APPROXIMATION THEORY IN TRANSCENDENTAL DYNAMICS

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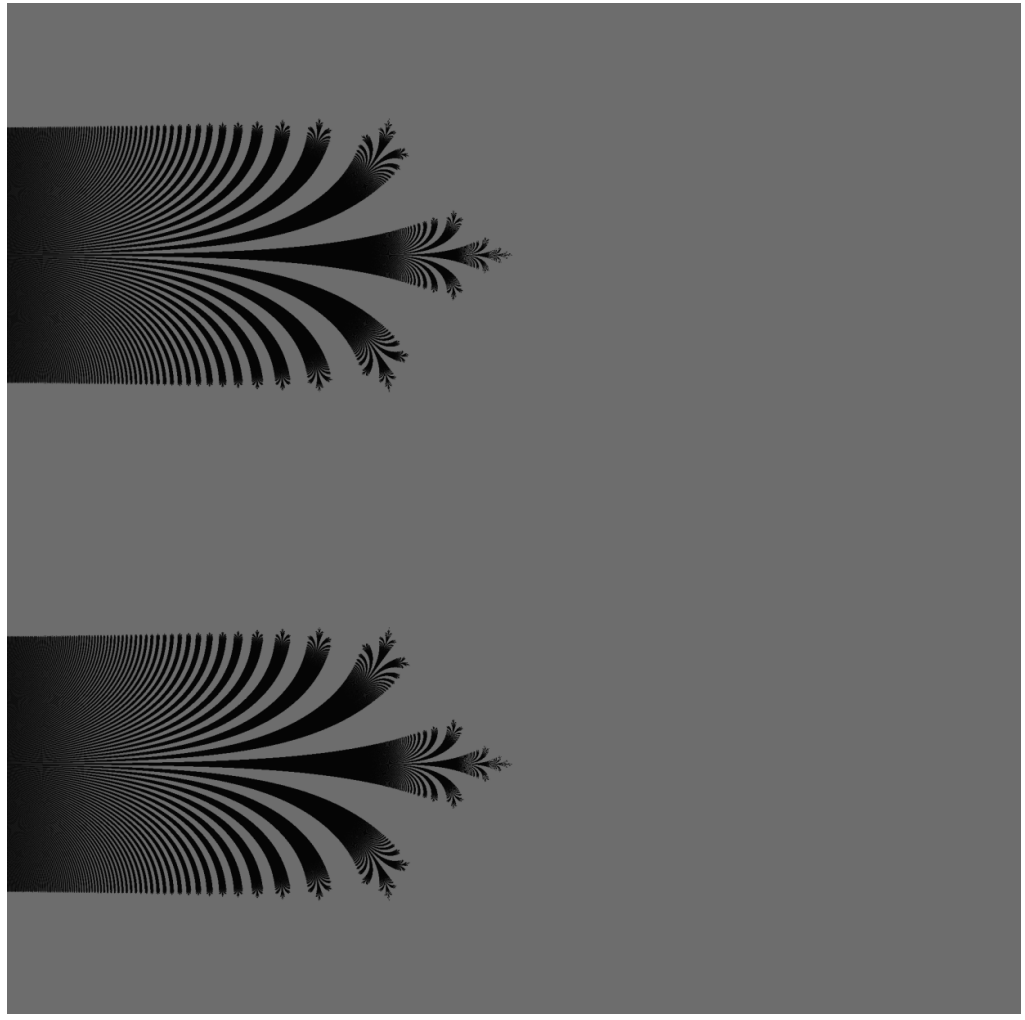
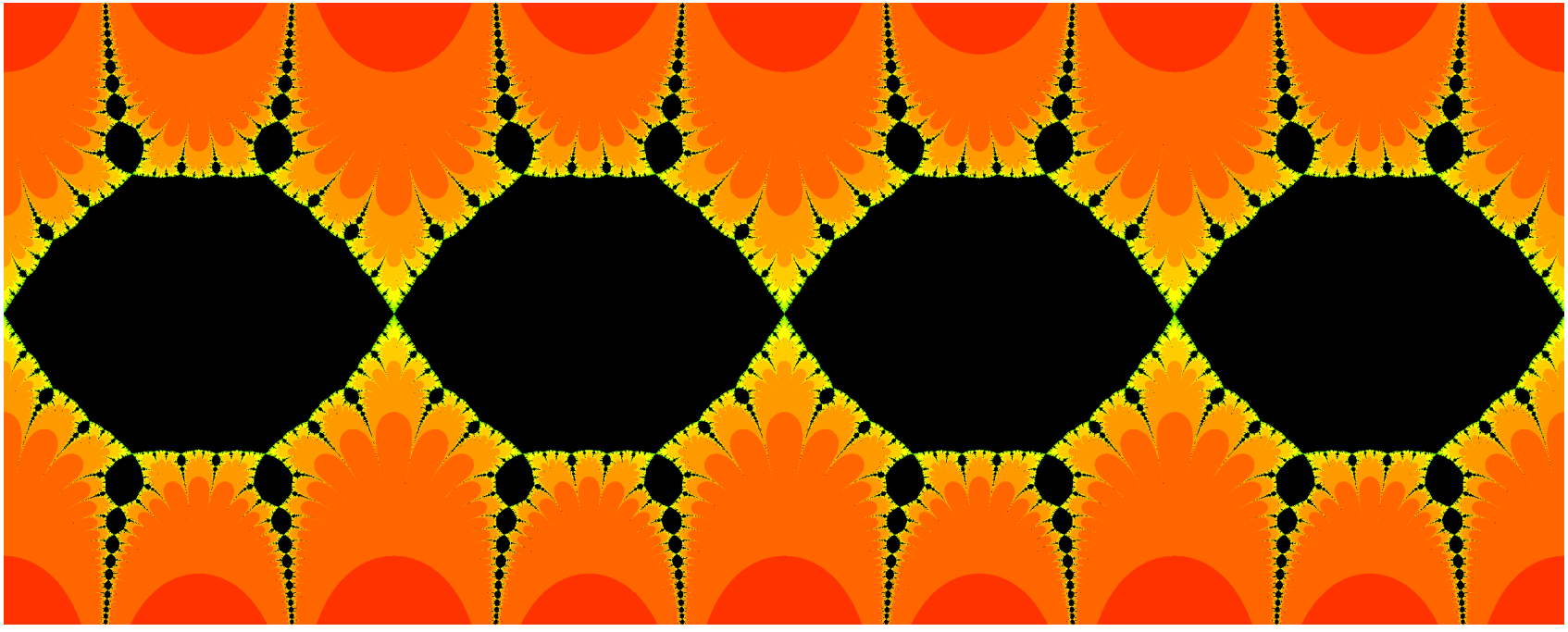
1. Introduction to approximation theory (David)
2. Early applications to transcendental dynamics (Vasiliki)
3. Constructing wandering domains with specific internal dynamics (Vasiliki)
4. Constructing wandering sets of entire functions (David)

