STOCHASTIC ANALYSIS ON INFINITE-DIMENSIONAL CURVED SPACES

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INFINITE-DIMENSIONAL LINEAR SPACE

W	a real separable Banach space
W^*	the continuous dual of W
$\mathcal{B}(W)$	the Borel σ-algebra on W

For a probability measure μ on $\mathcal{B}(W)$, its Fourier transform $\hat{\mu} : W^* \to \mathbb{C}$

$$\hat{\mu}(\alpha) := \int_W e^{i\alpha(x)} d\mu(x)$$

The measure μ is Gaussian if $W^* \subset L^2(\mu)$ and

$$\hat{\mu}(\alpha) = e^{-\frac{q_\mu(\alpha)}{2}} \text{ for any } \alpha \in W^*$$

$$q_\mu(\alpha, \beta) := \int_W \alpha(x)\, \beta(x)\, d\mu(x) \text{ for all } \alpha, \beta \in W^*$$

Fact. This definition is equivalent to saying that any $\alpha \in W^*$ is a Gaussian random variable relative to μ. The quadratic form q is continuous as a map $W^* \times W^* \to \mathbb{R}$.

Fernique's Theorem.

$$C_p := \int_W \|x\|_W^p d\mu(x) < \infty, \text{ for all } 1 \leqslant p < \infty.$$

$H = H_\mu$ the Cameron-Martin subspace

$$\left\{ h \in W : \|h\|_H := \sup_{\alpha \in W^*} \frac{|\alpha(x)|}{\sqrt{q(\alpha,\alpha)}} < \infty \right\}$$

Properties of (W, H, μ)

- $(H, \|\cdot\|_H)$ is a normed space such that

$$\|h\|_W \leqslant \sqrt{C_2}\|h\|_H \text{ for all } h \in H$$

- H is a separable Hilbert space
- For any orthonormal basis $\{h_k\}_{k=1}^{\infty}$ for H

$$q(\alpha, \beta) = \sum_{n=1}^{\infty} \alpha(h_k)\,\beta(h_k)$$

- If q is non-degenerate, then the Cameron-Martin space H is dense in W
- If $\dim W = \infty$, then $\mu(H) = 0$

- (Cameron-Martin-Girsanov Theorem) Let $\mu_h(A) := \mu(A-h)$ for all $h \in W$, and $A \in \mathcal{B}(W)$. Then $\mu_h \ll \mu$ iff $h \in H$, and then

$$\frac{d\mu_h}{d\mu} = \exp\left((h,\cdot)_H - \frac{\|h\|_H^2}{2}\right)$$

- (Integration by parts formula) For any $h \in H$, $f \in \mathfrak{F}C_c^\infty(W^*)$ such that f and $\partial_h f$ do not grow too fast at infinity. Then

$$\int_W \partial_h f d\mu = \int_W \exp\left((h,\cdot)_H - \frac{\|h\|_H^2}{2}\right) f d\mu$$

$\mathfrak{F}C_c^\infty(W^*)$	cylinder functions $f = F(\alpha_1, ..., \alpha_n) : W \to \mathbb{R}$, $F \in C_c^\infty(\mathbb{R}^n)$
$\partial_h f$	directional derivative

$$\partial_h f(w) := \left.\frac{d}{dt}\right|_{t=0} f(w+th)$$

Irving Segal's Work on Infinite Dimensional Integration Theory

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Irving Segal's work on infinite dimensional integration theory was driven by the mathematical needs of quantum field theory.

