THE DENJOY-WOLFF THEOREM: FROM $\mathbb{D}$ TO WANDERING DOMAINS

INTRODUCTORY WORKSHOP MSRI

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CRM CENTRE DE RECERCA MATEMÀTICA

MSRI Mathematical Sciences Research Institute

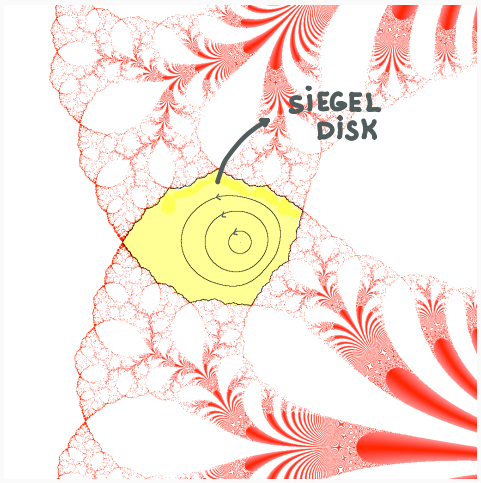
PERIODIC FATOU COMPONENTS

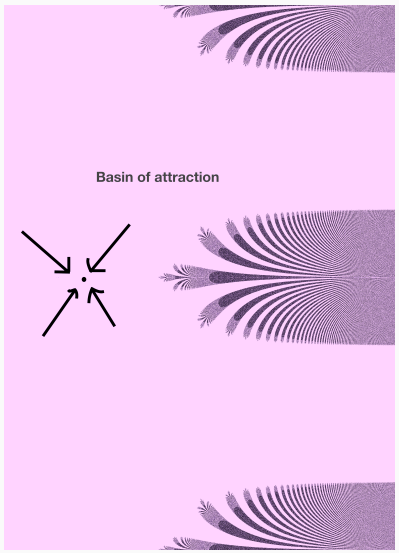
Fatou's Classification Theorem (1920-26)

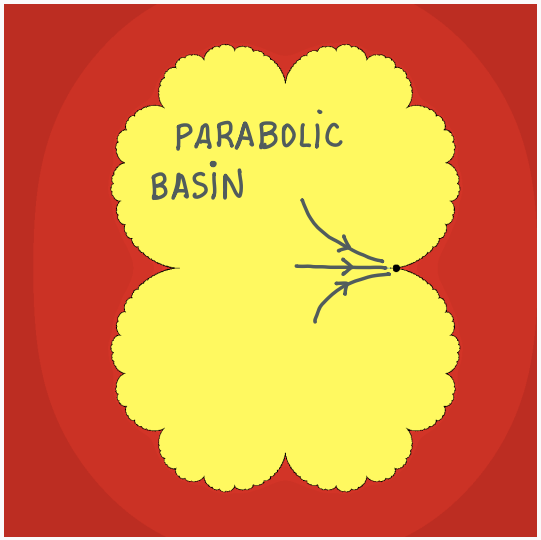
Suppose $f : \mathbb{C} \to \mathbb{C}$ is **entire**.

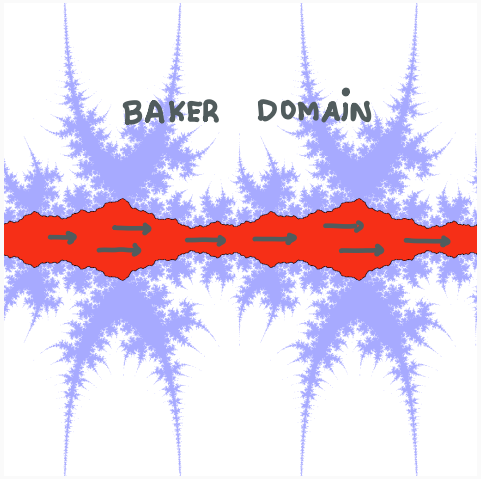
U periodic Fatou components, can be:

- **Basin of attraction** ($p \in U$)
- **Parabolic basin** ($p \in \partial U \cap \mathbb{C}$)
- **Baker domain** ($p = \infty$ ess. sing.)
- **Siegel disk** (isometric case - rotation)



