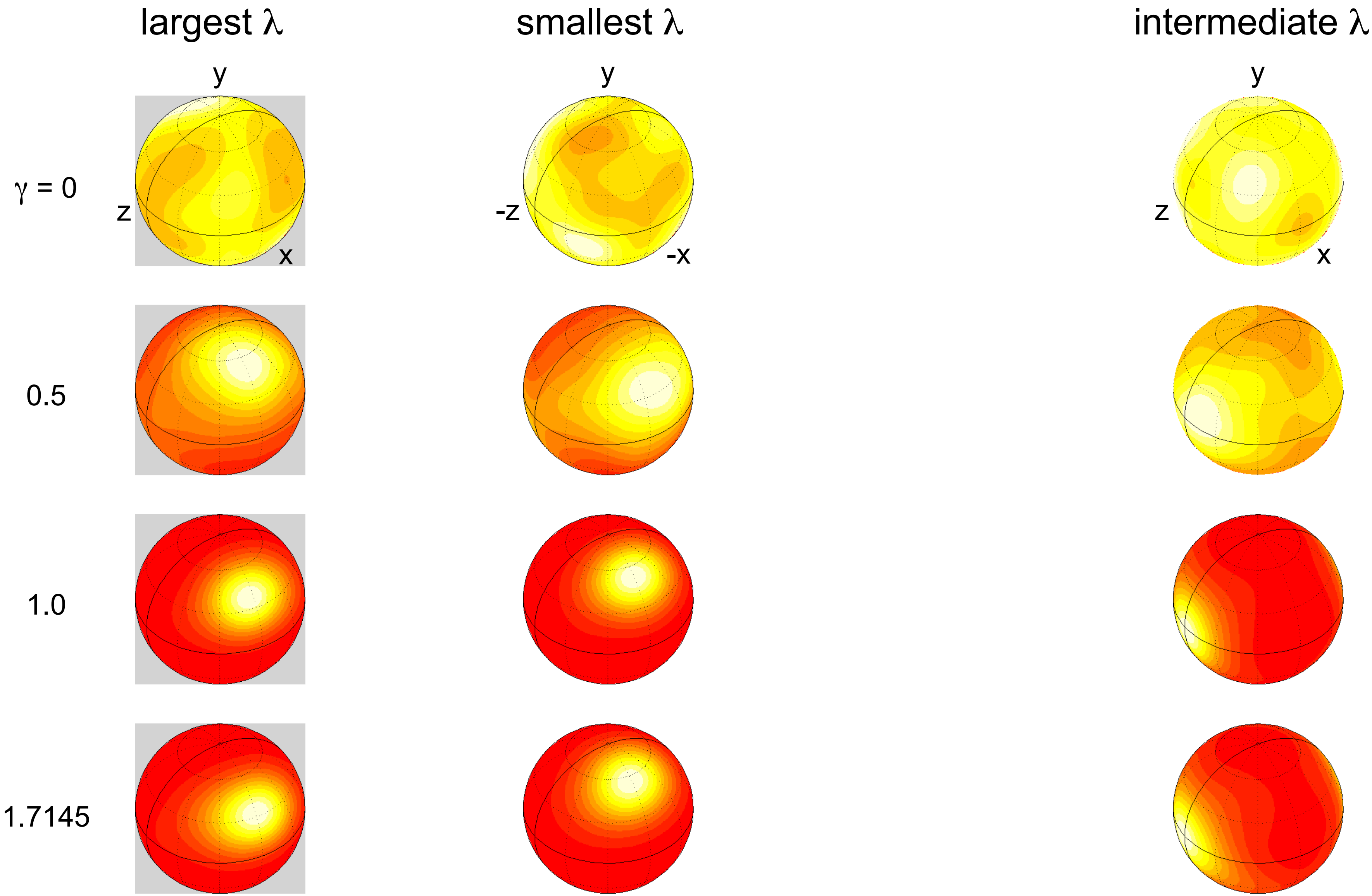
Eigenvectors of q_{ij} – orientation distribution