Irving's point of view on integration over an infinite dimensional real Hilbert space is easy to understand. Define Gauss measure on $\mathbb{R}^n$ by $\gamma_n = (2\pi)^{-n/2} e^{-|x|^2/2}\, dx$. Suppose that F is an n dimensional subspace of a real infinite dimensional Hilbert space H and that $P\colon H \to F$ is the orthogonal projection. If B is a Borel set in F then the set $C = P^{-1}(B) := \{x \in H;\ Px \in B\}$ is a cylinder in H with base B. Define $\gamma(C) = \gamma_n(B)$. The collection, $\mathscr{R}$, of all such cylinder sets, allowing F to run over all finite dimensional subspaces of H, is a field (but not a σ-field.) The set function γ is well defined on $\mathscr{R}$ because γ_n is a product measure in any orthogonal decomposition of $\mathbb{R}^n$. Unfortunately, although γ is finitely additive on $\mathscr{R}$, it is not countably additive (wherever possible) and consequently it has no countably additive extension to the Borel field of H. In fact the outer γ measure of H is zero. For this reason the theory of integration over an infinite dimensional Hilbert space H deviates significantly from that over $\mathbb{R}^n$. And yet the theory is also more structured than that for general finitely additive measures. Think of the set S of rational numbers in the interval $[0, 1]$ and the set function m which assigns measure $b-a$ to the subset $S \cap (a, b]$. m extends to a finitely additive measure on the field generated by these intervals. But the m outer measure of S is zero and m has no countably additive extension. Of course we know what to do about this: complete S in such a way as to obtain the real unit interval. The same formula used to define m now has a countably additive extension—Lebesgue measure on $[0, 1]$. Similarly there is a "completion" Ω of H and a countably additive probability measure Pr on Ω which extends the finitely additive measure γ in a useful way. The space Ω is not unique, however, and must be chosen in a technically convenient way for each application. Sometimes the choice is a natural one, as for example when H consists of the absolutely continuous functions on $[0, 1]$ which are zero at zero and have square integrable derivative. In this case the completion Ω of H—in the sup norm—consists of the continuous functions on $[0, 1]$ which vanish at zero. Pr is exactly Wiener measure on Wiener space. The important point that Irving emphasized in all of his work on infinite dimensional integration theory is that it is the Hilbert space H that controls both the heuristics and much of the analysis. The measure space Ω, Pr is just a convenient place "to hang your hat on," as he once told me. The orthogonal invariance of γ, along with its quasi-invariance under translations, should be kept front and center for the formulation and proofs of theorems of analysis over Wiener space and similar Gaussian measure spaces. This point of view makes much of the work of Cameron and Martin on analysis over Wiener space quite transparent.

$\{h_k\}_{k=1}^{\infty}$ orthonormal basis for H

$$L_\mu f := \sum_{k=1}^{\infty} \partial_{h_k}^2 f = \sum_{i,j=1}^{n} q(\alpha_i, \alpha_j) (\partial_j \partial_i F)(\alpha_1, ..., \alpha_n).$$

Theorem. (Mehler formula, heat equation) For any $f \in \mathfrak{F}P(W^*)$

$$F(t, x) := \int_W f\left(x + \sqrt{t}y\right) d\mu(y) = \left(e^{\frac{tL}{2}} f\right)(x),$$

If $f \in C^2(W)$ with first and second derivatives growing at most exponentially at infinity, then F satisfies the heat equation

$$\frac{\partial}{\partial t} F(t, x) = \frac{1}{2}(LF)(t, x)$$

Example (Classical Wiener space)

$\boldsymbol{W}$ $\{x \in C([0,T] \to \mathbb{R}) : x(0) = 0\}$

$\boldsymbol{H}$ the Cameron-Martin subspace $\Big\{h \in W : h$ is absolutely continuous, $\|h\|^2 := \int_0^T |h'(s)|^2 ds < \infty\Big\}$

$\boldsymbol{H}$ is a reproducing kernel Hilbert space with the reproducing kernel

$$G(s,t) = \min(s,t).$$

In particular, for any orthonormal basis $\{h_n\}_{n=1}^{\infty}$

$$\min(s,t) = \sum_{n=1}^{\infty} h_n(s) h_n(t)$$

Example (Brownian motion)

$W \quad \{x \in C([0,T] \to \mathbb{R}) : x(0) = 0\}$

B_t a Brownian motion

μ the law of $B.$ as a measure on $(W, \mathcal{B}(W))$.

