ANGELA TABIRI

Algebraic Structures in Group Theoretical Fusion Categories

Ostrik and Natale showed that ~~a~~ collection of twisted group algebras in a pointed fusion ~~category~~ form Morita equivalence class representatives of indecomposable separable algebras in this category. We give a generalisation of this result in the context of group theoretical fusion categories. We achieve this by constructing a Frobenius monoidal functor Φ. The "twisted Hecke algebra" which form Morita equivalence class representatives, is the image of the twisted group algebra $A(L,\psi)$ under Φ

Main Theorem: A collection of "twisted Hecke algebras" $A^{\xi,\beta}(L,\psi)$ serve as Morita equivalence class representatives of indecomposable separable algebras in the group theoretical fusion category $C(G,\omega,K,\beta)$

Joint with JP-GM Notation

Vec_G^ω — G-finite group, ω — 3-cocycle controlling associativity, category of G-graded vector spaces

$A(L,\psi)$: let $L \leq G$ with $\omega|_{L^{\times 3}}$ trivial, $\psi \in C^2(L,k^\times)$ a 2-cochain so that $d\psi = \omega|_{L^{\times 3}}$. The twisted group algebra $A(L,\psi)$ in Vec_G^ω is $\bigoplus_{g \in L} \delta_g$ with

$$\delta_g \otimes \delta_{g'} \longmapsto \psi(g,g')\,\delta_{gg'}$$

$C(G,\omega,K,\beta)$: A group theoretical fusion category is a category of bimodules of the form

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"Algebraic structures in comodule categories over weak bialgebras"

	Symmetries
classical	group acting on commutative algebra
Quantum	Hopf algebra (or bialgebra) acting on connected graded algebra
Weak?	weak Hopf algebra (or weak bialgebra) coacting on non-connected graded algebra

<u>Example.</u> Q = quiver, let $\mathbb{k}$ = field. There exists a weak bialgebra $\mathfrak{h}(Q)$ (Hayashi's face algebra associated to the quiver Q) that coacts on $\mathbb{k}Q$ in a very "nice" way.

What does "nice" mean?

Let's think about coactions.

If H is a weak bialgebra, $\mathcal{M}^H$ (= category of right comodules over H) is a monoidal category, but $\otimes \neq \otimes_{\Bbbk}$, $\mathbb{1} \neq \Bbbk$.

<u>Notion of symmetry:</u>

Categorical: algebras / coalgebras / Frobenius algebras in $\mathcal{M}^H$

Formulaic: H-comodule algebras / coalgebras / Frobenius algs. (in $Vect_{\Bbbk}$)

If H is a Hopf or bi-algebra, these notions are the same. If H is a weak bialgebra, they are <u>not</u> the same, but can get a categorical correspondence.

first proved by [Brzeziński-Caenepeel-Militaru]

<u>Theorem [Walton-W-Won].</u> Let H be a weak bialgebra. We have categorical isomorphisms:

$\boxed{Alg(\mathcal{M}^H) \cong \mathcal{A}^H}$ (category of H-comodule algebras)

$Coalg(\mathcal{M}^H) \cong \mathcal{C}^H$ (H-comodule coalgs)

$FrobAlg(\mathcal{M}^H) \cong \mathcal{F}^H$ (H-comodule Frob. algs.)

Example. $\Bbbk Q$ is a right H-comodule algebra and coalgebra. (Frobenius if Q has no arrows) "$h(Q)$

~~By~~ Theorem [Hayashi]. Every tensor category is equivalent to the category of finite-dim. comodules over a certain weak bialgebra.

Consequence. ~~Corollary [Walton-W-Won]~~. By the theorem, we have an explicit description of Frobenius algebras in any tensor category, using Frobenius algebras in $\mathrm{Vect}_{\Bbbk}$.

Given Hopf algebra L and strongly separable left L-module algebra B, there exists a weak Hopf algebra $H(L,B)$. (see Nikshych-Vainerman survey - "Finite Quantum Groupoids and their applications" - called a ~~the~~ quantum transformation groupoid)

Theorem [Walton-W-Won]. $\exists$ monoidal functor

$$L\text{-Bicomod} \longrightarrow \mathcal{M}^{H(L,B)}.$$

Corollary. ~~We have a func~~ Descends to a functor

$$\mathrm{Alg}(L\text{-Bicomod}) \to \mathrm{Alg}(\mathcal{M}^{H}).$$

This creates weak symmetries from quantum symmetries.