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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Malgorzata Marciniak **Email/Phone:** mmarciniak@lagcc.cuny.edu 5734620411

Speaker's Name: Eric Babson

Talk Title: Random Knots, Random Groups and DNA

Date: 10 / 04 /2018 **Time:** 11 :00 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences:
Connections between random topology and circular molecules such as plasmids of DNA can be made through the knot which the molecule forms in space. The second connection is through the space of possible chemical species. The process is a random walk on representatives of a fixed homotopy class in the fundamental group of the linear sequence tiling space with the short plasmid 2-cells.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Speaker: Eric Babson

Title: Random knoots, random groups and DNA

Note Taker: Małgorzata Marciniak

Plan: Suggest a relationship between homology of maps into tiling spaces and the dynamics of local reaction system

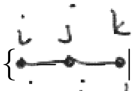
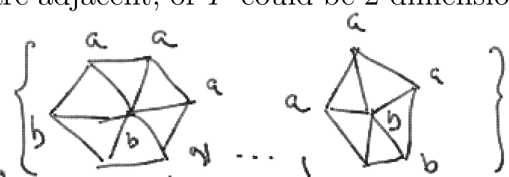
1. Toy examples fitting very well with the homotopy side (they are called plasmid examples).
2. Properties of the collections of systems that are related to homotopies
3. Theorems about the spaces arising here (random petro complexes) on the topology side and suggest phase transition
4. Suggest examples where one can try to use this approach to learn about systems that they care about.

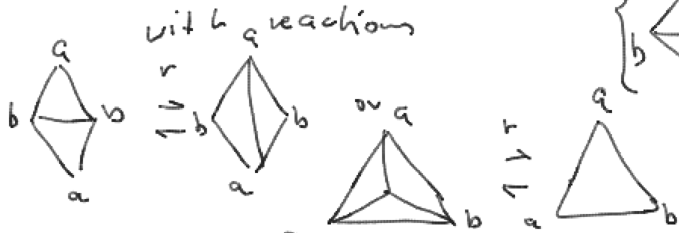
Basic Setup

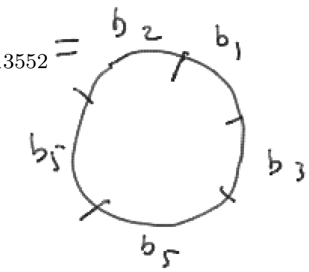
- P is a finite collection of allowed local structures (patches allowed in the system) e.g., triangulated discs with labeling of the vertices
- A is an infinite collection of assemblies which locally from P, e.g., triangulated surfaces covered by discs from P
- R a finite collection of local reactions e.g., $r \in R$ is associated to a pair of subdisks in disks in P with identical boundaries.

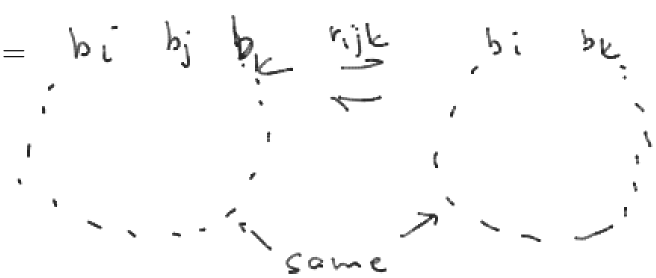
More carefully: patches ρ in P have a marked interior vertex p and every vertex v in some assembly $\alpha \in A$ has a neighborhood $N \subseteq \alpha$ with $(v, \alpha) \cong (p, \rho)$ (isomorphic).

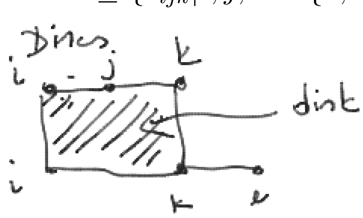
Plasmid toy example

- $P = \{$$| i, j, k \in \{1, \ldots n\}\}$ where each b_i is a linear short DNA strand  Variations (more general class of examples): P could have restrictions which pairs are adjacent, or P could be 2-dimensional triangulated disc such as $P =$ 



- $A = \{$all circular DNA strands consisting of $\{b_1, \ldots, b_n\}\} \ni \alpha_{13552} =$ 