Then H_μ is the classical Cameron-Martin space from the previous example

MOTIVATION AND APPLICATIONS

- QFT (log Sobolev inequality and hypercontractivity), GFF

- Quasi-invariance as smoothness of measures in infinite dimensions

- Regularity of invariant measures and degenerate semigroups in infinite dimensions

- Ergodicity of degenerate semigroups (Langevin dynamics)

- A unitary representation of a transformation group G on $L^2(X, d\mu)$

$$(\pi(g) f)(x) := \left(\frac{d\mu\left(g^{-1}x\right)}{d\mu(x)} \right)^{1/2} f\left(g^{-1}x\right)$$

for any $f \in L^2(X, d\mu)$ (Path groups, joint with S. Albeverio, B. Driver, A.M. Vershik)

FINITE-DIMENSIONAL MATRIX LIE GROUPS

G	finite-dimensional compact matrix Lie group with the identity I
$L^2(G, dg)$	square–integrable functions on G with respect to a right-invariant Haar measure dg
$\mathfrak{g}$	Lie algebra of G with the inner product $\langle \cdot, \cdot \rangle$
derivative	$\widetilde{\xi} f(g) = \frac{d}{dt}\Big\vert_{t=o} f\left(ge^{t\xi}\right)$, $\xi \in \mathfrak{g}$, $g \in G$, $f \in C^\infty(G)$
$\{\xi_i\}_{i=1}^d$	orthonormal basis of the Lie algebra $\mathfrak{g}$
Δ	Laplacian $\sum\limits_{j=1}^{d} \widetilde{\xi_j}^2 f$
P_t	the heat semigroup $e^{t\Delta}$ on $L^2(G, dg)$

heat kernel p_t is a left convolution kernel on G such that

$$P_t f(g) = f * p_t(g) = \int_G f(gk)\, p_t(k)\, dk, \text{ for all } f \in C^\infty(G)$$

The heat kernel measure $p_t(g)\, dg$ is the distribution of the process g_t satisfying the Stratonovich stochastic differential equation

$$\delta g_t = g_t \delta W_t, \; g_0 = I$$

or Itô's stochastic differential equation

$$dg_t = g_t dW_t + \frac{1}{2} g_t \sum_1^d \xi_i^2 dt, \; g_0 = I$$

W_t the Brownian motion on the Lie algebra $\mathfrak{g}$ with the identity operator as its covariance

$$W_t = \sum_{i=1}^{d} b_i^t \xi_i$$

where b_i^t are real-valued Brownian motions mutually independent on a probability space $(\Omega, \mathfrak{F}, P)$

$G = \mathbf{SU}\,(2)$ $\left\{ g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : g^* g = I, \det g = 1 \right\}$

$\mathfrak{g} = \mathbf{su}\,(2)$ $\left\{ A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} : A^* + A = 0, \operatorname{tr} A = 0 \right\}$

inner product $\langle A, B \rangle = 2 \operatorname{tr} B^* A$

$$\langle U^{-1} A U, U^{-1} B U \rangle = \langle A, B \rangle, \quad A, B \in \mathbf{su}\,(2)\,, \quad U \in \mathbf{SU}\,(2)$$

o.n.b. $\xi_j = \frac{i}{2} \sigma_j, \quad j = 1, 2, 3$

Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For any $\boldsymbol{U} \in \mathrm{SU}(2)$

$$\boldsymbol{U} = \boldsymbol{U}(\varphi, \theta, \psi) = \begin{pmatrix} \cos\frac{\theta}{2}\, e^{i\frac{(\varphi+\psi)}{2}} & i\sin\frac{\theta}{2}\, e^{i\frac{(\varphi-\psi)}{2}} \\ i\sin\frac{\theta}{2}\, e^{-i\frac{(\varphi-\psi)}{2}} & \cos\frac{\theta}{2}\, e^{-i\frac{(\varphi+\psi)}{2}} \end{pmatrix}$$