Baker domain



$$f(z) = z + 1 + e^{-z}$$

Wandering domain



$$f(z) = z + \sin z + 2\pi$$

1. Infinite products

$$f(z) = cz^2 \prod_{n=1}^{\infty} \left(1 + \frac{z}{a_n}\right)$$

- **Baker** (1976): transcendental entire function with a wandering domain
- **Sixsmith** (2012): transcendental entire function with a fast escaping simply connected wandering domain and no multiply connected wandering domains
- **Bishop** (2018): transcendental entire function with a Julia set of Hausdorff dimension 1

2. Approximation theory

- **Eremenko and Lyubich** (1987): transcendental entire functions with
  - (i) an oscillating wandering domain
  - (ii) a univalent Baker domain

- **Rempe, Rippon and Stallard** (2010): transcendental entire function with a Devaney hair that is not fast escaping

- **Boc Thaler** (2021): every bounded regular simply connected domain with a connected complement is a wandering domain of some transcendental entire function

3. Quasiconformal surgery

$$f = \phi \circ g \circ \phi^{-1} \text{ (dynamical)} \quad \text{or} \quad f = g \circ \phi^{-1} \text{ (non-dynamical)}$$

- **Fagella and Peter** (2012): every configuratiton of Herman rings of a rational function occurs for a transcendental *meromorphic* function
- **Bishop** (2015): transcendental entire function with a bounded set of singular values and a wandering domain (using quasiconformal folding)
- **Martí-Pete and Shishikura** (2020): transcendental entire function of order 1/2 with a bounded set of singular values and a wandering domain

4. Cauchy integrals

$$h(z) = \frac{1}{2\pi i} \int_\gamma \frac{g(\zeta)}{\zeta - z}\, d\zeta$$

- **Stallard** (1991): transcendental entire functions with a Julia set of Hausdorff dimension arbitrarily close to 1
- **Rottenfußer, Rückert, Rempe and Schleicher** (2011): transcendental entire function such that every path-connected component of the Julia set is a singleton (a counterexample to the strong Eremenko conjecture)
- **Rempe** (2014): a hyperbolic transcendental entire function of finite order such that its hyperbolic dimension equals 2