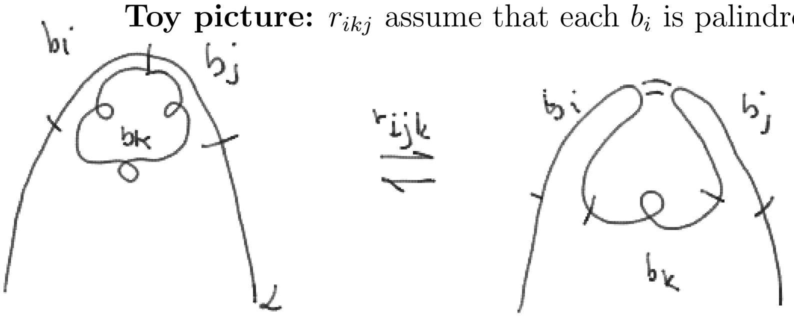
- $R \subseteq \{r_{ijk} | i, j, k \in \{1, \ldots, n\}\}$, where $\alpha =$ 

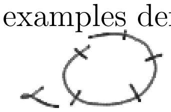


For convenience add some assumptions about R in the toy plasmid example (so it fits exactly with homotopy). $R = R_e \cup R_t$, where $R_e =$

$\{r_{iij}\} \cup \{r_{iji}\} \cup \ldots$ (with two indices equal) and $R_t \subseteq \{r_{ijk} | i \neq j \neq k\}$ (any subset closed with respect to permutations of indices i, j, k).

Toy picture: r_{ikj} assume that each b_i is palindromic

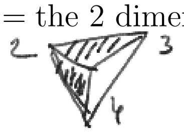


Notation: For the toy plasmid examples define $l(\alpha) = \#$ of pieces in α, $\alpha \in A$ (for example $l(\alpha_{12553}) = 5$) 

$\alpha \rightleftharpoons \tilde{\alpha}$ if there is a sequence of basic reactions in R changing α to $\tilde{\alpha}$. If $\alpha \rightleftharpoons \tilde{\alpha}$ then

$$d_R(\alpha, \tilde{\alpha}) = \text{minimum number of steps from } R \text{ changing } \alpha \text{ to } \tilde{\alpha}$$

If $\alpha \not\rightleftharpoons \tilde{\alpha}$ then $d_R(\alpha, \tilde{\alpha}) = \infty$.

If R is a toy plasmid example then $X_R =$ the 2 dimensional simplicial complex with trianglesR_t and all edges e.g., $R_t = \{r_{123} r_{124}\}$, $X_R =$ 

Definition 1 *A plasmid system R is λ-fast if for every $\alpha, \tilde{\alpha} \in A$ with $\alpha \overset{r}{\rightleftharpoons} \tilde{\alpha}$ there is*

$$d_R(\alpha, \tilde{\alpha}) \leq \lambda(l(\lambda) + l(\tilde{\alpha}))$$

For convenience assume that reaction rates preferring faster to shorten

Idea: Geometrically X_R is Gromov hyperbolic. In the toy example the system reaches the equilibrium quickly.

Definition 2 *A plasmid system R is dissolving if every pair $\alpha, \tilde{\alpha} \in A$ have $\alpha \rightleftharpoons \tilde{\alpha}$*

Idea: Geometrically $\Pi_1 X_r = \{1\}$. In the toy example only short plasmids remain

Definition 3 *A plasmid system R is l-stable if*

$$Prob_{\alpha\in A, l(\alpha)=l}\left(l = min_{\alpha=\tilde{\alpha}} l(\tilde{\alpha})\right) > \frac{1}{2}$$

Idea: Most long plasmids stay long

If $P \subseteq \{1, \dots n\}$ write $R_t[P] = \{r_{ijk} \in R_t | i, j, k \in P\}$

Theorems by Khale, Heffman (deterministic):

a) If for every $P \subseteq \{1, \dots n\}$ have $R_t[P] < 2|P|$ then R is fast (counterexamples exists for $R_t[P] = 2|P|$)

b) There is a function $M(e)$ such that $M \to \infty$ as $e \to 2$ (very hard to identify). If for some $e < 2$ and every $P \subseteq \{1, \dots n\}$ with $|P| \leq M(e)$ have $R_t[P] < e|P|$ then R is fast.

Note: There are examples (tori) with $n = k^2$ and $|R_t| = 2k^2$ but R is slow and is not dissolving or t-stable. This situation is rare.

Theorems (random):

a) If $\alpha > 2.5$ then

$$\lim_{n\to\infty} Prob_{|R_t|=n^\alpha}\left(R \text{ is fast and dissolving}\right) = 1$$

b) If $\alpha < 2.5$ then

$$\lim_{n\to\infty} Prob_{|R_t|=n^\alpha}\left(R \text{ is fast and } l\text{-stable}\right) = 1$$