Euler angles $\quad 0 \leqslant \varphi < 2\pi, \quad 0 \leqslant \theta \leqslant \pi, \quad -2\pi \leqslant \psi < 2\pi$

Theorem. (Laplacian on $\mathrm{SU}(2)$)

$$\begin{aligned} \Delta &= \widetilde{\xi_1}^2 + \widetilde{\xi_2}^2 + \widetilde{\xi_3}^2 \\ &= 2\frac{\partial^2}{\partial\theta^2} + \frac{2}{\sin^2\theta}\,\frac{\partial^2}{\partial\varphi^2} - 4\frac{\cos\theta}{\sin^2\theta}\frac{\partial^2}{\partial\varphi\partial\psi} \\ &\quad + \frac{2}{\sin^2\theta}\frac{\partial^2}{\partial^2\psi} + 2\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta} \end{aligned}$$

Let b_1^t, b_2^t and b_3^t be three real-valued independent Brownian motions. Then the Brownian motion on $\mathbf{SU}(2)$ is a solution to

$$\delta g_t = g_t \left(\delta b_1^t \xi_1 + \delta b_2^t \xi_2 + \delta b_3^t \xi_3 \right), \quad g_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$dg_t = g_t \left(db_1^t \xi_1 + db_2^t \xi_2 + db_3^t \xi_3 \right) - \frac{3}{2} g_t dt, \quad g_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

since

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 3I.$$

Theorem. The process $g_t \in \mathbf{SU}(2)$ with probability 1. Its generator is the Laplacian described above.

STOCHASTIC ANALYSIS ON INFINITE-DIMENSIONAL GROUPS

What happens on infinite-dimensional curved spaces

- What is a heat kernel (Gaussian) measure?

No Haar measure, no heat kernel, only a heat kernel measure

- Lie algebra and Laplacian

Different choices of a norm on a Lie algebra give different Lie algebras

$\Updownarrow$

the Lie algebra determines directions of differentiation

$\Updownarrow$

the Lie algebra and a norm on it determines a Laplacian and the Wiener process

- Cameron-Martin subspace

- metric and differential geometry

RESULTS FOR SOME INFINITE-DIMENSIONAL CURVED SPACES

Loop groups: heat kernel measures, the Cameron-Martin subgroup, **Ricci** is bounded from below, quasi-invariance of the heat kernel measures (S. Aida, B. Driver, S. Fang, P. Malliavin, I. Shigekawa ...)

Path spaces and groups: heat kernel measures, the Cameron-Martin subspace, Taylor map (S. Aida, M. Cecil, B. Driver, S. Fang, E. Hsu, O. Enchev–D. Stroock)

$\mathbf{Diff}(S^1)$**,** $\mathbf{Diff}(S^1)/S^1$**:** heat kernel measures, the Cameron-Martin subgroup, **Ricci** is bounded from below, quasi-invariance of the heat kernel measures (H. Airault, M. Bowick, A. A. Kirillov, G., P. Lescot, P. Malliavin, S.Rajeev, A. Thalmaier, M. Wu, D. V. Yur'ev, B Zumino...)

Hilbert-Schmidt groups: heat kernel measures, the Taylor map, a Cameron-Martin subgroup, $\mathbf{Ricci} = -\infty$, connection between $\mathbf{Diff}(S^1)/S^1$ and $\mathbf{Sp}_\infty$ (G. in PA '00, JFA '00, '05)

Infinite-dimensional nilpotent groups: heat kernel measures, quasi-invariance of the heat kernel measures, the Taylor map, holomorphic skeletons, Cameron-Martin subgroup, $\mathbf{Ricci}$ is bounded from below, log Sobolev inequality (Driver, G. *JFA'08*, *PTRF'10*, *JDG'09*, T. Melcher *JFA'09*)

Infinite-dimensional Heisenberg group: sub-Riemannian/hypoellipitc, $\mathbf{Ricci} = -\infty$ (Baudoin, G., Melcher, also Driver, Eldredge, Melcher)