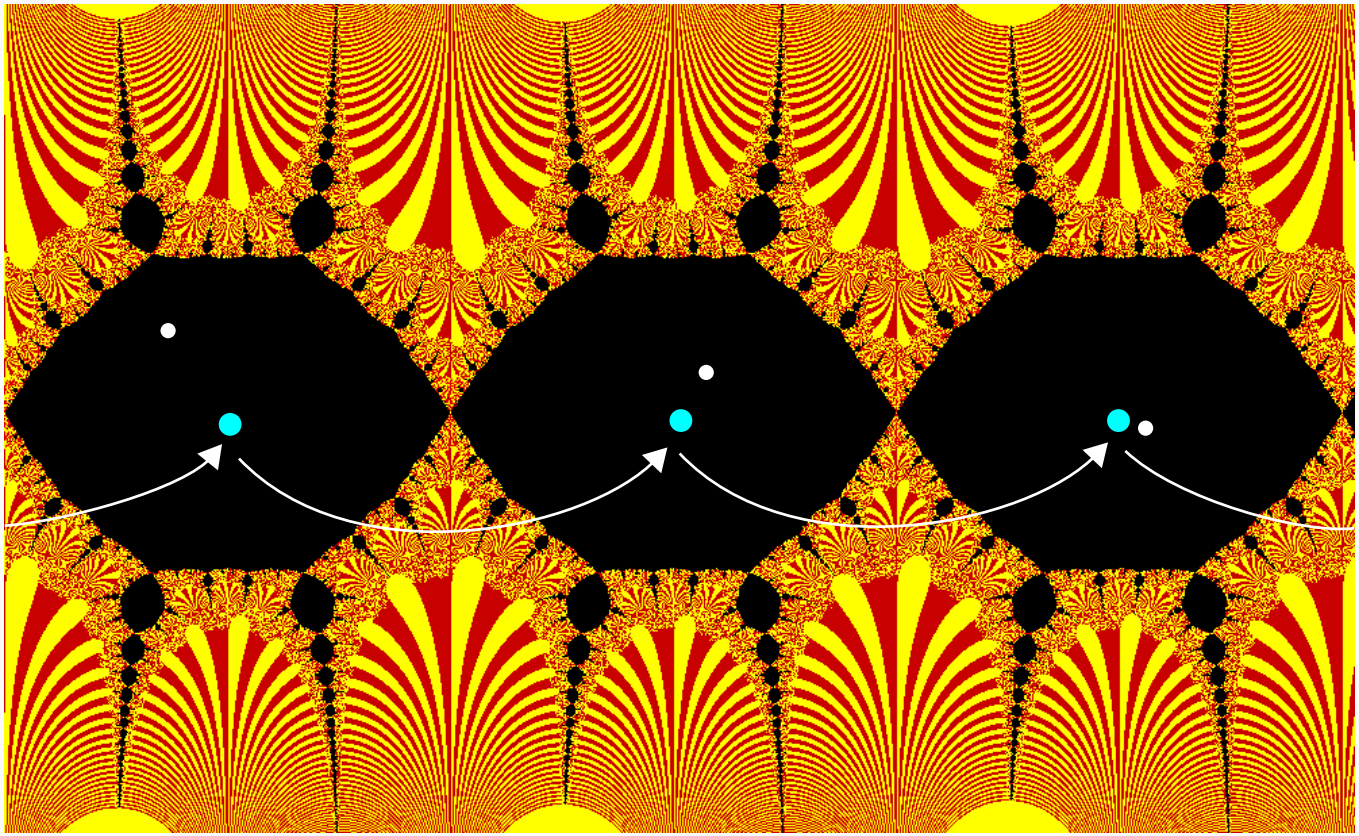






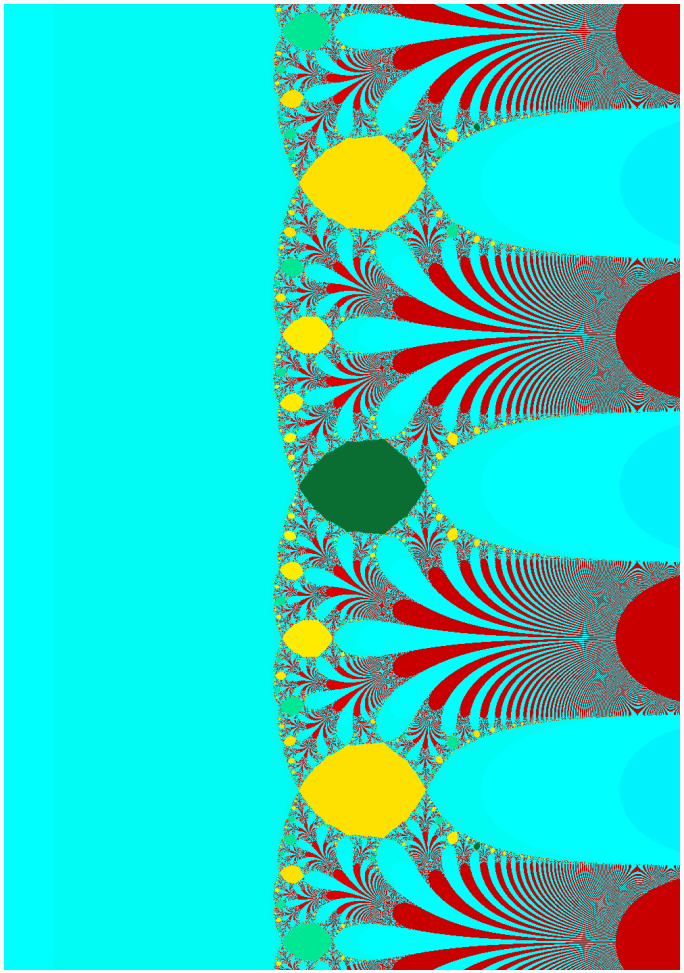
WANDERING DOMAINS

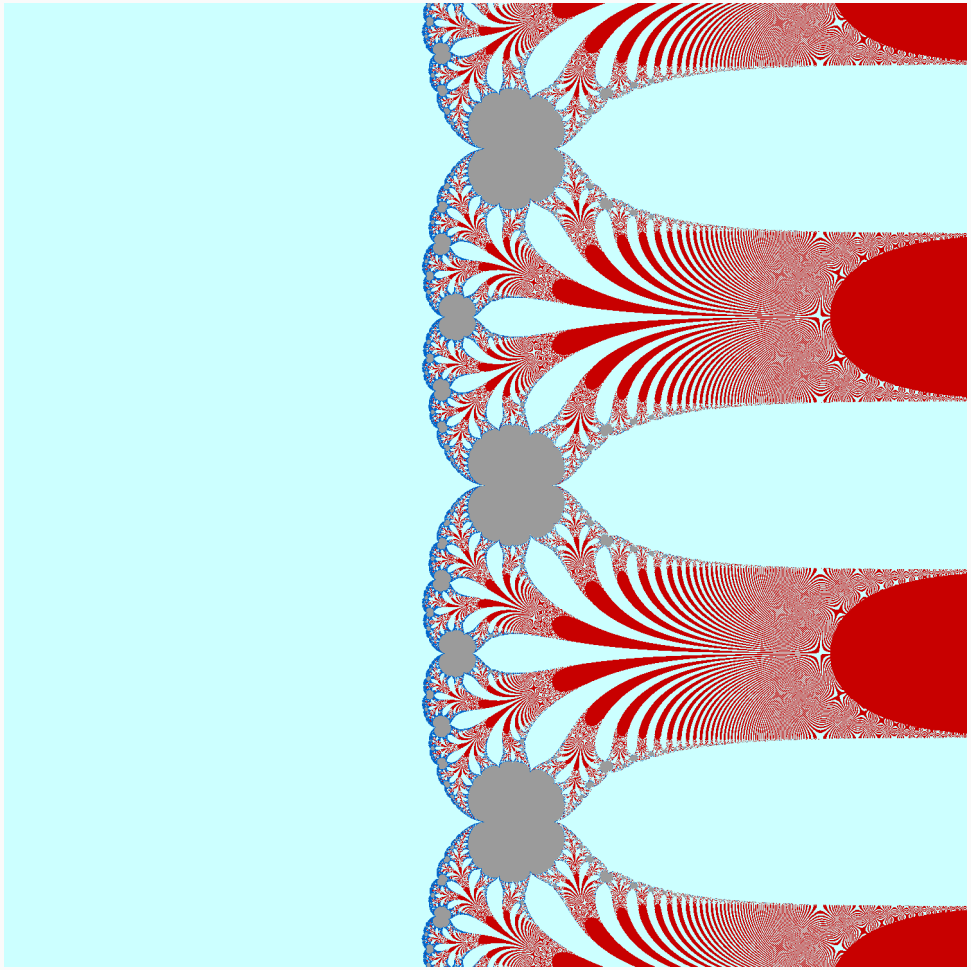
- U is a **wandering domain** if $f^n(U) \cap f^m(U) = \emptyset$ for all $n \neq m$.



$$z + 2\pi + \sin(z)$$

WANDERING DOMAINS





INTERNAL DYNAMICS

TWO WAY CLASSIFICATION: [BEFRS'21]

- **Hyperbolic clustering**:

$$\delta_n(z, w) = \text{dist}_{U_n}(f^n(z), f^n(w)) \longrightarrow ??$$

- **Convergence to the boundary**:

$$d_n(z) = \text{dist}(f^n(z), \partial U_n) \longrightarrow ??$$

- Conditions independent of $z \in U$!!!

	AWAY	BUNGEE	CONVERGING
Contracting	✓	✓	✓
Semi-contracting	✓	✓	✓
Ev. isometric	✓	✓	✓

THANK YOU FOR YOUR ATTENTION!