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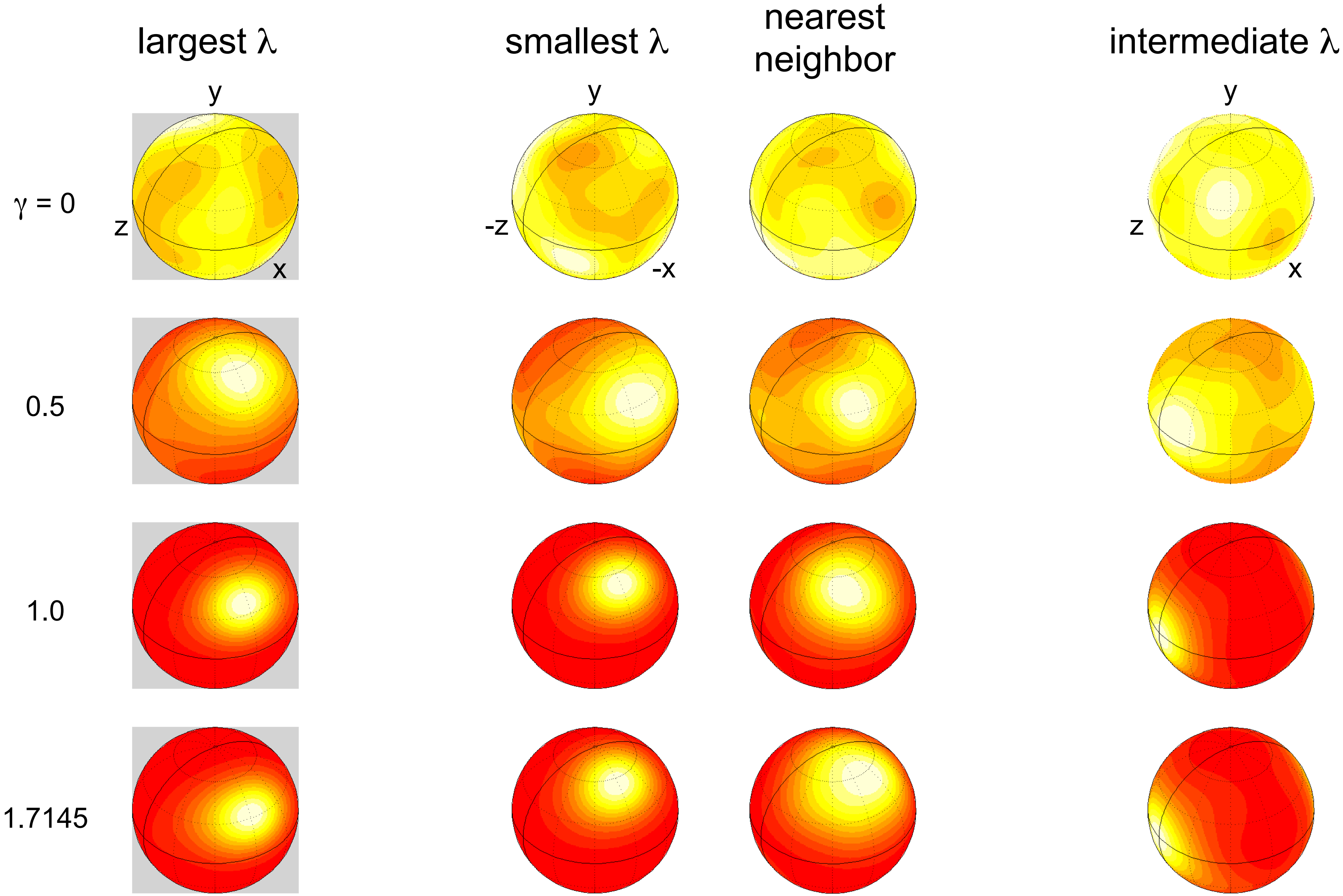
Eigenvectors of q_{ij} – orientation distribution



Claudia Redenbach

Michael Klatt