HILBERT-SCHMIDT GROUPS: INFINITE MATRIX GROUPS

$B(H)$	bounded linear operators on a complex Hilbert space H
$G = GL(H)$	invertible elements of $B(H)$
Q	a bounded linear symmetric nonnegative operator on HS
HS	Hilbert-Schmidt operators on H with the inner product $(A, B)_{HS} = TrB^*A$
$\mathfrak{g} = \mathfrak{g}_{CM} \subseteq HS$	an infinite-dimensional Lie algebra with a Hermitian inner product $(\cdot, \cdot)$, $\lvert A\rvert_{\mathfrak{g}} = \lvert Q^{-1/2}A\rvert_{HS}$

$G_{CM} \subseteq GL(H)$ Cameron-Martin group $\{x \in GL(H), d(x, I) < \infty\}$

$d(x, y)$ the Riemannian distance induced by $|\cdot|$

$$d(x, y) = \inf_{\substack{g(0)=x \\ g(1)=y}} \int_0^1 |g(s)^{-1} g'(s)|_{\mathfrak{g}} ds$$

HS as infinite matrices

HS= matrices $\{a_{ij}\}$ such that $\sum_{i,j} |a_{ij}|^2 < \infty$

$$e_{ij} = i \begin{pmatrix} & & & j & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}, \quad Qe_{ij} = \lambda_{ij} e_{ij}, \ \lambda_{ij} \geqslant 0$$

$$\xi_{ij} = \sqrt{\lambda_{ij}} e_{ij}$$

Q is a trace class operator $\iff \sum_{i,j} \lambda_{ij} < \infty$

For example, $\lambda_{ij} = r^{i+j}$, $0 < r < 1$

(i) The Hilbert-Schmidt general group

$$GL_{HS} = GL(H) \bigcap (I + HS)$$

Lie algebra $\mathfrak{gl}_{HS} = HS$, $\mathfrak{g}_{\mathrm{CM}} = Q^{1/2} HS$

(ii) The Hilbert-Schmidt orthogonal group SO_{HS} is the connected component of

$$\{B : B - I \in HS, \;\; B^T B = B B^T = I\}$$

Lie algebra $\mathfrak{so}_{HS} = \{A : A \in HS, \;\; A^T = -A\}$

$$\mathfrak{g}_{\mathrm{CM}} = Q^{1/2} \mathfrak{so}_{HS}$$

(iii) The Hilbert-Schmidt symplectic group

$Sp_{HS} = \{X : X - I \in HS, \;\; X^T J X = J\}$, where

$$J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

Lie algebra $\mathfrak{sp}_{HS} = \{X : X \in HS, \;\; X^T J + J X = 0\}$

$\mathfrak{g}_{\mathrm{CM}} = Q^{1/2} \mathfrak{sp}_{HS}$.

STOCHASTIC DIFFERENTIAL EQUATIONS ON HS, HEAT KERNEL MEASURES (G., JFA 2000)

W_t a Brownian motion in HS with the covariance operator Q

$$W_t = \sum_{i=1}^{\infty} W_t^i \xi_i$$

W_t^i one-dimensional independent real Brownian motions

$\{\xi_j\}_{j=1}^{\infty}$ an orthonormal basis of $\mathfrak{g}$ as a real space

$$T = \frac{1}{2} \sum_{j=1}^{\infty} \xi_j^2$$

Theorem. (G.)

Suppose that Q is a trace-class operator. Then

- [SDEs] the stochastic differential equations

$$dG_t = TG_t dt + dW_t G_t,\ G_0 = X$$

$$dZ_t = Z_t T dt - Z_t dW_t,\ Z_0 = Y$$

have unique solutions in HS

- [Inverse]. The solutions of these SDEs with $G_0 = Z_0 = I$ satisfy

$$Z_t G_t = I \text{ with probability } 1 \text{ for any } t > 0$$

• [Kolmogorov's backward equation] The function $v(t,X)=P_t\varphi(X)$ is a unique solution to the parabolic type equation

$$\frac{\partial}{\partial t}v(t,X)=\frac{1}{2}\sum_{j=1}^{\infty}D^2v(t,X)(\xi_jX\otimes\xi_jX)+(TX,Dv(t,X))_{HS}$$

Kolmogorov's backward equation=the group heat equation

Definition. **The heat kernel measure** μ_t on HS is the transition probability of the stochastic process G_t, that is, $\mu_t(A)=P(G_t\in A)$

Open question: quasi-invariance of the heat kernel measures

CAMERON-MARTIN GROUP AND EXPONENTIAL MAP

Definition. The Cameron-Martin group is

$G_{CM}=\{x \in GL(H),\ d(x,I) < \infty\}$, where d is the Riemannian distance induced by $|\cdot|$:

$$d(x,y)=\inf_{\substack{g(0)=x \\ g(1)=y}} \int_0^1 |g(s)^{-1}g'(s)|_{\mathfrak{g}} ds$$

Finite dimensional approximations:

$\mathfrak{g}_n$ ascending finite dimensional Lie subalgerbas of $\mathfrak{g}$,

G_n Lie groups with Lie algebras

Assumption: all $\mathfrak{g}_n$ are invariant subspaces of Q.

Theorem. (G.) If $|[X, Y]| \leqslant c|X||Y|$ then

1. $\mathfrak{g} = \overline{\bigcup \mathfrak{g}_n}$, and the exponential map is a local diffeomorphism from $\mathfrak{g}$ to G_{CM}.

2. $\bigcup G_n$ is dense in G_{CM} in the Riemannian distance induced by $|\cdot|$.

INFINITE-DIMENSIONAL HEISENBERG GROUPS

joint with Bruce Driver: *JFA '08, PTRF '10, JDG '09*

Tai Melcher (higher step nilpotent groups): *JFA '09*

(W, H_{CM}, μ) an abstract Wiener space

$\omega : W \times W \to \mathbb{C}$ a skew-symmetric continuous bilinear form on W

$G = G(\omega) = W \times \mathbb{C}$ with the group multiplication

$$(w, c) \cdot (w', c') = \left(w + w', c + c' + \frac{1}{2}\omega(w, w') \right)$$

$G_{CM} = H_{CM} \times \mathbb{C}$ the Cameron-Martin subgroup of G

$\boldsymbol{B(t)}$ a $\boldsymbol{W}$–valued Brownian motion with the variance determined by $\boldsymbol{H_{CM}}$

$\boldsymbol{B_0(t)}$ an independent $\mathbb{C}$-valued Brownian motion

$$\boldsymbol{g(t)} = \left(\boldsymbol{B(t), B_0(t) + \frac{1}{2}\int_0^t \omega\left(B(s), \delta B(s)\right)} \right)$$
$$= \left(\boldsymbol{B(t), B_0(t) + \frac{1}{2}\int_0^t \omega\left(B(s), dB(s)\right)} \right)$$

MAIN RESULTS

- Quasi-invariance: the path space measure and the heat kernel measure $\Rightarrow$ integration by parts formulae

- Ricci curvature: bounded from below

- log Sobolev inequality

- L^p–bounds of the Radon-Nikodym derivatives

- Taylor isomorphism, Itô-Wiener expansion

SUBELLIPTIC HEAT KERNEL MEASURES ON AN INFINITE-DIMENSIONAL HEISENBERG GROUP

joint with Fabrice Baudoin and Tai Melcher, *JFA '13*,
B. Driver, N. Eldredge, T. Melcher, *TAMS '16*

lower Ricci curvature bounds for the subelliptic Laplacian are not available. But still we can prove

- Reverse Poincaré inequality
- Reverse log Sobolev inequality
- Integrated Harnack inequality
- quasi-invariance of the subelliptic heat kernel measure

F. Baudoin, Q. Feng, G.: quasi-invariance on path space of a sub-Riemannian manifold, *JFA '19*

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